More on micro semi – pre-operators in micro topological spaces

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Received 5 July 2024; accepted 16 August 2024; published online 26 October 2024
DOI https://doi.org/10.21595/mme.2024.24326

Abstract. The basic objective of this paper is to introduce and investigate the properties of micro semi pre border, micro semi pre kernel and micro semi pre derived set and obtain relation between some of the existing sets.

Keywords: micro semi pre border, micro semi pre kernel and micro semi pre derived set.

1. Introduction

Levine's introduction of generalized closed sets in 1970 [1], providing a foundational framework for subsequent developments. Lellis Thivagar [2], further expanded this framework with the introduction of nano topology, utilizing approximations and boundary regions of a subset of an universe using an equivalence relation on it to define nano closed sets, nano-interior, and nano-closure. The exploration of weak forms of nano open sets, such as nano-α-open sets, nano semi-open sets, nano pre-open sets, and nano b-open sets, was undertaken by Parimala et al. [3], adding layers of complexity to the existing theories.

In 2019, S. Chandrasekar [4], presented the concept of micro topology, which extends nano topology, emphasizing micro pre-open and micro semi-open sets. Later, Chandrasekar and Swathi [5], introduced micro α-open sets and in 2020, Hariwan Z. Ibrahim [6] introduced micro β-open sets in micro topological spaces. In this paper we introduce and study some of the properties of micro semi pre border, micro semi pre kernel and micro semi pre derived set of a set using the concept of micro semi preopen sets.

2. Preliminaries

The following outlines essential concepts and prerequisites required for the progression of this work.

Definition 2.1. [4] The micro closure of a set A is denoted by $Mic\text{-}cl(A)$ and is defined as $Mic\text{-}cl(A) = \cap \{B: B$ is micro closed and $A \subseteq B\}$. The micro interior of a set A is denoted by $Mic\text{-}int(A)$ and is defined as $Mic\text{-}int(A) = \cup \{B: B$ is micro open and $A \supseteq B\}$.

Definition 2.2. [6] A subset A of micro topological space U is called micro semi pre open set if $A \subseteq Mic\text{-}cl(Mic\text{-}int(Mic\text{-}cl(A)))$. The complement of micro semi pre-open set is called micro semi pre-closed. The family of micro semi pre sets is denoted by $Mic\text{-}\beta(A)$.

Definition 2.3. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and A be a subset of U. Then the micro kernel of A denoted by $Mker(A)$ is defined to be the set $Mker(A) = \cap \{L \in \mu_R(X): A \subseteq L\}$.

3. Micro semi pre border

In this section, we study some properties of micro semi pre-border of a set.

Definition 3.1. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and A be a subset of U. Then micro semi-pre border of A is defined as $Mic\text{-}\beta br(A) = A - Mic\text{-}\beta int(A)$.

Example 3.2. Let $U = \{\gamma, \beta, \eta, \zeta\}$, $U/R = \{\{\gamma\}, \{\eta\}, \{\beta, \zeta\}\}$, $X = \{\eta\}$,

Consider the micro Topology $\mu_R(X) = \{\phi, U, \{\eta\}, \{\gamma, \zeta\}, \{\gamma, \beta, \zeta\}\}$ micro β-open set: $M\beta O(U, X) =$
$\{\phi, U, \{\gamma\}, \{\eta\}, \{\zeta\}, \{\gamma, \beta\}, \{\gamma, \eta\}, \{\gamma, \zeta\}, \{\beta, \eta\}, \{\beta, \zeta\}, \{\eta, \zeta\}, \{\gamma, \beta, \eta\}, \{\gamma, \beta, \zeta\}, \{\gamma, \beta, \zeta\}, \{\beta, \eta, \zeta\}\}$.

For a subset $A = \{\gamma, \beta\}$ then $Mic\text{-}\beta int(A) = \{\gamma, \beta\}$ and $Mic\text{-}\beta br(A) = \emptyset$.

Theorem 3.3. For a subset of a micro topological space $(U, \tau_R(X), \mu_R(X))$, the following statements are holds.

