Plane waves in an isotropic thermoelastic diffusive material using strain gradient theory

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Abstract. The governing equations of motion for an isotropic strain-gradient thermoelastic material with diffusion are formulated in context of Lord and Shulman generalization of thermoelasticity and are further specialised for a two dimensional plane. Plane harmonic solution of the governing equations in two-dimension suggests the existence of five plane waves which include four coupled longitudinal waves and a shear vertical wave. A numerical example is considered to illustrate graphically the effect of frequency, measure constant of diffusion, measure constant of thermo-diffusion, thermal relaxation time, diffusive relaxation time and the coefficients of hyperstress tensor on the phase speed and attenuation coefficients of the plane waves.

Keywords: plane wave, phase velocity, attenuation coefficient, thermal and diffusion relaxation times, hyperstress coefficients.

1. Introduction

Biot [1] gave the theory of coupled thermoelasticty, in which equation of motion and heat conduction equation are coupled. This theory eliminates the first shortcoming of uncoupled theory that elastic changes have no effect on the temperature. To overcome the second shortcoming of uncoupled theory, the non-classical theories of Lord and Shulman [2], Green and Lindsay [3], Green and Naghdi [4] and Tzou [5] came in to existence which are also referred as generalized thermoelastic theories in the literature. Ezzat and his coworkers [6-9] deveoped various new thermoelastic theories by considering various factors including the fractional order, memory-dependent, variable thermal conductivity and phase lag.

A spontaneous movement of particles, from high region concentration to low region concentration is called as diffusion. The coupling of temperature, mass diffusion and strain parameters in an elastic solid is called as thermodiffusion. The diffusion process has many application in fields of oil extractions, integrated circuit fabrication, geophysics and semiconductor industry. The process of thermodiffusion is applied in exploration of the field associated with the advent of semiconductor devices and the advancement of microelectronics. Nowacki [10-12] considered the coupling of elastic, thermal and diffusion parameters and developed a coupled theory of thermodiffusion. Further, Sherief et al. [13] derived the governing equations for a theory of generalized thermodiffusion in elastic solids. Aoudai [14] considered the uniqueness and reciprocity theorem in elastic media with generalized thermoelastic diffusion. Ezzat and Fayik [15] applied fractional calculus to derive an another theory of thermodiffusion in elastic solids. Karamany and Ezzat [16] introduced a theory of generalized thermodiffusion which was based on the memory-dependent derivatives. Recently, Li et al. [17] and Abouelregal [18] studied the effects of thermodiffusion in various anisotropic devices.

The gradient theory of elasticity is considered to be adequate for investigating important problems related to size effects, chiral materials and nanotechnology. The origin of the strain gradient theories of nonsimple elastic solids goes back to works of Toupin [19] and Mindlin and Eshel [20]. Aifantis [21] formulated gradient elasticity theories for infinitesimal deformations.

Recently, Iesan and Quintanilla [22] developed the strain gradient theory of thermoviscoelasticity. There are various applications of strain gradient theories in literature including Ahmadi and Firoozbakhsh [23], Aifantis ([24], [25]), Forest et al. [26], Forest and Aifantis [27], Iesan and Quintanilla ([28] and [29]). Aouadi et al. [30] developed a consistent nonlinear theory of strain gradient theory for thermoelastic diffusion materials and derived the basic equation of nonlinear strain gradient theory of thermoelasticity with mass diffusion effect.

Wave propagation in thermoelastic materials is a topic of interest for many researchers since last many decades. Chadwick and Sneddon [31] studied the plane wave in a thermoelastic solid. Puri [32] studied the plane waves in generalized thermoelastic solid in context of Lord Shulman theory. Agarwal [33] studied the plane waves in generalized thermoelastic solid in context of both Green Lindsay and Lord-Shulman theories. Sharma et al. [34] studied the plane harmonic waves in orthorhombic heat conducting materials. Verma and Hasebe [35] studied the wave propagation in plates of general anisotropic media. Singh ([36], [37]) explored the plane wave characteristics in a generalized thermoelastic medium with diffusion in context of Lord-Shulman, Green-Lindsay and Green-Nagdhi theories. Various other research works on wave propagation in thermoelastic diffusive materials are also available in literature, which include Othman et al. [38], Kumar and Gupta [39], Deswal et al. [40].

No theoretical/numerical study is traced in literature yet which investigated the characteristics of plane waves in a strain-gradient generalized thermoelastic medium with the effects of diffusion/mass transfer. Motivated by the strain gradient theory of thermoelastic diffusive material given by Aouadi et al. [30], the plane wave characteristics are investigated in an isotropic strain-gradient thermoelastic diffusive material. In Section 2, the governing equations of an isotropic model are derived with the help of Lord and Shulman [2] and Aouadi et al. [30]. In Section 3, the governing equations are specialized for a plane. These specialized equations are solved in Section 4 to explore the possibility of plane waves in the model. Two velocity equations for plane waves are derived which suggest the propagation of four coupled longitudinal waves and a shear vertical wave. Some special cases of these velocity equations are also derived in absence of diffusion parameters or hyperstress coefficients or both parameters. A numerical example of Magnesium is considered in Section 5 to illustrate graphically the phase speeds and attenuation coefficients of plane waves. The theoretical and numerical findings of the present work are presented in the last section.

2. Governing equations of an isotropic model

In absence of body forces and external heat sources, the governing equations of an isotropic strain-gradient thermoelastic diffusive material are formulated in context of Aouadi et al. [30] and Lord and Shulman [2] theories as.

