Vibrational analysis of orthotropic rectangular plate having combination of circular thickness and parabolic temperature

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Received 24 October 2023; accepted 6 November 2023; published online 27 November 2023
DOI https://doi.org/10.21595/vp.2023.23729

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67th International Conference on Vibroengineering in Udaipur, India, November 27, 2023

Abstract. The objective of this study is to investigate the natural vibration of a rectangular plate made of orthotropic material with circular thickness (two dimensions) and temperature variation on the plate is parabolic (two dimensions) in nature. The solution to the problem is obtained by utilizing the Rayleigh-Ritz technique and the first four frequency modes are obtained under clamped edge conditions. The study aims to provide numerical data that demonstrate how circular variation in tapering parameters of plate can effectively control and optimized vibrational frequencies of the plate. Orthotropic rectangular plate, thermal gradient, circular tapering, aspect ratio.

Keywords: orthotropic rectangular plate, thermal gradient, circular tapering, aspect ratio.

1. Introduction

To design structures or understand system characteristics, it becomes vital to investigate the vibrational properties of plates. Many systems and structures such as bridges, buildings, and aircraft wings consist of plates of various shapes. The vibration characteristics of a plate are influenced by plate parameters such as tapering, non-homogeneity (in the case of nonhomogeneous materials), and thermal gradient. A considerable number of studies in the literature have focused on various values of plate parameters.

The approach outlined in [1] was utilized to amalgamate solutions for plates with different geometries (such as circular, annular, circular sector, and annular sector plates) under various boundary conditions. In [2], the wave-based method (WBM) was utilized to forecast the flexural vibrations of orthotropic plates. In [3], a solution based on two-variable refined plate theory of Levy type was developed for free vibration analysis of orthotropic plates. In [4], a new analytical solution utilizing a double finite sine integral transform technique was introduced for the vibration response of plates reinforced by orthogonal beams. In [5], the Rayleigh Ritz method was utilized to determine the frequency of an orthotropic rectangular plate, whereas in [6], the time period of transverse vibration of a skew plate with different edge conditions was assessed. In [7], the influence of temperature on the frequencies of a tapered plate was discussed, while [8] investigated a non-uniform triangular plate subjected to a two-dimensional parabolic temperature distribution. The investigation of time period of rectangular plates with varying thickness and temperature was examined in [9]. Time period analysis of isotropic and orthotropic visco skew plate having circular variation in thickness and density at different edge conditions is discussed in [10] and [11].

It is noticeable from the literature that most of the authors have investigated either linear or parabolic variations in tapering parameters, but no one has focused on circular variation in tapering parameter. This study aims to fill this research gap by exploring the influence of two dimension circular thickness on the vibrational frequency of an orthotropic rectangular plate under a two dimension parabolic temperature profile. The circular variation examined in this paper results in a reduction in the variation in frequency modes, as shown in the numerical results section.

2. Problem geometry and analysis

Taking into account that the nonhomogeneous rectangular plate shown in Fig. 1 with sides a, b and thickness l.

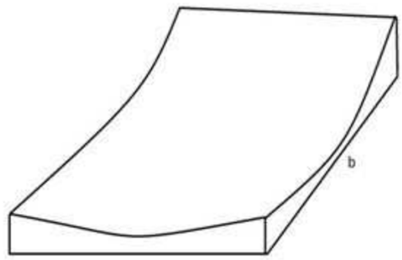

Fig. 1. Orthotropic rectangular plate with $2D$ circular thickness

The formulation of the kinetic energy and strain energy for plate vibration is given below, similar to the approach presented in [12]:

$$T_s = \frac{1}{2}\omega^2 \int_0^a \int_0^b l\Phi^2 d\psi d\zeta, \tag{1}$$

$$V_s = \frac{1}{2}\int_0^a \int_0^b \left[D_\zeta \left(\frac{\partial^2\Phi}{\partial\zeta^2}\right)^2 + D_\psi \left(\frac{\partial^2\Phi}{\partial\psi^2}\right)^2 + 2\nu_\zeta D_\psi \frac{\partial^2\Phi}{\partial\zeta^2}\frac{\partial^2\Phi}{\partial\psi^2} + 4D_{\zeta\psi}\left(\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\right)^2\right] d\psi d\zeta, \tag{2}$$

where ϕ is deflection function, ω is natural frequency, $D_\zeta = E_\zeta l^3/12(1-\nu_\zeta\nu_\psi)$, $D_\psi = E_\psi l^3/12(1-\nu_\zeta\nu_\psi)$, $D_{\zeta\psi} = E_\zeta l^3/12(1-\nu_\zeta\nu_\psi)$. Here, D_ζ and D_ψ is flexural rigidity in ζ and ψ directions respectively and $D_{\zeta\psi}$ is torsional rigidity.