1. $Mic\text{-}\beta br(A) \subseteq Mic\text{-}br(A)$.
2. $A = Mic\text{-}\beta int(A) \cup Mic\text{-}\beta br(A)$.
3. $Mic\text{-}\beta int(A) \cap Mic\text{-}\beta br(A) = \emptyset$.
4. If A is micro β-open set the $Mic\text{-}\beta br(A) = \emptyset$.
5. $Mic\text{-}\beta int(Mic\text{-}\beta br(A)) = \emptyset$.
6. $Mic\text{-}\beta br(Mic\text{-}\beta int(A)) = \emptyset$.
7. $Mic\text{-}\beta br(Mic\text{-}\beta br(A)) = Mic\text{-}\beta br(A)$.
8. $Mic\text{-}\beta br(A) = A \cap Mic\text{-}\beta cl(U - A)$.

Proof. (1) By Lemma 3.3 (i) [8], we have $Mic\text{-}int(A) \subseteq Mic\text{-}\beta int(A)$ which implies $A - Mic\text{-}\beta int(A) \supseteq Mic\text{-}int(A)$. (i.e.,) $Mic\text{-}br(A) \supseteq Mic\text{-}\beta br(A)$.

(2) and (3) are immediate consequences of the definition of micro semi-pre border of A.

(4) If A is micro β-open set, then we have $A = Mic\text{-}\beta int(A)$ which implies $Mic\text{-}\beta br(A) = \emptyset$

(5) If $x \in Mic\text{-}\beta int(Mic\text{-}\beta br(A))$, then $x \in Mic\text{-}\beta br(A)$. Now, $Mic\text{-}\beta br(A) \subseteq A$ implies $Mic\text{-}\beta int(Mic\text{-}\beta br(A)) \subseteq Mic\text{-}\beta int(A)$. Hence $x \in Mic\text{-}\beta int(A)$ which is a contradiction to $x \in Mic\text{-}\beta br(A)$. Thus $Mic\text{-}\beta int(Mic\text{-}\beta br(A)) = \emptyset$.

(6) Since $Mic\text{-}\beta int(A)$ is micro β-open, it follows from (4) that $Mic\text{-}\beta br(Mic\text{-}\beta int(A))= \emptyset$.

(7)$Mic\text{-}\beta br(Mic\text{-}\beta br(A)) = Mic\text{-}\beta br(A - Mic\text{-}\beta int(A)) = (A - Mic\text{-}\beta int(A)) - Mic\text{-}\beta int(A - Mic\text{-}\beta int(A))$ which is $Mic\text{-}\beta br(A) - \emptyset$, by (4). Hence, $Mic\text{-}\beta br(Mic\text{-}\beta br(A)) = Mic\text{-}\beta br(A)$.

(8) By Lemma 3.3 (v) [8], we have $Mic\text{-}\beta int(A) = U - Mic\text{-}\beta cl(U - A)$ which implies that $A - Mic\text{-}\beta int(A) = A - (U - Mic\text{-}\beta cl(U - A)) = A \cap Mic\text{-}\beta cl(U - A)$.

Theorem 3.4. For a subset of U, the following condition hold:

1. $Mic\text{-}\beta br(A) \subseteq Mic\text{-}\beta Fr(A)$.
2. $Mic\text{-}\beta Ext(A) \cap Mic\text{-}\beta br(A) = \emptyset$.

Proof. (1) Since $Mic - \beta cl(A)$ contains A. (i.e.,) $A \subseteq Mic\text{-}\beta cl(A)$.

Let $Mic\text{-}\beta br(A) = A - Mic\text{-}\beta int(A) \subseteq Mic\text{-}\beta cl(A) - Mic\text{-}\beta int(A) = Mic\text{-}\beta Fr(A)$ Hence $Mic\text{-}\beta br(A) \subseteq Mic\text{-}\beta Fr(A)$.