(a) Equations of motion:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \mu \nabla^2 u_i - \nu_1 \nabla^4 u_i + (\mu + \lambda_0) \frac{\partial u_j}{\partial x_j \partial x_i} - \nu_2 \nabla^2 \frac{\partial u_j}{\partial x_j \partial x_i} - \gamma_1 \frac{\partial \Theta}{\partial x_i} - \gamma_2 \frac{\partial P}{\partial x_i}. \tag{1}$$

(b) Heat conduction equation:

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\left(\gamma_1 \frac{\partial e_{kk}}{\partial t} + C_e \frac{\partial \Theta}{\partial t} + d \frac{\partial P}{\partial t}\right) = \kappa \nabla^2 \theta. \tag{2}$$

(c) Mass diffusion equation:

$$\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\left(\gamma_2 \frac{\partial e_{kk}}{\partial t} + d \frac{\partial \Theta}{\partial t} + r \frac{\partial P}{\partial t}\right) = h \nabla^2 P, \tag{3}$$

where:

$$\gamma_1 = \beta_1 + \frac{a\beta_2}{b}, \quad \gamma_2 = \frac{\beta_2}{b}, \quad \lambda_0 = \lambda - \frac{\beta_2^2}{b}, \quad C_e = \frac{\rho c_E}{T_0} + \frac{a^2}{b}, \quad d = \frac{a}{b}, \quad r = \frac{1}{b},$$
$$\beta_1 = (3\lambda + 2\mu)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu)\alpha_c, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2},$$

in which ρ is the density of material, λ, μ are the Lame's constants, κ is coefficient of thermal conductivity, h is the coefficient of mass diffusion conductivity, ν_1, ν_2 are constitutive coefficients of hyperstress tensor, a is the measure constant of thermo-diffusive effect, b is the measure constant of diffusive effect, τ_0 is the thermal relaxation time, τ_1 is the diffusion relaxation time, c_E is the specific heat at constant strain, α_t is coefficient of linear thermal expansion, α_c is the coefficient of linear diffusion expansion, P is the chemical potential per unit mass of diffusive material, Θ is increment in temperature, T_0 is the temperature of the body in natural state, u_i is the displacement vector, e_{ij} are the components of strain tensor, $e_{kk} = u_{k,k}$, x_i and x_j are dependent variables and ∇^2 is the Laplace operator.

3. Specialization in two-dimension

A half-space of an isotropic thermoelastic material with diffusion in strain gradient theory is considered in rectangular Cartesian coordinate system (x_1, x_2, x_3) having the surface of the half-space as the plane $x_3 = 0$. Therefore, the present analysis is restricted to $x_1 - x_3$ plane. The following components of displacement vector are taken as Eq. (4):

$$u_1 = u_1(x_1, x_3, t), \qquad u_3 = u_3(x_1, x_3, t), \qquad u_2 = 0. \tag{4}$$

Using the Helmholtz decomposition theorem on vectors, the components of displacement vectors are written in terms of scalar potentials functions ϕ_1, ϕ_3 as Eq. (5):

$$u_1 = \frac{\partial \phi_1}{\partial x_1} - \frac{\partial \phi_3}{\partial x_3}, \quad u_3 = \frac{\partial \phi_1}{\partial x_3} + \frac{\partial \phi_3}{\partial x_1}. \tag{5}$$

Using Eqs. (4) and (5), the Eqs. (1) to (3) are specialized in $x_1 - x_3$ plane as Eqs. (6-9):

$$\left((\lambda_0 + 2\mu)\nabla^2 - \rho\frac{\partial^2}{\partial t^2}\right)\phi_1 - \nu_1\nabla^4\phi_1 - \nu_2\nabla^4\phi_1 - \gamma_1\Theta - \gamma_2 P = 0, \tag{6}$$

$$\mu\nabla^2\phi_3 - \nu_1\nabla^4\phi_3 = \rho\frac{\partial^2\phi_3}{\partial t^2}, \tag{7}$$

$$\left(1 + \tau_0\frac{\partial}{\partial t}\right)\left(\gamma_1 T_0\nabla^2\frac{\partial\phi_1}{\partial t} + C_e T_0\frac{\partial\Theta}{\partial t} + dT_0\frac{\partial P}{\partial t}\right) = \kappa\nabla^2\theta, \tag{8}$$

$$\left(1 + \tau_1\frac{\partial}{\partial t}\right)\left(\gamma_2\nabla^2\frac{\partial\phi_1}{\partial t} + d\frac{\partial\Theta}{\partial t} + r\frac{\partial P}{\partial t}\right) = h\nabla^2 P. \tag{9}$$

Here, the Eqs. (6), (8) and (9) are coupled in ϕ_1, Θ and P and Eq. (7) is uncoupled in ϕ_3.

4. Plane wave solution

To solve the Eqs. (6) to (9), the plane harmonic waves propagating with wave normal lying in the $x_1 - x_3$ plane and making an angle θ with the x_3-axis is considered. Then, the following form of plane harmonic solutions are considered as Eq. (10):

$$\{\phi_1, \phi_3, \Theta, P\} = \{\bar{\phi}_1, \bar{\phi}_3, \bar{\Theta}, \bar{P},\} \exp(ik(x_1 \sin\theta + x_3 \cos\theta) - i\omega t), \tag{10}$$

where, $i = \sqrt{-1}$, k is wave number, ω is angular frequency, $\bar{\phi}_1$, $\bar{\Theta}$, $\bar{P}$ and $\bar{\phi}_3$ are arbitrary constants and the pair $(\sin\theta, \cos\theta)$ denote the projection of the wave normal on to $x_1 - x_3$ plane. The homogeneous system of Eqs. (11-14) is obtained by inserting Eq. (10) into Eqs. (6) to (9):

$$[k^4\, \nu + (k^2 c_1^2 - \omega^2)]\, \bar{\phi}_1 + \bar{\gamma}_1\, \bar{\theta}_1 + \bar{\gamma}_2\, \bar{P} = 0, \tag{11}$$
$$K_1\, k^2\, \omega^2\, \bar{\phi}_1 + (K_2\, k^2 - \omega^2)\, \bar{\theta} - K_3\, \omega^2\, \bar{P} = 0, \tag{12}$$
$$d_1\, k^2\, \omega^2\, \bar{\phi}_1 - d_2\, \omega^2\, \bar{\theta} + (d_3\, k^2 - \omega^2)\, \bar{P} = 0, \tag{13}$$
$$(b_1 k^4 + c_2^2 k^2 - \omega^2)\, \bar{\phi}_3 = 0, \tag{14}$$

where:

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \quad \bar{\gamma}_1 = \frac{\gamma}{\rho}, \quad \bar{\gamma}_2 = \frac{\gamma_2}{\rho}, \quad K_1 = \frac{\gamma_1}{C_e}, \quad K_2 = \frac{\kappa}{C_e\, T_0\, \tau_0^*},$$
$$\nu = \nu_1 + \nu_2, \quad K_3 = \frac{d}{C_e}, \quad d_1 = \frac{\gamma_2}{r}, \quad d_2 = \frac{d}{r}, \quad d_3 = \frac{h}{r\, \tau_1^*}, \quad \tau_0^* = \tau_0 + \frac{\iota}{\omega},$$
$$\tau_1^* = \tau_1 + \frac{\iota}{\omega}, \quad b_1 = \frac{\nu_1}{\rho}.$$

The system of Eqs. (11) to (13) admits non-trivial solution if and only if determinant of coefficient matrix vanishes, i.e.:

$$\begin{vmatrix} k^4 \nu + (k^2 c_1^2 - \omega^2) & \bar{\gamma}_1 & \bar{\gamma}_2 \\ K_1 k^2 \omega^2 & (K_2 k^2 - \omega^2) & -K_3 \omega^2 \\ d_1 k^2 \omega^2 & -d_2 \omega^2 & (d_3 k^2 - \omega^2) \end{vmatrix} = 0, \tag{15}$$

which is expanded as following bi-quadratic equation in k^2:

$$S_0 (k^2)^4 + S_1 (k^2)^3 + S_2 (k^2)^2 + S_3 (k^2) + S_4 = 0, \tag{16}$$

where:

$$S_0 = d_3\, K_2\, \nu, \quad S_1 = c_1^2\, d_3\, K_2 - (d_3 + K_2)\, \omega^2\, \nu - K_2\, \omega^2\, \nu,$$
$$S_2 = (\omega^2)^2\, \nu - c_1^2\, d_3\, \omega^2 - c_1^2\, K_2\, \omega^2 - d_3\, K_2\, \omega^2 - d_1\, \bar{\gamma}_2\, K_2\, \omega^2 - d_3\, \bar{\gamma}_1\, K_1\, \omega^2 - d_2\, K_3\, (\omega^2)^2 \nu,$$
$$S_3 = c_1^2\, (\omega^2)^2 + d_3\, (\omega^2)^2 + K_2\, (\omega^2)^2 + d_1\, \bar{\gamma}_2\, (\omega^2)^2 + \bar{\gamma}_1\, K_1\, (\omega^2)^2 - c_1^2\, d_2\, K_3\, (\omega^2)^2 - d_1\, \bar{\gamma}_1\, K_3\, (\omega^2)^2 - d_2\, \bar{\gamma}_2\, K_1\, (\omega^2)^2,$$
$$S_4 = \omega^6\, (d_2\, K_3 - 1).$$

The four roots of velocity Eq. (16) correspond to four coupled longitudinal plane waves. The phase speed V_i $(i = 1, 2, \ldots, 4)$ and the attenuation coefficients Q_i $(i = 1, 2, \ldots, 4)$ of the plane waves P_i $(i = 1, 2, \ldots, 4)$ may be obtained by using following formulae:

$$V_i = \frac{\omega}{Re(k_i)}, \quad Q_i = |Imag(k_i)|, \tag{17}$$

where $Re(k_i)$ and $Imag(k_i)$ are the real and imaginary part of k_i. The solution of Eq. (14) correspond to the SV wave with speed:

$$V_5 = \sqrt{c_2^2 + b_1 k^2}. \tag{18}$$

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4.1. In absence of hyperstress coefficients

In absence of hyperstress coefficients, the velocity Eq. (15) reduces to the Eq. (19):

$$\begin{vmatrix} (k^2c_1^2-\omega^2) & \bar{\gamma}_1 & \bar{\gamma}_2 \\ K_1k^2\omega^2 & (K_2k^2-\omega^2) & -K_3\omega^2 \\ d_1k^2\omega^2 & -d_2\omega^2 & (d_3k^2-\omega^2) \end{vmatrix} = 0. \tag{19}$$

The cubic Eq. (19) indicates the propagation of three coupled longitudinal waves. In absence of hyperstress coefficients, the speed of SV wave given in Eq. (18) will reduce to the speed of classical SV wave as given by Eq. (20):

$$V_5 = \sqrt{\frac{\mu}{\rho}}. \tag{20}$$

The reduced velocity Eqs. (19) and (20) are in agreement with Singh [36].

4.2. In absence of diffusion parameters

In absence of diffusion parameters, the velocity Eq. (15) reduces to Eq. (21):

$$\begin{vmatrix} k^4\nu+(k^2c_1^2-\omega^2) & \bar{\gamma}_1 \\ K_1k^2\omega^2 & (K_2k^2-\omega^2)=0 \end{vmatrix} = 0. \tag{21}$$

The cubic Eq. (21) indicates the propagation of three coupled longitudinal waves. The speed of SV wave given by equation (18) is independent of diffusion parameters.

4.3. In absence of both diffusion and hyperstress coefficients

In absence of diffusion and hyperstress coefficients, the Eq. (15) reduces to Eq. (22):

$$\begin{vmatrix} (k^2c_1^2-\omega^2) & \bar{\gamma}_1 \\ K_1k^2\omega^2 & (K_2k^2-\omega^2) \end{vmatrix} = 0. \tag{22}$$

The quadratic Eq. (22) suggests the propagation of two coupled plane waves. In absence of diffusion and hyperstress coefficients, the speed of SV wave given in Eq. (18) will reduce to the speed of classical SV wave as given by Eq. (20).

