# Common fixed-point theorem for commuting maps on a metric space

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**Abstract.** Several novel uses of theorems for fixed points in commuting mapping in a fully metric domain are presented. Several conclusions from full metric fixed point theory are improved and extended by our work. Our proofs are inspired by the study of commuting mappings [B. Fisher and S. Sessa, on a fixed point theorem of Gregus, 1986] and [P. Sumati Kumari, Fixed and periodic point theory in certain spaces, 2013].

**Keywords:** metric space, Cauchy sequence, common fixed-point theorem.

## 1. Introduction

The concept of a fixed point is essential to a number of different branches of mathematics. many fascinating and interesting branches of mathematics. The search for fixed points is under the purview of many subfields of analysis, such as classical analysis, operator theory, topology, functional analysis, topological algebra, and algebraic topology. Numerous fields make use of fixed-point theorems. different domains, including non-linear oscillations. Theoretical approximation, initial and boundary value difficulties, and the third flow for ordered and partial differential equations. In 1912, Brouwer [1] demonstrated that the mapping is continuous $\zeta$ in reference to the closed unit ball $R^l$ space having at least one fixed point, which we'll refer to as point $z$: a point where $\zeta_z = z$. In 1922, mathematicians Birkhoff and Kellogg [2] developed the first theory of fixed points in infinite dimensions. Using Brouwer's Birkhoff and Kellogg provided evidence for the existence of theorems in the theory of differential equations through their work on the fixed-point theorem. In the case where E is a convex compact set that exists in a normed vector space, Schaunder [3, 4] generalized Brouwer's fixed point theorem in 1927 and 1930, respectively. Tychnoff [5] was the first mathematician to generalize the Schaunder result from normed space to locally convex topological vector space considered by the authors in [35]. It was first proven in 1986 by Fisher and Sessa [6] that two self-maps have a fixed point if they are located on a convex subset of a Banach space. Refer to the work Sessa, [7] elaborated in which Das&Naik, [8] research was represented as a reference. Jungek [9] first generalized the Sessa concept to compatible mapping, and later to weakly compatible mapping (see [10]). Several authors have extended Jungek's approach to produce coincidence point findings for various types of mapping on Fatus-proper metric spaces (see [11,12]). Some generalizations points of coincidental occurrence and popular fixed points in fixed point theory were shown to be false by Haghi et al. (2011) in which they could be produced immediately from the fixed-point theorems that are linked to it.

Jleli and Samet [13] were the ones who came up with the idea of a generalized metric space, which has subsequently been used to the process of recovering a number of different topological spaces. Metric, b-metric, displaced, and modular spaces are examples (see [16-24, 30]).

Using the E. A property on metric space, authors L. Wangwe and Santosh Kumar [25] studied the common fixed-point theorem under implicit contractive conditions. An arbitrary binary relation has been developed and applied to second-order differential equations with boundary conditions. In his study of fixed point and continuity for a pair of contractive maps, Santosh Kumar [26] applied the nonlinear Volterra Integral equation and provided multiple examples proving the existence and uniqueness of the solution. In TVS valued cone metric space, Lucus Wangwe and Santosh Kumar [27] have investigated to generalize and extend several fixed points results that illustrate the common fixed pints theorem for F-Kanan- Suzuki type mapping. Dur-e-Shehwar Sangheer et al. [28] have investigated a novel multi valued F- contraction mapping incorporating α- admissibility with applications, where they have proved on existence solution of integral equations. Recently authors Lucus Wangwe [29] proved the common fixed points theorem for interpolative rational type of contraction mapping in complex valued metric space with application in existence and uniqueness solution of R-L-C differential equation.

