Secure metric dimension of new classes of graphs

Iqbal M. Batiha[1], Basma Mohamed[2], Iqbal H. Jebril[3]
[1, 3]Department of Mathematics, Al Zaytoonah University of Jordan, Amman, 11733, Jordan
[1]Nonlinear Dynamics Research Center (NDRC), Ajman University, Ajman 346, United Arab Emirates
[2]Giza Higher Institute for Managerial Sciences, Tomah, Egypt
[1]Corresponding author
E-mail: [1]*i.batiha@zuj.ed.jo*, [2]*bosbos25jan@yahoo.com*, [3]*i.jebril@zuj.ed.jo*

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Abstract. The metric representation of a vertex v of a graph G is a finite vector representing distances of v with respect to vertices of some ordered subset $S \subseteq V\ (G)$. If no suitable subset of S provides separate representations for each vertex of $V(G)$, then the set S is referred to as a minimal resolving set. The metric dimension of G is the cardinality of the smallest (with respect to its cardinality) minimal resolving set. A resolving set S is secure if for any $v \in V - S$, there exists $x \in S$ such that $(S - \{x\}) \cup \{v\}$ is a resolving set. For various classes of graphs, the value of the secure resolving number is determined and defined. The secure metric dimension of the graph classes is being studied in this work. The results show that different graph families have different metric dimensions.

Keywords: secure metric dimension, classes of graphs.

1. Introduction

Let $G = (V, E)$ be a connected, simple, finite graph. On which the ordering $(x_1, x_2, \dots, x_k)$ is imposed, let $\bar{S} = \{x_1, x_2, \dots, x_k\}$. The metric description of b with regard to $\bar{S}$ is defined as the ordered k-tuples $r(b\,|\bar{S}) = (d(x_1, b), d(x_2, b), \dots, d(x_k, b))$ for each $b \in V(G)$. If $r(x|\,\bar{S}) = r\ (b|\bar{S})$ implies $x = b$ for every $x, b \in V(G)$, then the set $\bar{S}$ is referred to as a resolving set of G. A minimal resolving set, also known as a basis, is a resolving set of G with minimum cardinality. The dimension of G, represented by $dim(G)$, is the cardinality of a minimum resolving set [1].

There is previous study in the literature on the location of sets in a connected graph [2, 3]. Nearly forty years ago, Slater introduced the idea of finding sets (resolving sets) and a reference set (metric dimension). Afterwards, the aforementioned theory [4] was independently discovered by Harary and Melter. They started referring to location numbers as metric dimensions. On resolving sets, resolving dominating sets, independent resolving sets, etc., many papers have been composed. In a graph, the concept of security is linked to many kinds of sets. A dominating set D of G is a secure set, for instance, if there is an $x \in D$ such that $(D - \{u\}) \cup \{v\}$ is a dominating set for any $v \in V - D$ [5, 6]. Farooq et al. [7] studied the metric dimension of the line graph of the Bakelite network and subdivided the Bakelite network. According to [8], a path graph is the only graph with a metric dimension of 1. For $n \geq 3$, the metric dimension of the cycle graph is 2. This idea is especially helpful for applications including chemistry and space routing. For instance, in space routing, the objective is to assign the smallest number of robots feasible to certain vertices so that they can visit each vertex exactly once. The problem can be resolved by applying the idea of metric dimension. The minimal landmarks needed for the hexagonal network $HX(n)$ and the honeycomb network $HC(n)$ are three and six, respectively, according to research by Abbas et al. [9]. The local metric dimension of a number of specific line graphs was examined by Yang et al. [10]. Jothi et al. [11] investigated the relation between the metric dimension of a bipartite graph and its projections. Additionally, they provided some realization results for the bounds on the metric dimension of a bipartite graph. The dominant metric dimension of the generalized Petersen graph was examined by Susilowati et al. [12]. Mazidah et al. [13] examined the resolving independent domination number of path graph, cycle graph, friendship graph, helm graph and fan graph. The maximum number of vertices in a bipartite graph with a particular diameter and metric dimension was computed by Dankelmann et al. [14]. Additional details can be discovered in the literature [15-19].