5. Numerical results and discussion

The objective of present numerical simulations is to check the variations of phase speeds and attenuation coefficients for a specific material. The following material constants of Magnesium at $T_0 = 300$ K from Singh [41] are considered to compute the phase speeds and attenuation coefficients as in Table 1.

Using above above physical constants and using Eq. (17), the velocity Eq. (16) is solved to obtain the speeds and attenuations coefficients of four coupled longitudinal waves. The velocity Eq. (18) is also solved to obtain the speed of the SV wave. The wave speeds and attenuation coefficients of four coupled longitudinal waves (P_1, P_2, P_3 and P_4) are plotted in Fig. 1 against the circular frequency ω for three different values of measure constant of thermo-diffusive effect a For all values of a, the illustrations in figure 1 shows that the speeds and attenuation coefficients of all coupled longitudinal increase nonlinearly as the circular frequency ω increases. For increasing value of frequency, the rate of increase in speeds of P_1 and P_2 waves become fast and the rate of increase in speeds of P_3 and P_4 waves becomes slow. The effect of measure constant of thermodiffusion a is observed on P_1 and P_3 waves as these waves become faster as value of a is increased at a given frequency ω. This effect of thermodiffusion on P_1 and P_3 waves becomes more considerable as the frequency ω increases. For the selected range of frequency, the speeds of P_2 and P_4 waves are not influenced considerably by changing the values of constant a. The variations of the attenuation coefficients of the coupled longitudinal waves except P_1 wave are also affected due to change in values of constant a at a given frequency.

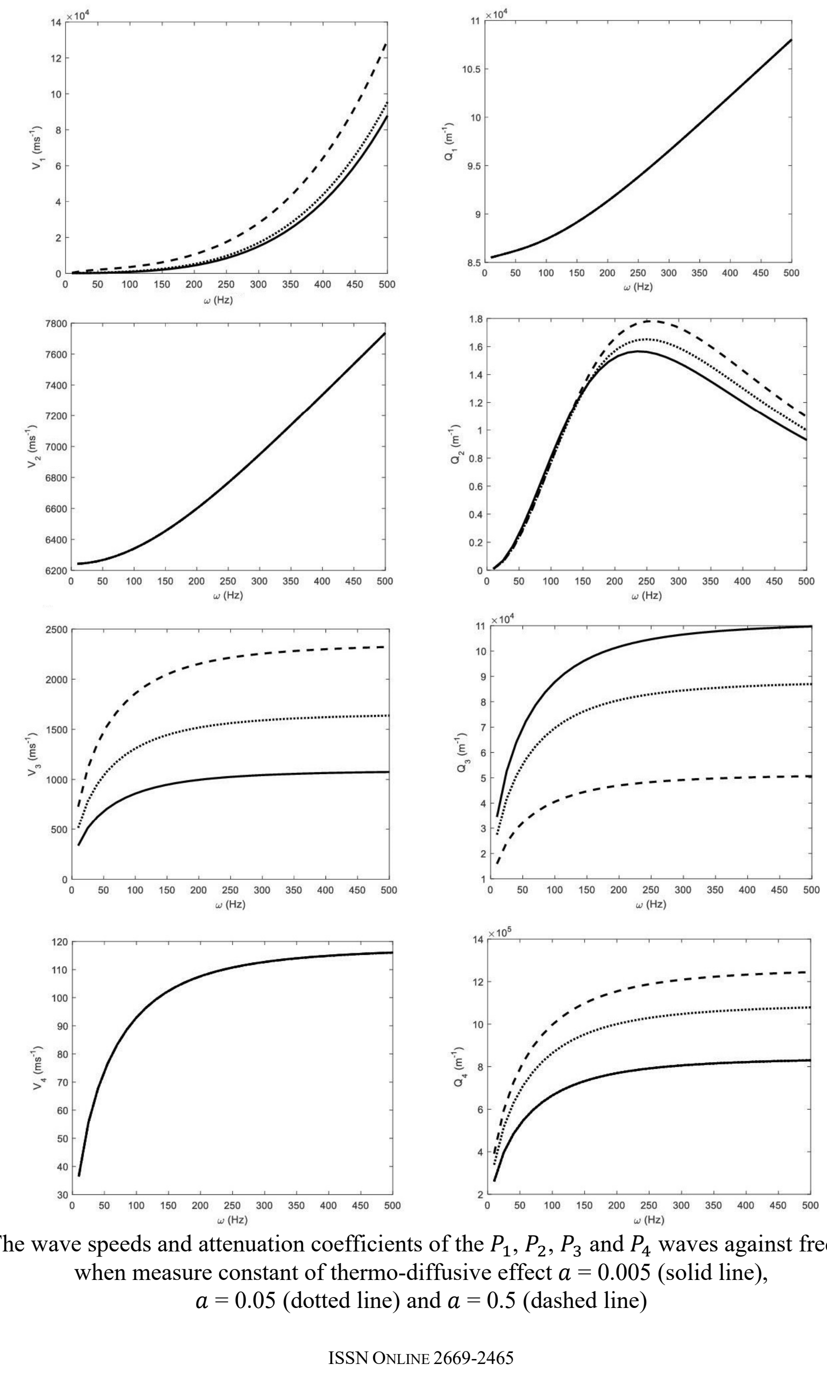

Fig. 1. The wave speeds and attenuation coefficients of the P_1, P_2, P_3 and P_4 waves against frequency ω, when measure constant of thermo-diffusive effect a = 0.005 (solid line), a = 0.05 (dotted line) and a = 0.5 (dashed line)