**Definition [14]:** Take the set $F$ to be non-empty. Where $m, n \in F$ and $(F, d)$ be metric space. A mapping $\xi: F \to F$ is said to be a generalized contraction if we find a positive real number $\delta_1$ satisfies the following conditions:

$$d(\xi m, \xi n) \leq \delta_1 max\,\{d(m,n), d(m, \xi m), d(n, \xi n), d(m, \xi n) + d(n, \xi m)\}. \tag{1}$$

**Known theorems:** In 1979, Fisher [15] the following theorem.

**Theorem**: Let $(R, d)$ be entire metric space, $\xi$ and $\zeta$ are continuous functions (signals) of $(R, d)$ into itself. Then both signals have a common stationary in the set $R$ if only if we find a continuous signal $F$ of $R$ into $\xi(R) \cap \zeta(R)$ commutes with two signals $G$ and $\zeta$ meet certain requirements $F(R) \subset \xi(R) \cap \zeta(R)$, $d(Fm, Fn) \leq \delta d(\xi m, \zeta n)$, where, $m, n \in R$, $\delta \in (0,1)$, in fact, $G$ and $\zeta$ having a single point of agreement.

Let's say that $\xi$, $\zeta$, and $F$ are three self-mapping of $R$ satisfying the following constraints:

$$F\xi = \xi F, \tag{2}$$
$$F\zeta = \zeta F, \tag{3}$$
$$F(R) \subset \xi(F) \cap \zeta(F), \tag{4}$$

where $\xi$ and $\zeta$ are continuous. For all $m, n \in F$ we find a positive integer $\delta_1$ such that:

$$d(Fm, Fn) \leq \delta_1 max\left\{d(\xi m, \zeta n), d(\xi m, Fm), d(\zeta n, Fn), \frac{d(\xi m, Fn), d(\zeta n, Fm)}{2}\right\}. \tag{5}$$

**Lemma**: If $F$, $\xi$, and $\zeta$ are self-mapping of $R$ satisfying when (2), (3), and (4) hold, then is what $R$ calls a Cauchy sequence.

**Proof**: Let $m_o$ be a random number in $R$. Given that $F(R)$ is encapsulated in $\xi(R)$, we choose a special point $m_1$ in $R$ such that $\xi m_1 = Fm_o$.

Since $F(R)$ is also contained in $\zeta(R)$, we choose another special point $m_2$ in $F$ such that $\xi m_2 = Fm_1$. In the same way, we choose a point $m_r$ such that $\xi m_r = Fm_{r-1}$ when $r$ is even.

Now we have to show that $\{Fm_r\}$ is a Cauchy sequence in $R$. Using the Eq. (5) we obtain:

$$\begin{aligned} d(Fm_1, Fm_2) &\leq \delta_1\, max\left\{d(\xi m_1, \zeta m_2), d(\xi m_1, Fm_2), d(\zeta m_2, Fm_2), \frac{d(\xi m, Fn), d(\zeta n, Fm)}{2}\right\} \\ &= \delta_1\, max\left\{d(Fm_o, Fm_1), d(Fm_1, Fm_2), \tfrac{1}{2} d(Fm_o, Fm_2)\right\}. \end{aligned} \tag{6}$$

Thus, we obtain $d(Fm_1, Fm_2) \leq d(Fm_o, Fm_1)$.

In the same way, we can easily obtain $d(Fm_r, Fm_{r+1}) \leq \delta_1^r d(Fm_o, Fm_1) \Rightarrow \{Fm_r\}$ for all $r \in N$, contains a Cauchy sequence in $R$.

**Theorem 1:** Allow a whole metric domain to map itself using $F$, $\xi$, and $\zeta$ If (2), (3), (4), and (6) hold, if this is the case, then $F$, $\xi$, and $\zeta$ all have a common fixed point.