Our main aim in this paper is to compute the secure resolving set of some graphs, including the join of (m, n) Kite graph, the 1-join of square of path P_n^2 graph, coconut tree $CT(m, n)$ and extended jewel graph.

Definition 1.1 The basis of the graph is the resolving set with the least vertices [7].

Definition 1.2 [20]: A (m, n) Kite graph is made up of a cycle of length m with n edges and a path connected to one of the cycle's vertices.

Definition 1.3 [21]: A Coconut tree $CT(m, n)$: for any positive integers n and $m \geq 2$ is derived from the path P_m by attaching n additional pendant edges at an end vertex of P_m.

Illustration 1.3. Consider Δ_5, for which $R = \{u_1, u_4\}$. Then, R is resolving, and for any $u \in V - R$, there exists $v \in R$ such that $(R - \{v\}) \cup \{u\})$ is a resolving set of Δ_5. It can be easily seen that $sdim\ (\Delta_5) = 2$.

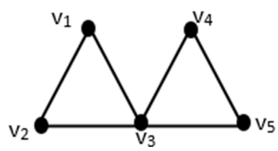

Fig. 1. An example on a Kite graph

2. Secure resolving dimension for several known graphs [5, 6]

1. $sdim(K_n) = n - 1 = dim(K_n)$.
2. $sdim(K_1, n) = n > dim(K_1, n)$.
3. $sdim\left(K_{m,n}\right) = m + n - 2 = dim\left(K_{m,n}\right)$, $(m, n \geq 2)$.
4. $sdim\ (P_n) = 2 > dim\ (P_n) = 1$, $(n \geq 3)$.
5. $sdim(C_n) = 2 = dim(C_n)$.
6. For Trapezoid graph Tr_n, $dim(Tr_n) = 2$ and $sdim(Tr_n) = 2$, for all $n \geq 6$.
7. For $Z - (P_n)$ graph, $dim(Z - (P_n)) = 2$ and $sdim(Z - (P_n)) = 2$.
8. For Tortoise graph To_n, $dim\ (To_n) = 2$ and $sdim(To_n) = 2$.
9. $sdim(P_{2n} \,\overline{\triangledown}\, P_n) = 2 = dim(P_{2n} \,\overline{\triangledown}\, P_n)$.

3. Main results

Here, we demonstrate that the secure metric dimension of including the join of (m, n) Kite graph, the 1-join of square of path P_n^2 graph, coconut tree $CT(m, n)$ and extended jewel graph. We also derive the explicit formulas for the secure metric dimension of shadow graph of path $D_2(P_n)$, Jelly fish $J_{n,n}$, Lilly graph L_n, Twig graph Tg_n, Joint sum of two copies of C_n, quadrilateral graphs Q_n and subdividing all the edges of $P_n \odot K_1$.

Theorem 3.1: Let G is Join of (m, n) Kite graph $K_{m,n}$ with n vertices and $\{v_1, v_{n/2+3}\}$ be a secure metric basis of G, then $sdim(K_{m,n}) = 2$.

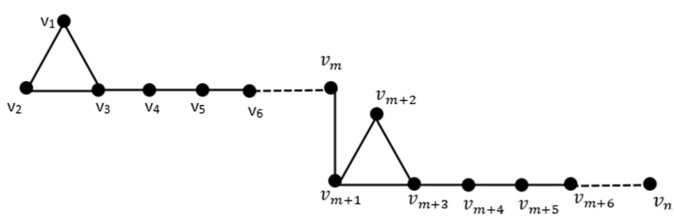

Fig. 2. Join of (m, n) Kite graph $K_{m,n}$

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Proof. The secure resolving set in general form is $\bar{S} = \{v_1, v_{m+3}\} \subset V(G)$. The following are representations of the vertices $v_i \in V(G)$ with respect to $\bar{S}$:

$$r(v_i|\bar{S}) = \begin{cases} \left(0, \frac{n}{2}\right), & i = 1, \\ \left(1, \frac{n}{2}\right), & i = 2, \\ (i-2, m-i+2), & 3 \le i \le m+1, \\ (m, 1), & i = m+2, \\ (m, 0), & i = m+3, \\ (i-3, i-m-3), & m+4 \le i \le n. \end{cases}$$

Since all vertices have unique representations, we obtain $sdim(K_{m,n}) = 2$.