Table 1. Material constants of magnesium

Symbol	Value	Unit	Symbol	Value	unit
λ	3.17×10^{10}	Nm^{-2}	μ	1.639×10^{10}	Nm^{-2}
ρ	1740	Kgm^{-3}	C_E	2361	$JKg^{-1}deg^{-1}$
κ	49.2	$Wm^{-1}deg^{-1}$	h	0.85	$Kgsm^{-3}$
α_t	0.005	deg^{-1}	α_c	0.005	m^3Kg^{-1}
a	0.005	$m^{-2}s^{-2}deg^{-1}$	B	0.005	$m^5kg^{-1}s^{-2}$
τ_0	0.005	s	τ_1	0.005	s
ν_1	0.002	$Kgm^{-1}s^{-2}$	ν_2	0.002	$Kgm^{-1}s^{-2}$

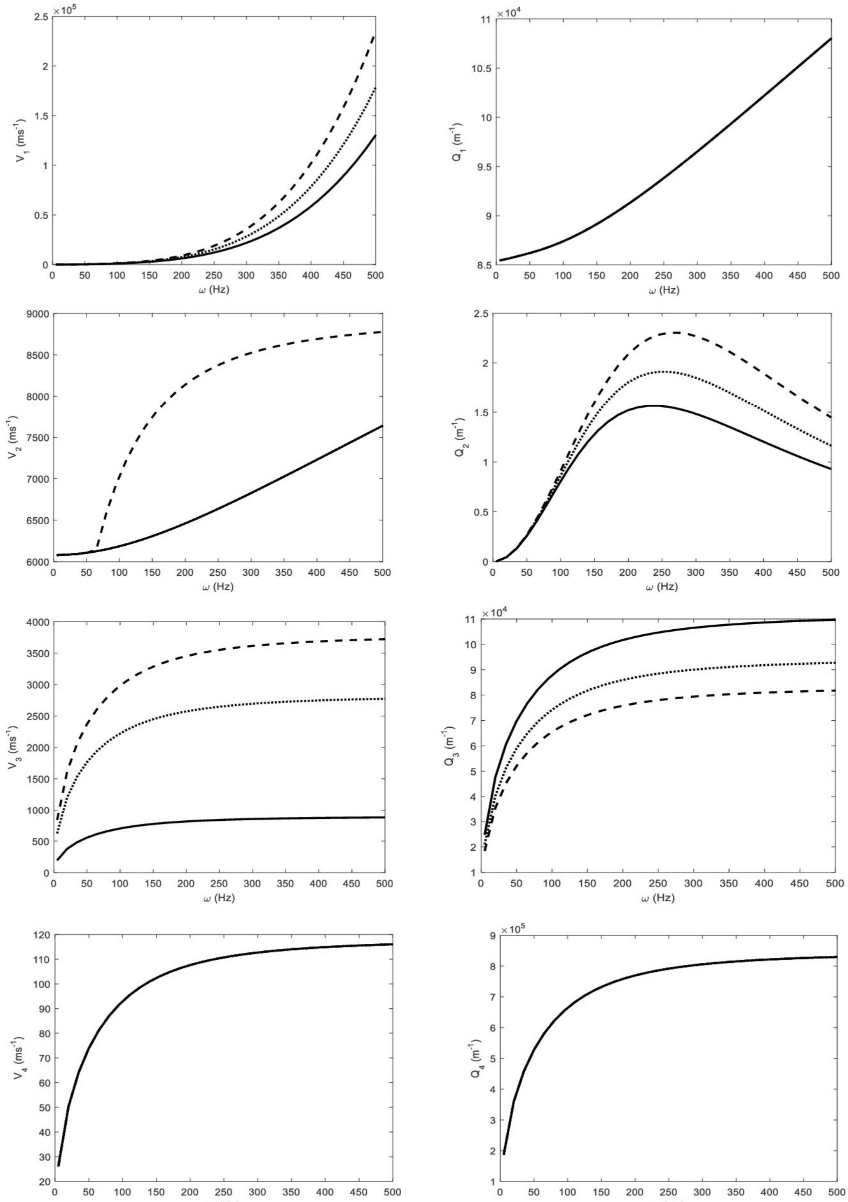

Fig. 2. The wave speeds and attenuation coefficients of the P_1, P_2, P_3 and P_4 waves against frequency ω, when measure constant of diffusive effect $b = 0.005$ (solid line), $b = 0.05$ (dotted line) and $b = 0.5$ (dashed line)

The wave speeds and attenuation coefficients of four coupled longitudinal waves (P_1, P_2, P_3 and P_4) are plotted in Fig. 2 against the circular frequency ω for three different values of measure constant of diffusive effect b. The effect of measure constant of diffusion b is observed on P_1, P_2 and P_3 waves as these waves become faster as value of b is increased at a given frequency ω. The diffusion effect on P_1, P_2 and P_3 waves becomes more considerable as the frequency ω increases. For the selected range of frequency, the speed P_4 wave is not influenced considerably by changing the values of constant b. The attenuation coefficients of the coupled longitudinal waves except P_1 and P_4 waves are also affected due to change in diffusion constant a at a given frequency.