**Proof:** Suppose $m_o$ is a chosen coordinate in $R$, and $\{Fm_r\}$ be a sequence satisfying all conditions of the given lemma. Then $\{Fm_r\}$ transforms into a Cauchy sequence. There is just one possible limit point for a convergent Cauchy sequence. Let us suppose that $\{Fm_r\} \to l$. Since the sequences, $\{\xi m_{2r+1}\}$ and $\{\zeta m_{2r}\}$are the subsequences of $\{Fm_r\}$. We know that every subsequence of a convergent sequence has the same limit point. Thus, we have $\{G\xi\} \to l$ and $\{\zeta m_{2r}\} \to l$ as $r \to \infty$.

Consequently, we write $\zeta(Fm_r) \to \zeta l$, $\xi Fm_{2r+1} \to \xi l$, $\zeta(\xi m_{2r+1}) \to \zeta l$, and $\xi(\zeta m_{2r}) \to \xi l$. We consider:

$$d(\zeta Fm_{2r}, \xi Fm_{2r+1}) = d(F\zeta m_{2r}, F\xi m_{2r+1})$$
$$\leq \delta_1 \, max \left\{ \begin{matrix} d(\xi\zeta m_{2r}, \zeta\xi\, m_{2r+1}), d(\xi\zeta m_{2r}\,, F\zeta\, m_{2r}), d(\zeta\xi m_{2r+1}\,, F\xi m_{2r+1}), \\ d(\zeta\xi m_{2r}, F\xi m_{2r+1})\,, \frac{1}{2}\left(d(\zeta\xi m_{2r}\,, F\xi m_{2r+1}), d(F\zeta m_{2r}\,, \zeta\xi m_{2r+1})\right) \end{matrix} \right\}.$$

Using $r \to \infty$ then $d(\zeta l, \xi l) \leq \delta_1\, max\{d(\xi l, \zeta l), d(\xi l, \zeta l), d(\zeta l, \xi l)\} \Rightarrow \zeta l = \xi l$.

Again, from the Eq. (5):

$$d(\zeta Fm_{2r}, Fl) = \left\{d(F\zeta m_{2r}, Fl), d(\xi\zeta m_{2r}, \zeta l), d(\zeta l, Gl), \frac{1}{2}\left(d(\xi\zeta m_{2r}, Fl), d(F\zeta m_{2r}, Fl)\right)\right\}.$$

Using $\to \infty$ , we get $d(\zeta l, Fl) \leq d(\zeta l, Fl) \Rightarrow \zeta l = Fl$.

Thus, $Fl = \zeta l = Gl$.

Next, we consider:

$$d(\zeta m_{2r}, Fl) = d(Fm_{2r-1}, Fl) \leq max \left\{ \begin{matrix} d(\xi m_{2r-1}, \zeta l), d(\xi m_{2r-1}, Fm_{2r-1}), \\ d(\zeta l, Fl), \frac{1}{2}\left(d(\xi m_{2r-1}, Fl), d(Fm_{2r-1}, \zeta l)\right) \end{matrix} \right\}.$$

Using $r \to \infty$, we obtain $d(l, Fl) \leq \delta_1\, d(l, Fl) \Rightarrow l = Fl$.

At every condition, we have seen that $l$ is a fixed point of $\xi$, $F$, and $\zeta$.

**Theorem 2:** Assume the following about the compact metric space $(R, d)$: $F$, $\xi$, and $\zeta$ are all self-mappings satisfying:

$$d(Fm, Fn) < max \left\{ d(\xi\, m\,, Fn), d(\xi\, n, \zeta\, n), d(\zeta\, n, Fm), \frac{d(\xi\, m, Fn) + d(\zeta\, n, Fm)}{2} \right\}, \tag{7}$$

for all $m, n \in R$ and R.H. $S$ of Eq. (7) is a positive number.

If the self-mapping $F$, $\xi$ and $\zeta$ are continuous then they have a unique common fixed point.