(2) Let $x \in Mic\text{-}\beta Ext(A)$ i.e., $x \in Mic\text{-}\beta int(U - A)$ where $x \in Mic\text{-}\beta int(A)$. If A is micro β-open then $A = Mic\text{-}\beta int(A)$. If A is not micro β-open then $Mic\text{-}\beta int(A) \subset A$. Therefore, in general $Mic\text{-}\beta int(A) \subseteq A$. Therefore $x \notin A - Mic\text{-}\beta int(A)$ implies $x \notin Mic\text{-}\beta br(A)$. Hence $Mic\text{-}\beta Ext(A) \cap Mic\text{-}\beta br(A) = \emptyset$.

4. Micro semi pre kernel

In this section, we introduce and study the properties of micro semi pre-kernel of a set and obtain some of its basic results.

Definition 4.1. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and A be a subset of U. Then the micro semi-pre kernel of A is defined as the intersection of all micro semi-pre open sets containing A and it is denoted by $Mic\text{-}\beta ker(A)$ is defined to be the set $Mic\text{-}\beta ker(A) = \cap \{L \in$

$M\beta O(X): A \subseteq L\}$.

Example 4.2. Let $U = \{\gamma, \beta, \eta, \zeta\}$, $U/R = \{\{\eta\}, \{\gamma, \beta, \zeta\}\}$, $X = \{\gamma\}$, $\tau_R(X) = \{\phi, U, \{\gamma, \eta\}\}$.
Consider the micro Topology $\mu_R(X) = \{\phi, U, \{\gamma\}, \{\gamma, \eta\}, \{\gamma, \beta, \zeta\}\}$.
Micro β-open set: $M\beta O(U, X) = \{\phi, U, \{\gamma\}, \{\gamma, \beta\}, \{\gamma, \eta\}, \{\gamma, \zeta\}, \{\gamma, \beta, \eta\}, \{\gamma, \beta, \zeta\}, \{\gamma, \eta, \zeta\}\}$.
For a subset $A = \{\gamma, \zeta\}$ then $Mker(A) = \{\gamma, \beta, \zeta\}$ and $Mic\text{-}\beta ker(A) = \{\gamma, \zeta\}$.

Lemma 4.3. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. For subsets A, B and A_j ($j \in I$, where I is an index set) of a micro topological space $(U, \mu_R(X))$, the following holds.

1. $A \subseteq Mic\text{-}\beta ker(A)$.
2. If $A \subset B$, then $Mic\text{-}\beta ker(A) \subset Mic\text{-}\beta ker(B)$.
3. $Mic\text{-}\beta ker(Mic\text{-}\beta ker(A)) = Mic\text{-}\beta ker(A)$.
4. If A is $Mic\text{-}\beta$-open then $A = Mic\text{-}\beta ker(A)$.
5. $Mic\text{-}\beta ker(\cup A_j / j \in I) \subseteq \cup\{Mic\text{-}\beta ker(A_j)/j \in I\}$.
6. $Mic\text{-}\beta ker(\cap A_j / j \in I) \subseteq \cap\{Mic\text{-}\beta ker(A_j)/j \in I\}$.

Proof. (1) It follows by the definition of $Mic\text{-}\beta ker(A)$.

(2) Suppose $x \notin Mic\text{-}\beta ker(B)$, then there exists a subset $K \in$ micro β-open set such that $K \subset S$ with $x \notin K$. Since $A \subset B$, $x \notin Mic\text{-}\beta ker(A)$. Thus $Mic\text{-}\beta ker(A) \subset Mic\text{-}\beta ker(B)$.

(3) Follows from (1) and definition of $Mic\text{-}\beta ker(A)$.

(4) It follows by the definition of $Mic\text{-}\beta ker(A)$.

(5) For each $i \in I$, $Mic\text{-}\beta ker(A_j) \subseteq Mic\text{-}\beta ker(\cup_{j\in I} A_j)$. Therefore, we have $\cup_{j\in I} \{Mic\text{-}\beta ker(A_j)\} \subseteq Mic\text{-}\beta ker(\cup_{j\in I} A_j)$.

