

# Automatic modal identification of bridges based on free vibrations and advanced signal decomposition techniques

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**Abstract.** Free vibration tests are attractive because they can be performed by means of a network consisting of few sensors temporarily installed on the structure in such a way to limit duration and cost of the experimental campaign. Additionally, free vibration tests are usually performed by introducing an initial perturbation that can induce a structural response significantly higher than the ambient excitation. This, in turn, allows to reduce the noise-to-signal ratio in the final measurements and/or to consider less stringent requirements about the technical specifications of the sensors. Since free vibration tests can provide accurate estimates of the modal parameters while being rather cheap and easy to implement, they have been performed in many applications, such as the experimental dynamic characterization of base-isolated buildings, masonry towers, ancient tie-rods, and bridges. An efficient and automatic computational framework is thus presented for the modal identification of bridges based on their free vibrations. The novel procedure proposed in the current work combines advanced signal decomposition techniques and a robust approach for damping identification. Two advanced signal decomposition techniques are considered, namely the Variational Mode Decomposition and the Empirical Fourier Decomposition. Experimental applications are then illustrated for roadway and railway bridges.

**Keywords:** bridge, damping, free vibrations, modal identification, signal decomposition.

## 1. Introduction

Free vibration tests are often performed for the experimental dynamic characterization of the structures. Such strategy is usually implemented at some point in time, periodically, and even after exceptional events. To this end, a sparse network consisting of few sensors is deployed on the structure to measure its free response following an applied load or an imposed displacement. Although such strategy does not allow to track continuously the evolution of the current structure conditions, it has some advantages. First, the larger intensity of the dynamic structural response makes possible the use of a sensor network designed to fulfill less stringent technical requirements. Moreover, the monitoring campaign takes a short time span, and a limited amount of data has to be collected and processed. In light of this, free vibrations tests have been reported for a large variety of applications [1-5]. In most field applications, free vibrations are extracted from the dynamic response induced in the structural system after an applied load or an imposed displacement. The original contribution of the current research consists with a novel procedure that aims at enabling the robust and automatic modal identification of bridges based on their free vibrations.

## 2. Proposed framework for modal identification using free vibrations

According to the proposed framework, all embedded relevant components are initially extracted from the recorded structural response. Among the available signal decomposition techniques, the Hilbert-Huang transform [6], also known as empirical mode decomposition, is widely adopted in structural monitoring applications [7]. Nonetheless, it has been found in previous applications that such technique is rather sensitive to noise and sampling. Therefore, two alternative advanced signal decomposition techniques are herein considered, namely the variational mode decomposition (VMD) technique [8] and the empirical Fourier decomposition (EFD) technique [9]. The decomposition of the recorded dynamic response into its relevant components allows to estimate the natural frequencies of the structure. Next, suitable strategies are implemented for the identification of damping ratios and mode shapes.

### 2.1. Variational mode decomposition

Let  $t$  and  $K$  be the time variable and the total number of components  $f_k(t)$  to be extracted from the recorded signal  $f(t)$  by means of the VMD technique, respectively. In the time domain, the VMD technique aims at extracting the relevant components  $f_k(t)$  from  $f(t)$ , also known as modes or intrinsic mode functions (IMFs), and estimating the corresponding central pulsation  $\omega_k$  through the following steps:

- 1) the Hilbert-Huang transform is applied to compute the analytical signal for each  $k$ th IMF.
- 2) the frequency spectrum of the mode is shifted to baseband by mixing with an exponential function tuned to the respective estimated central frequency.
- 3) the  $H_1$  Gaussian smoothness of the demodulated signal (i.e., the squared  $L^2$ -norm of the gradient) is performed to estimate the bandwidth for each  $k$ th IMF.

Considering these steps, the signal decomposition according to the VMD technique is thus performed by looking for the solution of the following variational problem:

$$\min_{\substack{f_1(t), \dots, f_K(t) \\ \omega_1, \dots, \omega_K}} \left\{ \sum_{k=1}^K \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) * f_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\}, \quad (1)$$

$$\text{s. t. } \sum_{k=1}^K f_k(t) = f(t),$$

where  $\partial_t$  and  $\|\cdot\|_2$  are the gradient operator and the  $L^2$ -norm operator, respectively. The variational problem in Eq. (1) can be solved by using a quadratic penalty term and enforcing the constraints by means of the Lagrange multipliers technique. This leads to the following augmented Lagrangian function:

$$L(f_1(t), \dots, f_K(t), \omega_1, \dots, \omega_K, \lambda) = \alpha \sum_{k=1}^K \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) * f_k(t) \right] e^{-j\omega_k t} \right\|_2^2 + \left\| f(t) - \sum_{k=1}^K f_k(t) \right\|_2^2 + \langle \lambda, f(t) - \sum_{k=1}^K f_k(t) \rangle, \quad (2)$$

where  $\alpha$  is the quadratic penalty factor and  $\lambda$  is the Lagrangian multiplier whereas  $\langle \cdot \rangle$  is the  $L^2$ -inner product. The solution of the problem given in Eq. (1) thus corresponds to the saddle point of  $\mathcal{L}$ , which is obtained through a sequence of iterative sub-optimizations named alternate direction method of multipliers. From a computational standpoint, however, the complete final algorithm is more efficiently posed in the spectral domain rather than in the time domain. It is

noted that the number of IMFs to be extracted  $K$  and the penalty factor  $\alpha$  rule the performance of the VMD technique. Therefore, their value must be carefully determined for the accurate decomposition of the signal. In the present work, the procedure proposed by Mazzeo et al. [10] has been implemented for the automatic optimal tuning of both control parameters.

## 2.2. Empirical Fourier decomposition

The EFD technique is an adaptive decomposition method introduced recently by Zhou et al. [9] that consists of two main steps. First, a segmentation procedure produces  $N$  frequency partitions of the frequency spectrum of the signal to be analyzed. Next, a zero-phase filter bank is required to perform the actual decomposition.

The segmentation process is carried out within a normalized frequency domain  $[0, \pi]$ . Therefore, signal frequency lines must be also normalized. Initially, the boundaries of the  $N$  contiguous frequency partitions  $\Lambda_n = [\omega_{n-1}, \omega_n]$  are detected. The frequencies  $\omega_0$  and  $\omega_N$  are not required to be equal to 0 and  $\pi$ , respectively. The first  $N$  frequencies corresponding to the largest maximum magnitudes detected in the signal spectrum are then sorted in descending order and are denoted as  $\{\Omega_1, \dots, \Omega_N\}$ , whereas  $\Omega_0=0$  and  $\Omega_{N+1} = \pi$ . For each pair of consecutive frequencies  $\Omega_n$  and  $\Omega_{n+1}$ , the partition is determined by picking the frequency value  $\omega_n$  at which a global minimum is attained as follows:

$$\omega_n = \begin{cases} \underset{\omega}{\operatorname{argmin}} \hat{X}_n(\omega), & 0 \leq n \leq N, \quad \Omega_n \neq \Omega_{n+1}, \\ \Omega_n, & 0 \leq n \leq N, \quad \Omega_n = \Omega_{n+1}, \end{cases} \quad (3)$$

where  $\hat{X}_n(\omega)$  is the Fourier spectrum amplitude between  $\Omega_n$  and  $\Omega_{n+1}$  whereas  $\omega$  is the frequency variable. The second step of the procedure is the construction of a filter bank. Zero-phase filters are considered to avoid possible interference due to the transition phase, because it can cause mode-mixing effects. The zero-phase filter bank is based on the frequency partitions obtained after the segmentation (i.e., the boundary frequencies of each partition identify the cut-off frequencies). Therefore, let  $\hat{f}(\omega)$  and  $\hat{\mu}(\omega)$  be the Fourier transform of the signal to be analyzed and the zero-phase filter in the frequency domain, respectively. The generic filtered component has the following expression in the frequency domain:

$$\hat{f}_n(\omega) = \hat{\mu}(\omega)\hat{f}(\omega) = \begin{cases} \hat{f}(\omega), & \omega_{n-1} \leq |\omega| \leq \omega_n, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

The modal component  $\hat{f}_n(\omega)$  can be finally expressed in the time domain by means of the inverse Fourier Transform  $\mathcal{F}^{-1}[\cdot]$  operator, i.e.  $f_n(t) = \mathcal{F}^{-1}[\hat{f}_n(\omega)]$ . Compared to the VMD technique, only one control parameter is involved into the EMD technique, which is the number of the frequency partitions  $N$ . In principle, its value can be obtained from the frequency spectrum of the signal. Unfortunately, a simple counting of the peaks in the Fourier Transform of the signal is likely to produce a wrong segmentation because the signal spectrum is always noisy in practical applications. Therefore, an iterative procedure has been proposed to automatically tune the proper value of  $N$ , in such a way to avoid both mode-mixing and mode-splitting phenomena [11].