**Proof:** Let us Let's pretend the answer to the right side of the problem is Eq. (7) is positive. Then we define a function $\eta(m, n)$ be defined by:

$$\eta(m, n) = \frac{d(Fm, Fn)}{max\left\{d(\xi m, Fm), d(\xi m, \zeta m), d(\zeta n, Fn), \frac{1}{2}\left(d(\xi\, m, Fn) + d(\,\zeta n, Fm)\right)\right\}}$$

is continuous real valued mapping on the compact metric space and attains a maximum value of $\delta$. From the Eq. (7) we choose a number $\delta$ such that $\delta > 0$ and hence by using theorem 1, the mapping $F$, $\xi$ and $\zeta$ have a single, unifying point of reference.

Let's pretend the answer to Eq. (7) on the right is zero since $m, n \in R$.

Thus $\xi m = \zeta n = Fm = Fn \Rightarrow F\xi\, m = F^2 m = \xi Fm$.

If $F^2 m \neq Fn$ then:

$$d(F^2 m, Fn) < max\left\{d(\xi Fm, F^2 m), d(\xi Fm, \zeta n), d(\zeta n, Fn), \tfrac{1}{2}[d(\xi Fm, Fn) + d(\zeta n, F^2 m)]\right\}$$
$$\therefore d(F^2 m, Fn) < d(F^2 m, Fn),$$

which gives a contradiction.

Thus, $F^2 m = Fn = F(Fm) = F(Fn)$. $\therefore Fn = l$ is a constant value that $F$.

And $\xi l = \xi(Fn) = \xi(Fm) = F(\xi m) = F(Fn) = Fl = l$, and $\zeta l = \zeta(Fn) = F(\zeta n) = F(Fn) = Fl = l$.

As a result, we can declare that l is the place where $F$, $\xi$, and $\zeta$ all meet.

**Corollary 1:** Let $F$ and $\zeta$ be compact metric $(R, d)$ mappings that commute into themselves such that:

$$d(\zeta n, Fn) < max\left\{d(\zeta m, \zeta n), d(\zeta m, Fm), d(\zeta n, Fn), \frac{d(\zeta m, Fn) + d(\zeta n, Fm)}{2}\right\}$$

holds for all $m, n \in R$ for which its R.H.S. is non-negative.

If $F(R) \subset \zeta(R)$, $F$ and $\zeta$ is an extended result of the authors [15] and states that if two maps are continuous, then they must share a fixed point.

**Corollary 2:** Let $F$ be a continuous function of a compact metric space $(R, d)$ into itself such that:

$$d(Fm, Fn) < max\left\{d(m, n), d(m, Fm), d(n, Fn), \frac{d(m, Fn) + d(n, Fm)}{2}\right\}$$

holds for all $m, n \in R$ for which its R. H.S. of above inequality is non-negative, if $F$ has a shared fixed point, then. (This finding generalizes the author's previous findings; see [16]).

**Corollary 3:** If $\zeta$ a continuous function of a compact metric space $(R, d)$ into itself which satisfy the following condition:

$$d(m, n) < max\left\{d(\zeta m, \zeta n), d(\zeta m, m), d(\zeta n, n), \frac{d(\zeta m, n) + d(\zeta n, m)}{2}\right\},$$

$\forall\, m, n \in R$ for which its R. H. S of its inequality is non-negative then the continuous mapping $\zeta$ has a unique fixed point.

**Proof:** If $F$ be a continuous identity mapping and $F(R) \subset \zeta(R)$. With the help of corollary Eq. (2), we get our result.

**Theorem 3:** Let $F$, $\xi$, and $\zeta$ be self-mappings of a complete metric space which satisfy the following conditions:

1) $F\zeta = \zeta F$, $F\xi = \xi F$.
2) $F(R) \subset \zeta^a(R) \cap \xi^b(R)$.
3) $\xi^b$ and $\zeta^a$ are continuous.
4) $F^c$, $\xi^b$ and $\zeta^a$ satisfy:

$$d(F^c m, F^c n) < \delta_1 max\left\{d(\xi^b m, \zeta^a n), d(\xi^b m, F^c m), d(\zeta^a n, F^c n), \frac{d(\xi^b m, F^c n) + d(\zeta^a n, F^c m)}{2}\right\},$$


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where, $a$ and $b$ are some positive integers and $c > 0$ Therefore, $F$, $\xi$, and $\zeta$ have a common fixed point.