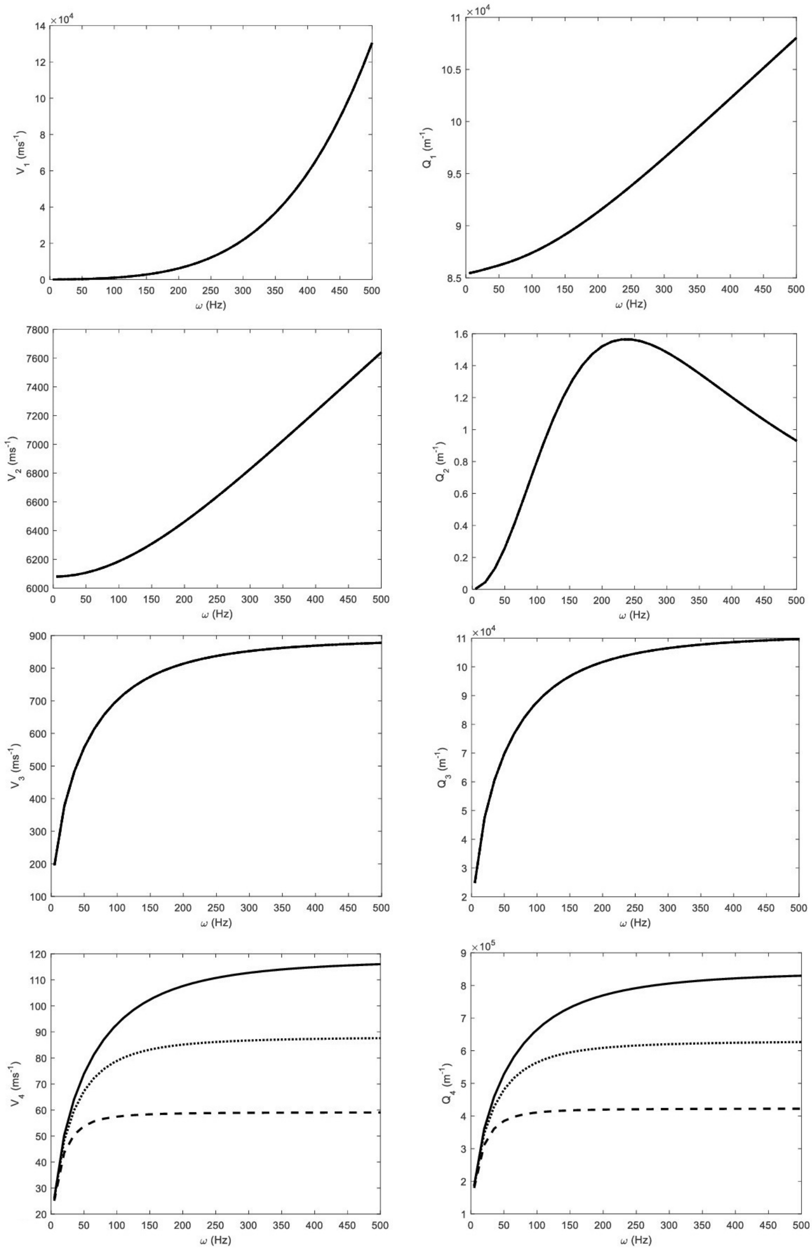

Fig. 3. The wave speeds and attenuation coefficients of the P_1, P_2, P_3 and P_4 waves against frequency ω, when thermal relaxation time τ_0 = 0.005 (solid line), τ_0 = 0.05 (dotted line) and τ_0 = 0.5 (dashed line)

The wave speeds and attenuation coefficients of four coupled longitudinal waves (P_1, P_2, P_3 and P_4) are plotted in Fig. 3 against the circular frequency ω for three different values of thermal relaxation time τ_0. The effect of thermal relaxation is observed on the speed and attenuation coefficient of P_4 wave only. The speed of P_4 wave becomes slow as the value of τ_0 increases at a given frequency. The effect of thermal relaxation time on speed and attenuation of P_4 wave becomes more considerable as the frequency increases.

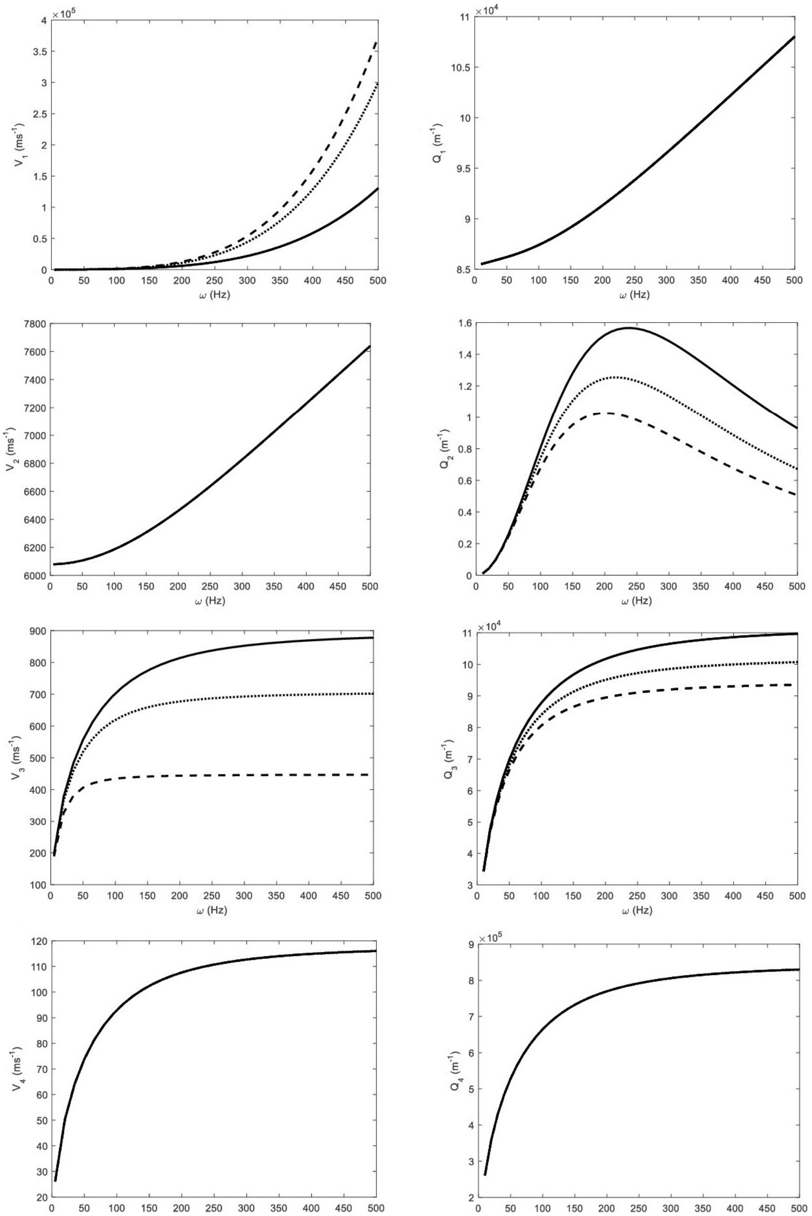

Fig. 4. The wave speeds and attenuation coefficients of the P_1, P_2, P_3 and P_4 waves against frequency ω, when diffusion relaxation time τ_1 = 0.005 (solid line), τ_1 = 0.05 (dotted line) and τ_1 = 0.5 (dashed line)