Theorem 3.2: Let G is 1-join of square of path P_n^2 with n vertices and $\{v_1, v_{n/2+3}\}$ be a secure metric basis of G, then $sdim(P_n^2) = 2$.

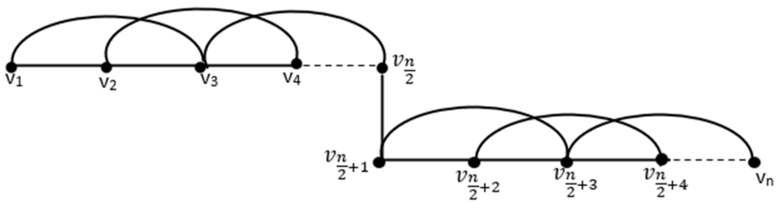

Fig. 3. 1-join of square of path P_n^2

Proof. We label the 1-join of square of path P_n^2 as shown in Fig. 2 such that n is the vertices number. It is clear that $|V(G)|$ is $n = k + 4$. Let $\overline{S} = \{v_2, v_{n-1}\}$.

Begin

$d(v_1, \bar{S}) = \left(1, \frac{n}{2}\right), d(v_2, \bar{S}) = \left(0, \frac{n}{2} - 1\right)$

for $(i = 3;\ i <= \frac{n}{2};\ i = i + 2)$

$d(v_i, \bar{S}) = \left(\frac{i-1}{2}, \frac{n-i+1}{2}\right)$

end

for $(i = 4;\ i <= \frac{n}{2} - 1;\ i = i + 2)$

$d(v_i, \bar{S}) = \left(\frac{i}{2}, \frac{n-i+2}{2}\right)$

end

for $(i = \frac{n}{2} + 1;\ i <= n;\ i = i + 2)$

$d(v_i, \bar{S}) = \left(\frac{i}{2}, \frac{n-i}{2}\right)$

end

for $(i = \frac{n}{2} + 2;\ i <= n - 1;\ i = i + 2)$

$d(v_i, \bar{S}) = \left(\frac{i+1}{2}, \frac{n-i+1}{2}\right)$

end

end

This completes the proof.

Theorem 3.3: If $CT(m, n), n \ge 2, m \ge 4$ is coconut tree, then $sdim(CT(m, n)) = m$.

Proof. The secure resolving set in general form is $\overline{S} = \{v_1, v_2, v_{m-1}, v_n\} \subset V(CT(m, n))$. The representations of vertices $v_i \in V(CT(m, n))$ in regard to $\overline{S}$ are as follow.

We choose a subset $\overline{S} = \{v_1, v_2, v_3, \dots, v_{m-1}, v_n\}$, and we must demonstrate that $sdim\ (CT\ (m, n)) = m$ for $m \ge 4$ and $n \ge 3$. We obtained the representations of vertices in

graph $CT(m,n)$ with respect to $\overline{S}$ are:
$r(v_1|\overline{S}) = (0,2,2,\ldots,2,m+2)$
$r(v_2|\overline{S}) = (2,0,2,\ldots,2,m+2)$
$r(v_3|\overline{S}) = (2,2,0,\ldots,2,m+2)$
$\vdots$
$r(v_m|\overline{S}) = (2,2,2,\ldots,2,m+2)$
$r(v_{m+1}|\overline{S}) = (1,1,1,\ldots,1,m+1)$
$r(v_{m+2}|\overline{S}) = (i-m,i-m,\ldots,i-m,2m-i+2)$
$\vdots$
$r(v_n|\overline{S}) = (i-m,i-m,\ldots,2m-i+2)$