**Proof:** Since $F$ commutes with the mapping $\xi$ and $\zeta$, $F^c$ commutes with $\xi^b$ and $\zeta^a$. Further $F^c(R) \subset F(R) \subset \xi^b(R)$. In the same line, we may write $F^c(R) \subset \zeta^a(R)$.

Also, since $F^c$, $\xi^b$ and $\zeta^a$ are continuous mapping then by using theorem 1, we find a point $l \in R$ such that $l = F^c l = \xi^b l = \zeta^a\, l$.

Also, $\xi^a(Fl) = F(\xi^a l) = Fl = F(F^c l) = F^c(Fl) \Rightarrow Fl$ is a common fixed of $\xi^b$ and $F^c$.

In the same line previous work, we define $Fl$ as the fixed point in common between $\zeta^a$ and $F^c$.

By the definition of uniqueness of a point $l \Rightarrow l = Fl$.

Let $\xi^{b+1} l = \xi l$ and $\zeta^a l = \zeta l$ and if we substitute $m = Fl$ and $n = Fn$ in Eq. (5) we easily obtain $\xi l = \zeta l$. $\therefore \xi l = \zeta l$ is also a common point for $F^c$, $\xi^b$ and $\zeta^a$.

Thus, by uniqueness of common fixed point l, we obtain $l = Fl = \xi l = \zeta l$.

This result the extends the result of author [15].

**Some Applications.** If we slight change in our theorems then we can easily obtain same famous results of periodic point theory in certain spaces. For this, we study.

**Definition 1:** Assume $\xi$ is a self-map of $R$, and $m \in R$ and $a \in N$. If $\xi^a\, m = m$ then $m$ is known as periodic point of the function $\xi$.

**Definition 2:** Let $m \in R$, $a$, $b \in N$, and $\xi$ and $\zeta$ are both self mappings of acomplete setric space $(R, d)$. If $\xi^a\, m = \zeta^a m = m$ then a given point $m$ is known as common periodic point of our function $\xi$ and $\zeta$.

**Definition 3:** Given a non-empty set $R$, we say that $m, n, l \in R$. A metric for a collection that is not empty as a function, $Rd: R \times R \to [0, \infty)$ such that:

1) $d(l, l) = 0$.
2) $d(l, m) = d(m, l) = 0 \Leftrightarrow m = l$.
3) $d(l, m) = d(m, l)$.
4) $d(l, m) \leq d(l, n) + d(n, m)$.

**Theorem 4:** Let $\xi$ and $\zeta$ are continuous functions of complete metric space $(R, d)$ satisfying:

$$d(\xi^b, \zeta^a n) < max\left\{d(m, n), \frac{d(\xi^b m, m) + d(\zeta^a n, n)}{2}, \frac{d(\zeta^a n, m) + d(\xi^b m, n)}{2}\right\}, \tag{8}$$

for all $m, n \in R$ ( $m \neq n$) in which R.H. S. of Eq. (8) is non-zero and $a, b \in N$. Then $r \in R$ is common periodic of mappings $\xi$ and $\zeta$ point if and only if $r$ is the single fixed point in both mappings.

**Proof:** It is obvious that only any periodic point is referred to as a fixed point. For conversely, we put $\xi^b = L$ and $\zeta^a = M$ in our Eq. (8) then we have:

$$d(Lm\,, M\,n) < max\left\{\frac{d(m, n), d(Lm, m) + d(n, Mn)}{2}, \frac{d(Mn, l) + d(n, Lm)}{2}\right\}.$$

Setting $a = b = 1$ in the theorem Eq. (8) we represent the following is an illustration.