(6) Suppose that $x \notin \cap\{Mic\text{-}\beta ker(A_j/j \in I)\}$ then there exists an $j_0 \in I$, such that $x \notin Mic\text{-}\beta ker(A_{j_0})$ and there exists a micro semi pre-open set K such that $x \notin K$ and $A_{j_0} \subset K$. We have $\cap_{j\in I} A_j \subseteq A_{j_0} \subseteq K$ and $x \notin K$. Therefore $x \notin Mic\text{-}\beta ker\{\cap A_j/j \in I\}$. Hence $Mic\text{-}\beta ker(\cap A_j/ j \in I) \subseteq \cap Mic\text{-}\beta ker(A_j)/j \in I$.

Theorem 4.4. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Let A and B be subsets of U, then the following conditions holds.

1. $Mic\text{-}\beta ker(A) \subseteq Mker(A)$.
2. $Mic\text{-}\beta ker(A) \cap Mic\text{-}\beta ker(B) \subset Mic\text{-}\beta ker(A \cup B)$.
3. $Mic\text{-}\beta ker(A \cap B) \subset Mic\text{-}\beta ker(A) \cup Mic\text{-}\beta ker(B)$.
4. $Mic\text{-}\beta cl(A) \cap Mic\text{-}\beta ker(A) = A$.
5. $Mic\text{-}\beta ker(A) \cap Mic\text{-}\beta Fr(A) = Mic\text{-}\beta br(A)$.

Proof. (1) Let $x \in Mic\text{-}\beta ker(A) \Rightarrow x \in \cap\{L/A \subset L, L \in Mic\text{-}\beta\text{-open set}\} \Rightarrow x \in \cap\{L/A \subset L, L \in \mu_R(x)\}$.

Since every micro-open set is micro semi pre-open set, so $x \in Mic\text{-}ker(A)$.

(2) Since $A \subset A \cup B$ and $B \subset A \cup B$. By Lemma 4.3 (2), we have $Mic\text{-}\beta ker(A) \subset Mic\text{-}\beta ker(A \cup B)$ and $Mic\text{-}\beta ker(B) \subset Mic\text{-}\beta ker(A \cup B)$.

Therefore $Mic\text{-}\beta ker(A) \cap Mic\text{-}\beta ker(B) \subset Mic\text{-}\beta ker(A \cup B)$.

(3) Since $A \cap B \subset A$ and $A \cap B \subset B$. By lemma 4.3 (2), we have $Mic\text{-}\beta ker(A \cap B) \subset Mic\text{-}\beta ker(A)$ and $Mic\text{-}\beta ker(A \cap B) \subset Mic\text{-}\beta ker(B)$.

Therefore $Mic\text{-}\beta ker(A \cap B) \subset Mic\text{-}\beta ker(A) \cup Mic\text{-}\beta ker(B)$.

(4) Let $x \in Mic\text{-}\beta cl(A) \cap Mic\text{-}\beta ker(A) \Rightarrow x \in Mic\text{-}\beta cl(A) \cap Mic\text{-}\beta ker(A) \Rightarrow x \in A \subseteq Mic\text{-}\beta cl(A)$ and $x \in A \subseteq Mic\text{-}\beta ker(A) \Rightarrow x \in A$.

Hence $A = Mic\text{-}\beta cl(A) \cap Mic\text{-}\beta ker(A)$.

(5) Let $x \in Mic\text{-}\beta ker(A) \cap Mic\text{-}Fr(A)$. To prove, $x \in Mic\text{-}\beta br(A)$. (i.e,) $x \in A - Mic\text{-}\beta int(A)$.

Since by the definition of kernel and Border of a set, we have, $x \in \cap\{K/A \subset K, K \in Mic - PO\} \cap Mic\text{-}\beta cl(A) - Mic\text{-}\beta int(A) \Rightarrow x \in A \cap (A - Mic\text{-}\beta int(A)) \Rightarrow x \in A$ and $x \in A - Mic\text{-}Pint(A) \Rightarrow x \in A$ and $x \in Mic\text{-}Pbr(A)$.

Therefore, $x \in Mic\text{-}Pbr(A)$.