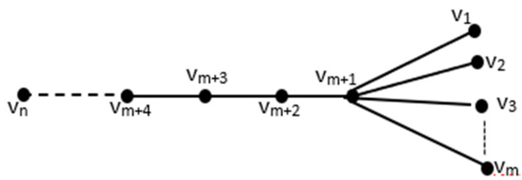

Fig. 4. Coconut tree $CT(m,n)$

The representations of vertices in graph $CT\ (m,n)$ are distinct as seen above. This implies that $\bar{S}$ is secure resolving set, but this does not prove that it is the lower bound. As a result, the upper bound is $sdim(CT(m,n)) \leq m$. Now, we demonstrate that $sdim(CT(m,n)) \geq m$. Let $\overline{S} = \{v_1, v_2, v_3, \ldots, v_{m-1}, v_n\}$ be a secure resolving set with $|\overline{S}| = m$. Assume that $\overline{S_1}$ is another minimal resolving set, or $|\overline{S_1}| < m$.

If we select an ordered set $\overline{S_1} \subseteq \overline{S} - \{v_i, v_j\}$, $1 \leq i,j \leq m$, $i \neq j$, so that there exist two vertices $v_i, v_j \in CT(m,n)$ such that $r(v_i|\overline{S}) = r(v_j|\overline{S}) = (i-m, i-m, \ldots, i-m)$. $\overline{S_1}$ is not a secure resolving set, which is contrary to assumption. As a result, $sdim(CT(m,n)) \geq m$ is the lower bound. From the above proving, we conclude that $sdim(CT(m,n)) = m$.

Theorem 3.4: If $E\ (j_n)$, $n \geq 7$, is extended jewel graph, then $sdim(E(j_n)) = n-4$.

Proof. We choose a subset $\overline{S} = \{v_1, v_2, v_3, \ldots, v_{n-5}, v_n\}$, and we must demonstrate that $sdim(E(j_n)) = n-4$ for $n \geq 7$. We obtained the representations of vertices in graph $E(j_n)$ with respect to $\overline{S}$ are:
$r(v_1|\overline{S}) = (0,2,2,\ldots,2,2)$
$r(v_2|\overline{S}) = (2,0,2,\ldots,2,2)$
$r(v_3|\overline{S}) = (2,2,0,\ldots,2,2)$
$\vdots$
$r(v_n - 5|\overline{S}) = (2,2,2,\ldots,2,0,2)$
$r(v_n - 4|\overline{S}) = (2,2,2,\ldots,2,2)$
$r(v_n - 3|\overline{S}) = (3,1,\ldots,1,1)$
$r(v_n - 2|\overline{S}) = (1,3,1,\ldots,1,1)$
$r(v_n - 1|\overline{S}) = (1,1,3,3,\ldots,3,3)$
$r(v_n|\overline{S}) = (2,2,\ldots,2,0)$

The representations of vertices in graph $E(j_n)$ are distinct as seen above. This implies that $\overline{S}$ is secure resolving set, but this does not prove that it is the lower bound. As a result, the upper bound is $sdim(E(j_n)) \leq n-4$. Now, we demonstrate that $sdim(E(j_n)) \geq n-4$. Let $\overline{S} = \{v_1, v_2, v_3, \ldots, v_{n-5}, v_n\}$ be a secure resolving set with $|\overline{S}| = n-4$. Assume that $\overline{S_1}$ is another minimal resolving set, or $|\overline{S_1}| < n-4$.

If we select an ordered set $\overline{S_1} \subseteq \overline{S} - \{v_i, v_j\}$, $1 \leq i,j \leq m$, $i \neq j$, so that there exist two vertices $v_i, v_j \in E(j_n)$ such that $r(v_i|\overline{S}) = r(v_j|\overline{S}) = (2,2,\ldots,2)$. It should be noted that $\overline{S_1}$ is not a secure resolving set, which is contrary to assumption. As a result, $sdim(E\ (j_n)) \geq n-4$ is

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the lower bound. From the above proving, we conclude that $sdim(E(j_n)) = n - 4$.