The wave speeds and attenuation coefficients of four coupled longitudinal waves (P_1, P_2, P_3 and P_4) are plotted in Fig. 4 against the circular frequency ω for three different values of diffusion relaxation time τ_1. The effect of diffusion relaxation time is observed on the speeds of P_1 and P_3 waves and on attenuation coefficient of P_2 and P_3 waves. For a given frequency, the speed of P_1 becomes faster as the value of τ_1 increases, whereas the speed of P_3 waves becomes slower. This effect of diffusion relaxation time on these coupled waves becomes more considerable as the value of circular frequency increases.

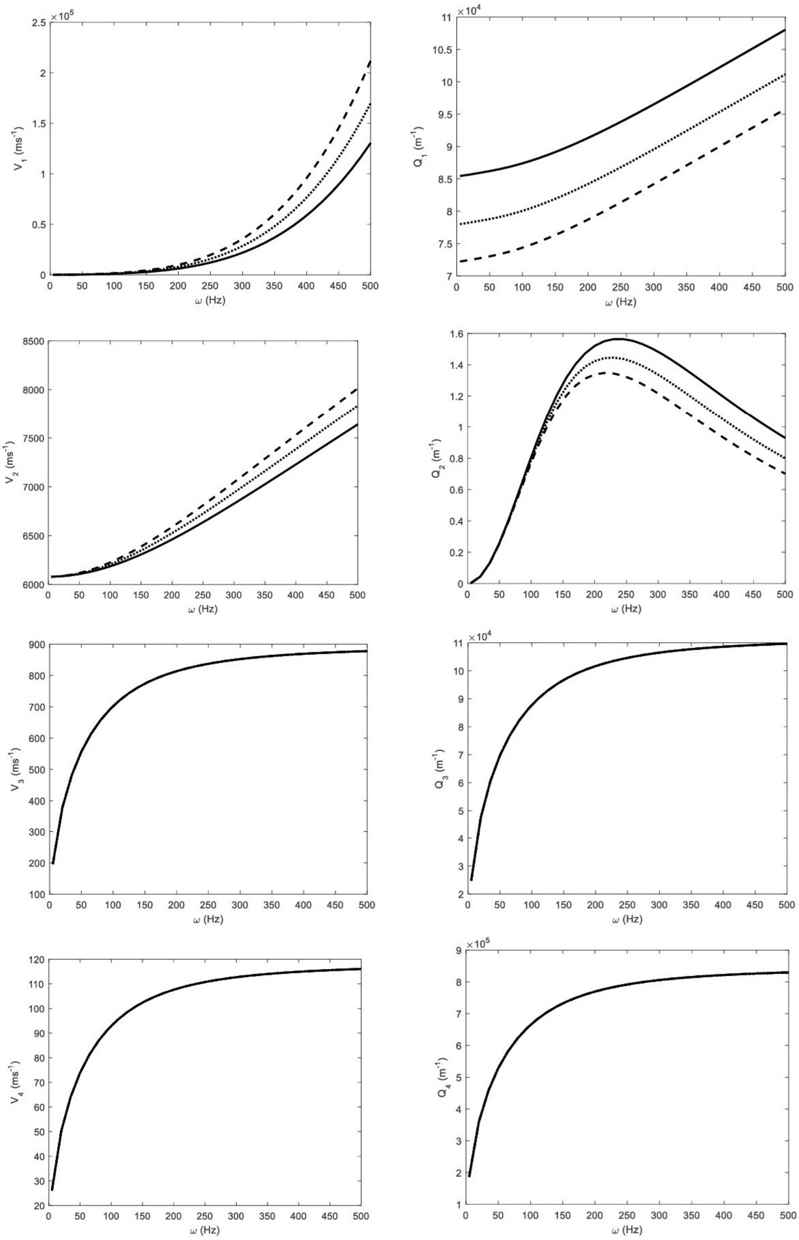

Fig. 5. The wave speeds and attenuation coefficients of the P_1, P_2, P_3 and P_4 waves against frequency ω, when the coefficient of hyperstress tensor ν = 0.005 (solid line), ν = 0.05 (dotted line) and ν = 0.5 (dashed line)

The wave speeds and attenuation coefficients of four coupled longitudinal waves (P_1, P_2, P_3 and P_4) are plotted in Fig. 5 against the circular frequency ω for three different values of hyperstress coefficient ν. The effect of hyperstress is observed on the speeds and attenuations of P_1 and P_2 waves. For a given frequency, the speeds of P_1 and P_2 becomes faster as the value of ν increases. This effect of hyperstress on these coupled waves becomes more considerable as the value of circular frequency increases.

The wave speeds and attenuation coefficients of coupled longitudinal waves are also plotted in Fig. 6 (in absence of hyperstress) where P_2 wave will not propagate in absence of hyperstress. The wave speeds and attenuation coefficients of coupled longitudinal waves are plotted in Fig. 7 (in absence of diffusion) where P_3 wave will not propagate in absence of diffusion. In Fig. 8, the wave speeds and attenuation coefficients of coupled longitudinal waves are shown graphically in absence of both diffusion and hyperstress, where the P_1 and P_4 waves will propagate.