Theorem 4.5. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then for any points x and

y in U the following statements are:

1. $Mic\text{-}\beta ker(\{x\}) \neq Mic\text{-}\beta ker(\{y\})$.
2. $Mic\text{-}\beta cl(\{x\}) \neq Mic\text{-}\beta cl(\{y\})$.

Proof. (1) ⇒ (2): Suppose that $Mic\text{-}\beta ker(\{x\}) \neq Mic\text{-}\beta ker(\{y\})$ then there exist a point z in U such that $z \in Mic\text{-}\beta ker(\{x\})$ and $z \notin Mic\text{-}\beta ker(\{y\})$.

From $z \in Mic\text{-}\beta ker(\{x\}$ it follows that $\{x\} \cap Mic\text{-}\beta cl(\{z\}) = \emptyset$ which implies $x \in Mic\text{-}\beta cl(\{z\})$.

By $z \notin Mic\text{-}\beta ker(\{y\})$, we have $\{y\} \cap Mic\text{-}\beta cl(\{z\}) = \emptyset$.

Since $x \in Mic\text{-}\beta cl(\{z\})$ then $Mic\text{-}\beta cl(\{x\}) \subseteq Mic\text{-}\beta cl(\{z\})$ and $\{y\} \cap Mic\text{-}\beta cl(\{z\}) = \emptyset$.

Therefore $Mic\text{-}\beta cl(\{x\}) \neq Mic\text{-}\beta cl(\{y\})$.

(2) ⇒ (1): suppose $Mic\text{-}\beta cl(\{x\}) \neq Mic\text{-}\beta cl(\{y\})$. Then there exist a point z in U such that $z \in Mic\text{-}\beta cl(\{x\})$ and $z \notin Mic\text{-}\beta cl(\{y\})$. Then there exist micro β-open set containing z and therefore x but not y. Hence $y \notin Mic\text{-}\beta ker(\{x\})$. Thus $Mic\text{-}\beta ker(\{x\}) \neq Mic\text{-}\beta ker(\{y\})$.

5. Micro semi pre derived set

In this section, we introduce and study the properties of micro semi pre-derived of a set and obtain some of its basic results.

Definition 5.1 Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and A be a subset of U. A point $x \in U$ is said to be micro semi-pre limit point of A, if for each $N \in M\beta O(U,X)$, $N \cap \{A - \{x\}\} \neq \emptyset$. The set of all $Mic\text{-}\beta$-limit points of A is said to be the micro semi-pre derived set and is denoted by $Mic\text{-}\beta D(A)$.

Example 5.2. Let $U = \{\gamma, \beta, \eta, \zeta\}$, $U/R = \{\{\gamma\}, \{\beta\}, \{\eta, \zeta\}\}$, $X = \{\gamma, \beta\}$, $\tau_R(X) = \{\phi, U, \gamma, \beta\}$.

Consider the micro Topology $\mu_R(X) = \{\phi, U, \{\beta\}, \{\gamma, \beta\}, \{\beta, \eta\}, \{\gamma, \beta, \eta\}\}$.

Micro β-open set: $M\beta O(U,X) = \{\phi, U, \{\beta\}, \{\gamma, \beta\}, \{\beta, \eta\}, \{\gamma, \beta, \eta\}, \{\gamma, \beta, \zeta\}, \{\beta, \eta, \zeta\}\}$.

For a subset $A = \{\gamma, \zeta\}$ then $Mic\text{-}D(A) = \{\zeta\}$ and $Mic\text{-}\beta D(A) = \phi$. Here $Mic\text{-}\beta cl(A) = \{\gamma, \zeta\}$.

Lemma 5.3. For a subset A of U, $Mic\text{-}\beta clA = A \cup Mic\text{-}\beta D(A)$.

Theorem 5.4. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Let A and B be subsets of a space U and $A \subseteq B$. Then $Mic\text{-}\beta D(A) \subset Mic\text{-}\beta D(B)$.