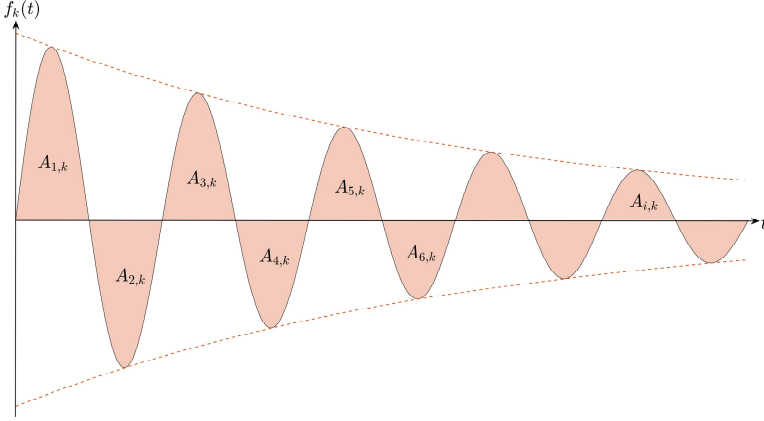
## 2.3. Robust damping identification

Once the  $k$ th component  $f_k(t)$  has been extracted by means of either the VMD technique or the EMD technique, the associated modal damping ratio is evaluated by means of the area ratio-based method [12]. Let  $2Z_k + 1$  be the number of zero-crossing points in  $f_k(t)$ . It is possible to detect a total of  $2Z_k$  regions enclosed between  $f_k$  and the time axis  $t$  with areas  $A_{1,k}, A_{2,k}, \dots, A_{2Z_k,k}$  (see Fig. 1). So doing, the damping ratio  $\xi_k$  is estimated as follows [12]:

$$\xi_k = \frac{1}{\sqrt{1 + (2Z_k\pi/R_k)^2}} \quad (5)$$

where  $R_k = \ln \left[ \frac{\sum_{i=1}^{Z_k} A_{i,k}}{\sum_{i=Z_k+1}^{2Z_k} A_{i,k}} \right]$ .

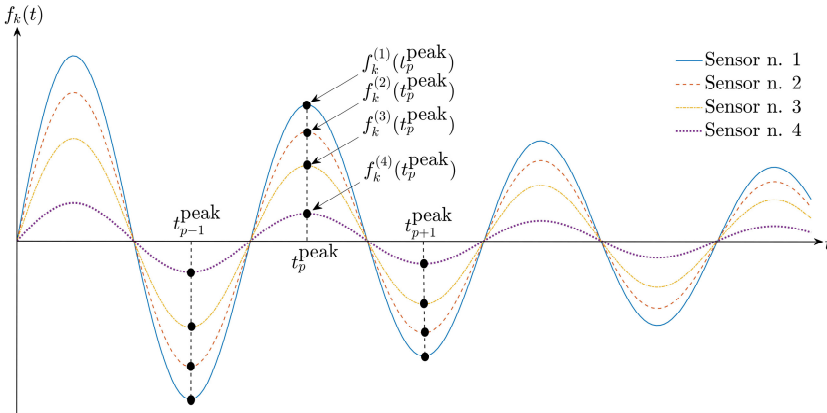
The application of the area ratio-based damping identification method is recommended since it is more robust against the noise as compared to the conventional logarithmic decrement technique.



**Fig. 1.** Detection of the regions with area  $A_{i,k}$  defined by the  $k$ th signal component for the application of the area ratio-based damping identification technique

## 2.4. Mode shapes identification

The mode shapes are estimated by processing the free vibration response recorded from all  $S$  available sensors after the decomposition accomplished via VMD technique or EMD technique [13].



**Fig. 2.** A schematic diagram of a vibration separator

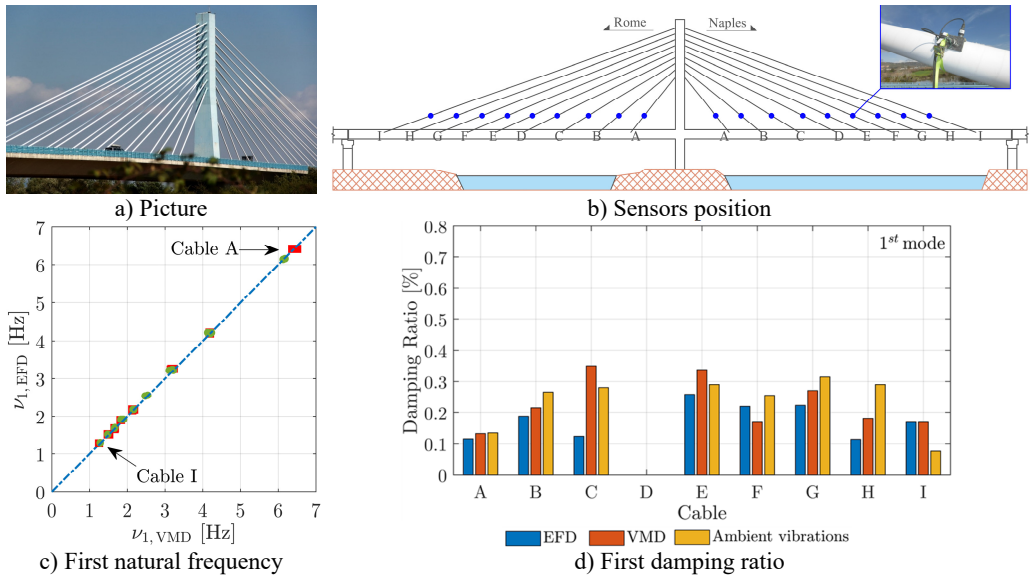
Particularly, the normalized  $k$ th mode shape vector  $\Phi_k$  is expressed as follows (see Fig. 2):