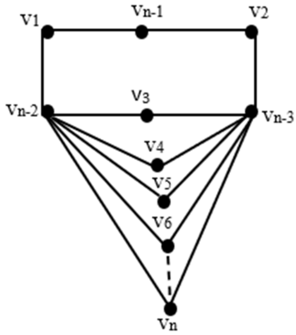

Fig. 5. Extended jewel graph $E(j_n)$

4. Secure resolving dimension for special classes of graphs

Corollary 4.1 If G is a shadow graph of path $D_2(P_n)$ of order $n \geq 3$, then $sdim(D_2(P_n)) = \frac{n}{2}$.
Corollary 4.2 If G is Jelly fish $J_{n,n}$ of order $n \geq 3$, then $sdim(J_{n,n}) = n - 5$.
Corollary 4.3 If G is Lilly graph L_n, $n \geq 5$, then $sdim(L_n) = \frac{n+1}{2}$.
Corollary 4.4 If G is Twig graph Tg_n, $n \geq 4$, then $sdim(Tg_n) = \frac{n+4}{3}$.
Corollary 4.5 If G is Joint sum of two copies of C_n, $n \geq 4$, then $sdim\ (G) = 2$.
Corollary 4.6 If G is quadrilateral graphs Q_n, $n \geq 6$, then $sdim\ (Q_n) = \frac{n-2}{2}$.
Corollary 4.7 If G is subdividing all the edges of $P_n \odot K_1$, $n \geq 7$, then $sdim(G) = 2$.

5. Conclusions

The secure metric dimension of a graph is an NP-complete problem. The present study starts with the task of finding the secure metric dimension of new graph types. The secure metric dimensions of the join of the (m, n) kite graph and the 1-join of the square of the path graph have the same secure metric dimensions. The secure metric dimension of the coconut tree and the extended jewel graph have different secure metric dimensions. Additionally, we deduced the exact formulas for the joint total of two copies of C_n, quadrilateral graphs Q_n, Jelly fish $J_{n,n}$, Lilly graph L_n, Twig graph Tg_n, and subdividing all the edges of $P_n \odot K_1$.

In the future, we plan to determine the secure metric dimension of many graphs, such as subdivisions of crown graphs, twig graph, Lilly graph and jelly fish graph. Many other ideas can be inspired from the references [22-25].

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Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Author contributions

Iqbal M. Batiha: conceptualization, validation, data curation. Basma Mohamed: methodology, formal analysis, software, investigation. Iqbal H. Jebril: resources, visualization, supervision.

Conflict of interest

The authors declare that they have no conflict of interest.

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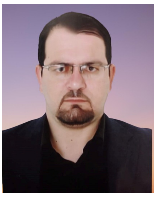

Iqbal M. Batiha holds a M.Sc. in Applied Mathematics (2014) from Al Al-Bayt University and a Ph.D. (2019) from The University of Jordan. He is a founding member of the International Center for Scientific Research and Studies (ICSRS, Jordan), and he is currently working as an Assistant Professor at the Department of Mathematics at Al-Zaytoonah University of Jordan, as well as at the Nonlinear Dynamics Research Center (NDRC) that was recently established at Ajman University. He has published several papers in different peer-reviewed international journals. Iqbal M. Batiha was awarded several prizes, including the Riemann-Liouville Award, which was presented at the International Conference on Fractional Differentiation and its Applications (ICFDA’18) that was held in Amman in July 2018, and the Oliviu Gherman Award, which was presented at the First Online Conference on Modern Fractional Calculus and Its Applications that was held in Turkey in December 2020.

Basma Mohamed received the B.S. and M.S. degrees and Ph.D. degrees in computer science from the Faculty of Science, Menoufia University, in 2011, 2017, and 2023, respectively. In addition, she has over nine years of teaching and academic experience. Her research interests include graph theory, discrete mathematics, algorithm analysis, and meta-heuristic algorithms.

Iqbal H. Jebril is Professor at the Department of Mathematics, Al-Zaytoonah University of Jordan, Amman, Jordan. He obtained his Ph.D. in 2005 from the National University of Malaysia (UKM). His fields of interest include functional analysis, operator theory, and fuzzy logic. He had several prestigious journal and conference publications and was on various journals and conferences’ committees.