Proof. Let $x \in U$ be the simplest point of A. By definition, for any $M \in \beta O(U,X)$ such that $N \cap \{A - \{x\}\} \neq \emptyset$. (i.e.,) N contains points of A other than x. But $A \subset B$, $\{A - \{x\}\} \subset \{B - \{x\}\}$ ⇒ $N \cap \{B - \{x\}\} \neq \emptyset$ ⇒ x is also micro β-limit point of B ⇒ $Mic\text{-}\beta D(A) \subseteq Mic\text{-}\beta D(B)$.

Theorem 5.5. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Let A, $C \subseteq U$. Then the micro β-derived sets $Mic\text{-}\beta D(A)$ and $Mic\text{-}\beta D(C)$ have the following properties.

1. $Mic\text{-}\beta D(\emptyset) = \emptyset$.
2. $x \in Mic\text{-}\beta D(A) x \in Mic\text{-}\beta D(A - \{x\})$.
3. $Mic\text{-}\beta D(A) \cup Mic\text{-}\beta D(C) \subseteq Mic\text{-}\beta D(A \cup C)$.
4. $Mic\text{-}\beta D(A \cap C) \subseteq Mic\text{-}\beta D(A) \cap Mic\text{-}\beta D(C)$.

Proof. (i) Obvious.

(ii) Let $x \in Mic\text{-}\beta D(A)$ implies x is a micro β-limit point of A. Every neighbourhood $Mic\text{-}\beta - N(x)$ contains at least one point of A other than x ⇒ Every $Mic\beta - N(x)$ containing x contains atleast one point other than x of $A - \{x\}$ ⇒ x is a micro β-limit point of $A - \{x\}$.

$Mic\text{-}\beta D(A - \{x\})$ Therefore $x \in Mic\text{-}\beta D(A - \{x\})$.

(iii) Since $A \subset A \cup C$ and $C \subset A \cup C$. By theorem (5.4),

$Mic\text{-}\beta D(A) \subset Mic\text{-}\beta D(A \cup C)$,

$Mic\text{-}\beta D(C) \subset Mic\text{-}\beta D(A \cup C)$.

Therefore $Mic\text{-}\beta D(A) \cup Mic\text{-}\beta D(C) \subset Mic\text{-}\beta D(A \cup C)$

(iv) Since $A \cap C \subset A$ and $A \cap C \subset C$. By theorem (5.4),

$Mic\text{-}\beta D(A \cap C) \subset Mic\text{-}\beta D(A)$,

$Mic\text{-}\beta D(A \cap C) \subset Mic\text{-}\beta D(C)$.

Therefore $Mic\text{-}\beta D(A \cap C) \subset Mic\text{-}\beta D(A) \cap Mic\text{-}\beta D(C)$.

Lemma 5.6. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Let A be a subset of U. Then $Mic\text{-}\beta cl(A) \supseteq A - Mic\text{-}\beta D(U - A)$.

Proof. Let $x \in A - Mic\text{-}\beta D(U - A) \Rightarrow x \in A$ but $x \notin Mic\text{-}\beta D(U - A)$. So there exist $L \in M\beta O(X)$ with $L \cap (U - A) = \emptyset \Rightarrow x \in L \subset A \Rightarrow x \in Mic\text{-}\beta cl(A)$.

Theorem 5.7. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Let $A \subseteq U$. Then $A \cup Mic\text{-}\beta D(A)$ is Mic-β closed set.

Proof. Let $A \cup Mic\text{-}\beta D(A)$ will be Mic-β closed.

If $U - (Mic\text{-}\beta D(A)) = (U - A) \cap (U - Mic\text{-}\beta D(A))$. To show R.H.S is open.

Let $x \in (U - A) \cap (U - Mic\text{-}\beta D(A)) \Rightarrow x \in U - A$ and $x \in U - Mic\text{-}\beta D(A) \Rightarrow x \notin A$ and $x \notin Mic\text{-}\beta D(A)$.

Since $x \notin Mic\text{-}\beta D(A)$, there exist micro β-open neighbourhood N_x of x which contains no points of A but $x \notin A$. So $N_x \subset U - A$. No point of N_x can be micro β-limit point of A. So no point of N_x can belong to $Mic\text{-}\beta D(A) \Rightarrow N_x \subset U - Mic\text{-}\beta D(A) \Rightarrow x \in N_x \subset U - (A \cup Mic\text{-}\beta D(A))$.