$$\Phi_k = \{f_k^{(1)}(t_p^{\text{peak}}) \quad \dots \quad f_k^{(s)}(t_p^{\text{peak}}) \quad \dots \quad f_k^{(S)}(t_p^{\text{peak}})\}^T / \max_{0 \leq s \leq S} f_k^{(s)}(t_p^{\text{peak}}). \quad (6)$$

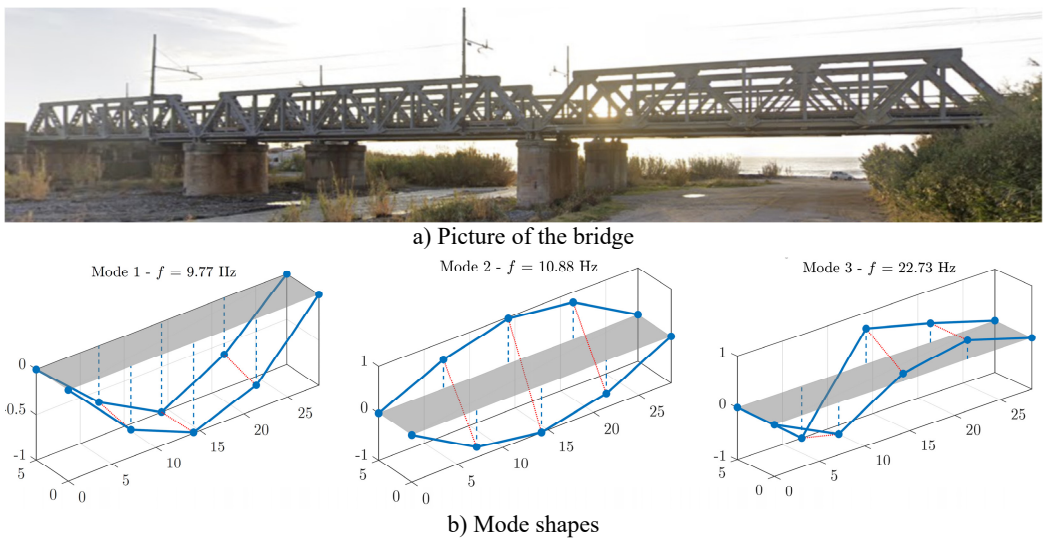
The effect of the noise can be mitigated by averaging the values in Eq. (6) for all peaks.

### 3. Experimental applications

The effectiveness of the presented framework for the modal identification of bridges based on free vibration response is investigated through preliminary applications to two experimental case-studies. The first application deals with the identification of natural frequency, damping ratio and force of the cables in the cable-stayed bridge crossing the Garigliano river (Fig. 3) [10, 14]. The second case-study is about the modal identification of a railway bridge (Fig. 4) [11]. Both the VMD technique and the EMD technique are employed for a comparative assessment of the final results.



**Fig. 3.** Some preliminary results about the dynamic identification of the cables in a cable-stayed bridge



**Fig. 4.** Some preliminary results about the modal identification of a railway bridge

### 4. Conclusions

This contribution has addressed the modal identification of structures from free vibration tests.

Special attention has been paid to the decomposition of the recorded dynamic response of the structure, which is accomplished via Variational Mode Decomposition and Empirical Fourier Decomposition. The proposed procedure is also complemented with robust strategies for the identification of damping ratios and mode shapes. Preliminary experimental applications related to roadway and railway bridges have demonstrated the feasibility of the proposed identification methodology for practical applications.

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## Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

## Conflict of interest

The authors declare that they have no conflict of interest.

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