In order to address the investigated problem, the Rayleigh-Ritz method is utilized, which necessitates:

$$L = \delta(V_s - T_s) = 0. \tag{3}$$

Using Eqs. (1), (2), we have:

$$L = \frac{1}{2}\int_0^a \int_0^b \left[D_\zeta \left(\frac{\partial^2\Phi}{\partial\zeta^2}\right)^2 + D_\psi \left(\frac{\partial^2\Phi}{\partial\psi^2}\right)^2 + 2\nu_\zeta D_\psi \frac{\partial^2\Phi}{\partial\zeta^2}\frac{\partial^2\Phi}{\partial\psi^2} + 4D_{\zeta\psi}\left(\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\right)^2\right] d\psi d\zeta - \frac{1}{2}\omega^2 \int_0^a \int_0^b \Phi^2 d\psi d\zeta = 0. \tag{4}$$

Proposing non-dimensional variable as $\zeta_1 = \zeta/a$, $\psi_1 = \psi/a$ along with two dimension circular thickness as:

$$l = l_0\left(1 + \beta_1\left\{1 - \sqrt{1-{\zeta_1}^2}\right\}\right)\left(1 + \beta_2\left\{1 - \sqrt{1 - \frac{a^2}{b^2}\psi_1^2}\right\}\right), \tag{5}$$

where l_0 is thickness at origin and $\beta_1, \beta_2 \leq 1$ are tapering parameters.

The two-dimensional parabolic temperature distribution, as presented in Eq. (6):

$$\tau = \tau_0(1-\zeta_1^2)\left(1-\frac{a^2\psi_1^2}{b^2}\right), \tag{6}$$

where τ and τ_0 represent the temperature at a given point and at the origin respectively.

For orthotropic materials, modulus of elasticity is evaluated by:

$$E_\zeta = E_1(1-\gamma\tau), \quad E_\psi = E_2(1-\gamma\tau), \quad G_{\zeta\psi} = G_0(1-\gamma\tau), \tag{7}$$

where E_ζ and E_ψ are the Young's modulus in ζ and ψ directions, $G_{\zeta\psi}$ is shear modulus and γ is called slope of variation.

Using Eq. (6), Eq. (7) becomes:

$$\begin{aligned} &E_\zeta = E_1\left(1-\alpha(1-\zeta_1^2)\left(1-\frac{a^2{\psi_1}^2}{b^2}\right)\right), \quad E_\psi = E_2\left(1-\alpha(1-\zeta_1^2)\left(1-\frac{a^2{\psi_1}^2}{b^2}\right)\right), \\ &G_{\zeta\psi} = G_0\left(1-\alpha(1-\zeta_1^2)\left(1-\frac{a^2{\psi_1}^2}{b^2}\right)\right), \end{aligned} \tag{8}$$

where $\alpha = \gamma\tau_0$ $(0 \leq \alpha < 1)$ is called thermal gradient.

Using Eqs. (5), (8) and non dimensional variable, the functional in Eq. (4) become:

$$\begin{aligned} L = \frac{D_0}{2}\int_0^1\int_0^{\frac{b}{a}} &\left[\left(1-\alpha(1-\zeta_1^2)\left(1-\frac{a^2{\psi_1}^2}{b^2}\right)\right)\left(1+\beta_1\left\{1-\sqrt{1-{\zeta_1}^2}\right\}\right)^3\right. \\ &\cdot\left(1+\beta_2\left\{1-\sqrt{1-\frac{a^2}{b^2}{\psi_1}^2}\right\}\right)^3\left\{\left(\frac{\partial^2\Phi}{\partial\zeta_1^2}\right)^2+\frac{E_2}{E_1}\left(\frac{\partial^2\Phi}{\partial\psi_1^2}\right)^2+2\nu_\zeta\frac{E_2}{E_1}\left(\frac{\partial^2\Phi}{\partial\zeta_1^2}\right)\left(\frac{\partial^2\Phi}{\partial\psi_1^2}\right)\right. \\ &\left.\left.+4\frac{G_0}{E_1}(1-\nu_\zeta\nu_\psi)\left(\frac{\partial^2\Phi}{\partial\zeta_1\partial\psi_1}\right)^2\right\}\right]d\psi_1 d\zeta_1 - \lambda^2\int_0^1\int_0^{\frac{b}{a}}\left[\left(1+\beta_1\left\{1-\sqrt{1-{\zeta_1}^2}\right\}\right)\right. \\ &\left.\cdot\left(1+\beta_2\left\{1-\sqrt{1-\frac{a^2}{b^2}{\psi_1}^2}\right\}\right)\right]\Phi^2 d\psi_1 d\zeta_1 = 0, \end{aligned} \tag{9}$$

where $D_0 = \frac{1}{2}\left(\frac{E_1 l_0^3}{12\ (1-\nu_\zeta\nu_\psi)}\right)$ and $\lambda^2 = \frac{12a^4\rho\omega^2(1-\nu_\zeta\nu_\psi)}{E_1{h_0}^2}$.

The deflection function that meets all the edge conditions is taken as in [13]:

$$\Phi(\zeta,\psi) = \left[(\zeta_1)^e(\psi_1)^f(1-\zeta_1)^g\left(1-\frac{a\psi_1}{b}\right)^h\right]\times\left[\sum_{i=0}^{n}\Psi_i\left\{(\zeta_1)(\psi_1)(1-\zeta_1)\left(1-\frac{a\psi_1}{b}\right)\right\}^i\right], \tag{10}$$

where Ψ_i, $i = 0,1,2\ldots n$ are unknowns and the value of e, f, g, h can be 0, 1 and 2, corresponding to given edge condition.

Eq. (10) can be minimized by imposing the following condition:

$$\frac{\partial L}{\partial \Psi_i} = 0, \quad i = 0,1,\dots n. \tag{11}$$

Solving Eq. (11), we have frequency equation:

$$|P - \lambda^2 Q| = 0, \tag{12}$$

where $P = [p_{ij}]_{i,j=0,1,..n}$ and $Q = [q_{ij}]_{i,j=0,1,..n}$ are square matrix of order $(n+1)$.

3. Numerical results and discussion

In this study, the first four natural frequencies of a clamped orthotropic rectangular plate with two dimension circular thickness and two dimension parabolic temperature variations are investigated corresponding to various plate parameters aspect ratio a/b, tapering parameters β_1 and β_2, and thermal gradient α. The numerical calculations are based on the subsequent parameter values:

$$E = 0.04, \quad E_1 = 1, \quad E_2 = 0.32, \quad G = 0.09, \quad \rho = 2.80\ 10^3\ \text{kg/m}^3, \quad \nu = 0.345.$$

Table 1. Modes of frequency of clamped orthotropic rectangular plate corresponding to β_1

β_2	$\alpha = 0.2$				$\alpha = 0.4$				$\alpha = 0.6$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	17.002	65.265	146.471	335.761	17.085	64.734	145.409	344.526	17.173	64.058	144.568	353.913
0.2	17.747	67.704	151.913	351.645	17.848	67.198	151.149	359.343	17.951	66.545	150.178	372.482
0.4	18.548	70.301	157.829	367.708	18.665	69.815	156.993	378.568	18.782	69.169	156.353	390.070
0.6	19.400	73.032	163.844	388.066	19.531	72.556	163.396	397.071	19.660	71.918	162.620	412.912
0.8	20.295	75.877	170.500	405.951	20.439	75.414	169.982	417.520	20.579	74.769	169.561	432.019
1.0	21.228	78.822	177.057	430.024	21.385	78.367	176.579	443.749	21.534	77.713	176.455	457.574

Table 2. Modes of frequency of clamped orthotropic rectangular plate corresponding to β_2