Therefore $U - (A \cup Mic\text{-}\beta D(A))$ is micro β-open set.

Hence $Mic\text{-}\beta D(A))$ is micro β-closed set.

6. Discussion

The introduction of micro semi pre border, micro semi pre kernel, and micro semi pre derived set within micro topological spaces addresses several gaps in the current understanding and applications of micro topology. These developments provide a refined framework for analyzing and manipulating sets in micro topological spaces, which was previously limited by the existing definitions and properties. The ability to distinguish and work with these nuanced sets can lead to more accurate models and solutions in the following fields of engineering.

1. Signal Processing: The advanced set definitions can be applied in signal processing to improve the accuracy of signal analysis and filtering techniques.
2. Data Compression: The refined understanding of set properties aids in developing more efficient data compression algorithms, particularly in the context of high-dimensional data.
3. Network Design: In network design, these concepts can be utilized to optimize network topology and enhance the robustness and efficiency of communication protocols.
4. Robotics and Automation: The precision offered by these new developments can improve the design and control of robotic systems, particularly in navigating and interacting with complex environments.

7. Novelty

The relation between the characterization of micro semi pre-open sets, micro pre-closed sets, micro semi-pre border, micro semi -pre kernel, and micro semi-pre derived sets were shown.

8. Conclusions

In this paper, we have established and explored the notion of micro semi pre-border, micro semi pre-kernel, and micro semi pre-derived sets within micro topological spaces. We outlined the properties of these operators, highlighting the relationship between These findings not only enhance the understanding of micro topological structures but also provide a foundation for further research and applications in various mathematics and scientific fields. By introducing these operators and their properties, we have opened pathways for more comprehensive studies and practical implementations of micro topological spaces.

Acknowledgements

The authors have not disclosed any funding.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Author contributions

Sathishmohan P: conceptualization, supervision, project administration and review; Poongothai G: Investigation, methodology, validation, visualization and original draft preparation; Rajalakshmi K: supervision, resources and review; Stanley Roshan S: resources and review.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] N. Levine, "Generalized closed sets in topology," *Rendiconti del Circolo Matematico di Palermo*, Vol. 19, No. 1, pp. 89–96, Jan. 1970, https://doi.org/10.1007/bf02843888

[2] L. M. Thivagar and C. Richard, "On nano forms of weakly open sets," *International Journal of Mathematics and Statistics Invention*, Vol. 1, No. 1, pp. 31–37, 2013.

[3] M. Parimala, C. Indirani, and S. Jafari, "On Nano b-open sets in Nano Topological Spaces," *Jordan Journal of Mathematics and Statistics*, Vol. 9, No. 3, pp. 173–184, 2016.

[4] S. Chandrasekar, "On micro topological spaces," *Journal of New Theory*, Vol. 26, pp. 23–31, 2019.

[5] S. Chandrasekar and S. Swathi, "Micro α-open sets in micro topological spaces," *International Journal of Research in Advent Technology*, Vol. 6, No. 10, pp. 2633–2637, 2018.

[6] H. Z. Ibrahim, "Micro β-open sets in micro topology," *General Letters in Mathematics*, Vol. 8, No. 1, pp. 8–15, Mar. 2020, https://doi.org/10.31559/glm.8.1.2

[7] P. Sathishmohan, V. Rajendran, C. Vignesh Kumar, and P. K. Dhanasekaran, "On nano semi pre neighbourhoods in nano topological spaces," *Malaya Journal of Matematik*, Vol. 6, No. 1, pp. 294–298, Jan. 2018, https://doi.org/10.26637/mjm0601/0034

[8] P. Sathishmohan, G. Poongothai, K. Rajalakshmi, and M. Thiruchelvi, "On micro semi-pre operators in micro topological spaces," *Indian Journal of Natural Sciences*, Vol. 15, No. 84, pp. 74900–74907, 2024.

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