Let $R = [0, 1)$ and $d(m, n) = |m - n|$. Let $G, \zeta: R \to R$ by $Gm = m/3$ and $\zeta m = 0$.

The value 0 is the only common fixed point that exists. $\Rightarrow$ 0 is the unique common periodic point.

Here $\xi$ and $\zeta$ satisfy the aforementioned inequality, and verification may be obtained by looking at the following:

1) $m = 0$, $0 < n < 1$ $\therefore$ L.H.S. = 0 and R. H. S. = $|n|$.
2) $n = 0$, $0 < m < 1$, L.H.S = $|m|/3$ and R. H. S. = $|m|$.
3) for $1 > n > m > 0$ (or for $1 > m > n > 0$) we obtain L.H.S and R.H.S. = $max\{|m -$

$n|, |2m| + |n|/3, |m| + \left|n + \frac{m}{3}\right|/2\}$.

Thus L.H.S. = $|m|/3 < \frac{|2m|/3+|n|}{2} \leq$ R.H.S.

The self-mappings $\xi$ and $\zeta$ satisfy above inequality when $a = 2$ and $b = 3$.

**Theorem 5:** Let $\xi$ and $\zeta$ are continuous function of complete metric space $(R, d)$ satisfying:

$$d(\xi^b m, \zeta^a n) < max\left\{d(m,n), \frac{d(\xi^b m, m) + d(\zeta^a n, n)}{2}, \frac{d(\zeta^a n, m) + d(\xi^b m, n)}{2}\right\}, \tag{9}$$

for all $m, n \in R$ $(m \neq n)$ in which R.H.S. of Eq. (9) is non- zero and $a, b \in N$. Then $r \in R$ is $r$ is common periodic point of mappings if and only if r is considered to be the common fixed point of mappings $\xi$ and $\zeta$.

**Proof:** If we put $b = a$ in the given theorem Eq. (8), we easily obtain above theorem 5.

**Theorem 6:** Consider the function $\xi$ which is continuous mapping from a complete metric space $(R, d)$ into itself satisfying:

$$d(\xi^b m, \xi^a n) < max\left\{d(m,n), \frac{d(\xi^b m, m) + d(\xi^a n, n)}{2}, \frac{d(\xi^b m, n) + d(\xi^a n, m)}{2}\right\}, \tag{10}$$

for all $m, n \in R$ $(m \neq n)$ and $a, b \in N$ in which R.H.S of Eq. (10) is non-negative numbers. Then $r \in R$ is unique fixed point of the function the function $\xi$ if $u$ is a periodic point of the function $\xi$.

**Proof:** If we put $\zeta = \xi$ in Eq. (8), we can readily establish theorem Eq. (10).

**Theorem 7:** Let $\xi$ be a continuous function a complete metric space $(R, d)$ into itself satisfying:

$$d(\xi^b m, \xi^b n) < max\left\{d(m,n), \frac{d(\xi^b m, m) + d(\xi^b n, n)}{2}, \frac{d(\xi^b n, m) + d(\xi^b m, n)}{2}\right\}, \tag{11}$$

for all $m, n \in R$ $(m \neq n)$ and $b \in \boldsymbol{N}$ in which R.H S of Eq. (11) is non-negative numbers. Then $r \in R$ is unique fixed point of the function $\xi$ if and only if $r$ is the periodic point of function $\xi$.

**Proof:** Put $a = b$ in theorem Eq. (10), we get the proof of the theorem 7.

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**Data availability**

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

**Author contributions**

Dr. Suresh Kumar Sahani contributed as author for preparing the manuscript and edited the article incorporated the comments of reviewers. Dr. Sahani, (Prof). Vijay Vir Singh, Dr. Krishnapal Singh Sisodia, and Dr. Kusum Sharma have revised the manuscript with their own insights to submitting the final revision.

**Conflict of interest**

The authors declare that they have no conflict of interest.

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