α	$\beta_1 = \beta_2 = 0.2$				$\beta_1 = \beta_2 = 0.4$				$\beta_1 = \beta_2 = 0.6$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	18.539	70.998	159.217	364.788	20.306	76.727	172.199	403.590	22.244	82.942	186.104	449.270
0.2	17.747	67.705	151.912	351.645	19.507	73.367	164.790	391.249	21.429	79.480	178.659	437.433
0.4	16.912	64.226	144.209	338.017	18.665	69.816	156.984	378.585	20.571	75.821	170.826	425.369
0.6	16.027	60.520	136.028	323.885	17.772	66.023	148.867	365.262	19.660	71.917	162.636	412.879
0.8	15.079	56.527	127.337	309.053	16.816	61.942	140.183	351.597	18.684	67.710	153.955	400.094
1.0	14.050	52.161	118.026	293.397	15.777	57.482	130.883	337.433	17.620	63.109	144.730	386.914

Table 3. Modes of frequency of clamped orthotropic rectangular plate corresponding to α

α	$\beta_1 = \beta_2 = 0.2$				$\beta_1 = \beta_2 = 0.4$				$\beta_1 = \beta_2 = 0.6$			
	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0.0	18.539	70.998	159.217	364.788	20.306	76.727	172.199	403.590	22.244	82.942	186.104	449.270
0.2	17.747	67.705	151.912	351.645	19.507	73.367	164.790	391.249	21.429	79.480	178.659	437.433
0.4	16.912	64.226	144.209	338.017	18.665	69.816	156.984	378.585	20.571	75.821	170.826	425.369
0.6	16.027	60.520	136.028	323.885	17.772	66.023	148.867	365.262	19.660	71.917	162.636	412.879
0.8	15.079	56.527	127.337	309.053	16.816	61.942	140.183	351.597	18.684	67.710	153.955	400.094

Table 1 presents the modes of frequency (first four modes) corresponding to β_1. Specifically, the values of $\beta_2 = \alpha$ chosen were 0.2, 0.4, and 0.6 respectively. Based on the results presented in Table 1, it can be inferred that:

1. The frequency modes increase in all four modes as β_1 rises from 0.0 to 1.0.
2. As both β_1 and α increase from 0.2 to 0.6 (i.e., $\beta_2 = \alpha = 0.2$ to $\beta_2 = \alpha = 0.6$), the modes of frequency also show an increment.

 ISSN PRINT 2345-0533, ISSN ONLINE 2538-8479

3. The modes of frequency (rate of increment) are predominantly influenced by β_1, as opposed to the α and β_2.

Table 2 presents the modes of frequency (first four modes) corresponding to β_2, with fixed values of tapering parameter β_1 and thermal gradient α i.e., $\beta_1 = \alpha = 0.2$, $\beta_1 = \alpha = 0.4$, and $\beta_1 = \alpha = 0.6$, respectively. Based on the findings in Table 2, it is evident that:

1. The modes of frequency exhibit an increase as β_2 varies from 0.0 to 1.0.

2. The modes of frequency decrease as β_1 and α increase from 0.2 to 0.6 (i.e., $\beta_1 = \alpha = 0.2$ to $\beta_1 = \alpha = 0.6$), when β_2 changes from 0.0 to 0.6. However, the modes of frequency increase as β_1 and α increase from 0.2 to 0.6 (i.e., $\beta_1 = \alpha = 0.2$ to $\beta_1 = \alpha = 0.6$), when β_2 varies from 0.8 to 1.0.

3. In terms of the rate of change in modes of frequency, β_2 exerts a stronger influence compared to α and β_1.

4. The tables 1 and 2 indicate that the modes of frequency are primarily influenced by β_2 as compared to β_1.

Table 3 presents the modes of frequency (first four modes) for various values of α, while keeping both β_1 and β_2 fixed at 0.2, 0.4, and 0.6, respectively. By examining Table 3, the subsequent key findings can be observed:

1. The frequency modes also increase with an increase in β_1 and β_2 from 0.0 to 1.0, while the modes of frequency decrease with an increase in the value of α from 0.0 to 1.0.

2. Compared to β_1 and β_2, α has a greater influence on the modes of frequency (i.e., rate of change in the modes of frequency).

4. Conclusions

The above observation suggests that the plate parameters have a significant impact on the frequency modes of the plate. The selection of appropriate plate parameters can enable the manipulation of frequency modes and their variations as per the system requirements. Thus, it can be inferred that circular variation in plate parameters can be effective in minimizing and regulating frequency modes and their variations.

Acknowledgements

The authors have not disclosed any funding.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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