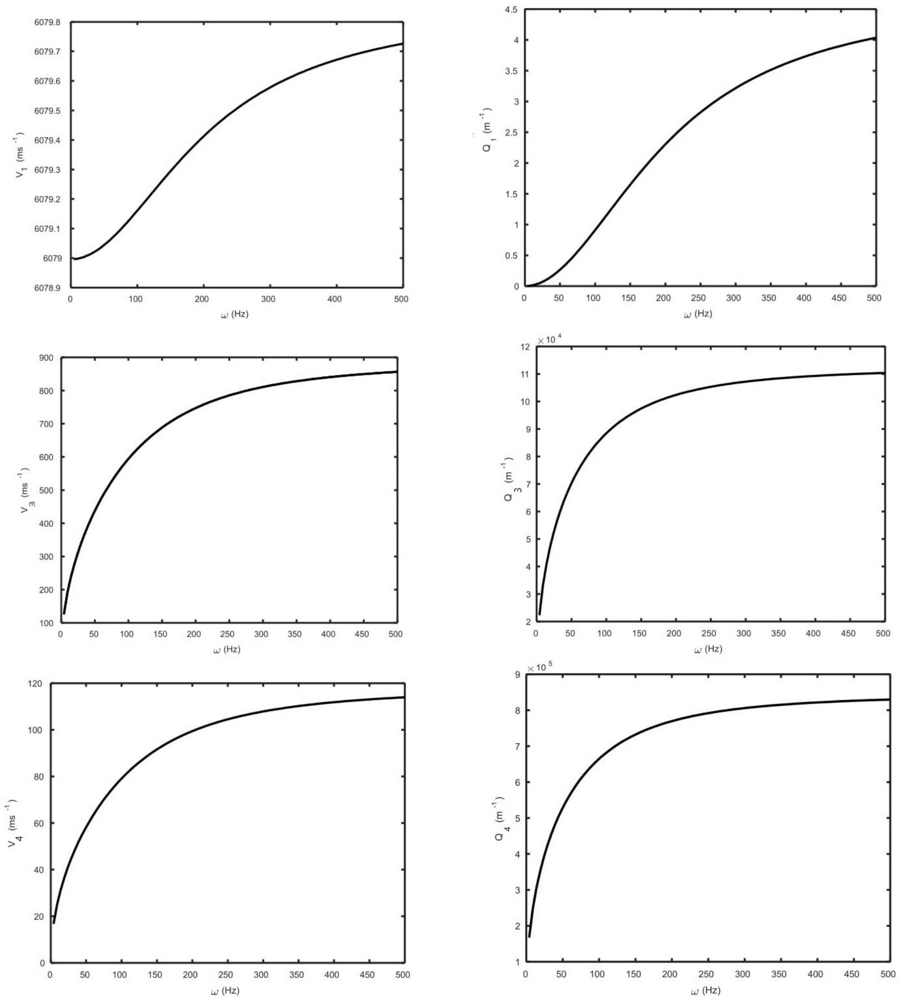

Fig. 6. The wave speeds and attenuation coefficients of the P_1, P_3 and P_4 waves against frequency ω (in absence of hyperstress)

The wave speed of SV wave is plotted in Fig. 9 against the circular frequency ω for three different values of hyperstress coefficient ν. In presence of hyperstress, the speed of SV wave increases nonlinearly as the frequency increases. For a given frequency, the speed of SV becomes faster as the value of ν increases. This effect of hyperstress on the SV wave is observed more considerable as the value of circular frequency increases.

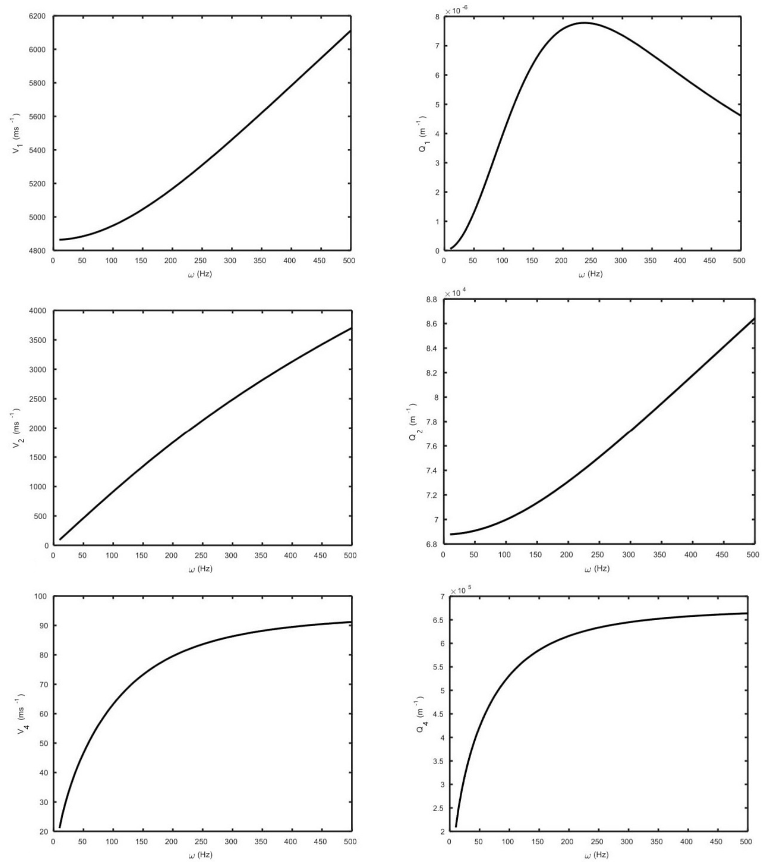

Fig. 7. The wave speeds and attenuation coefficients of the P_1, P_2 and P_4 waves against frequency ω (in absence of diffusion)

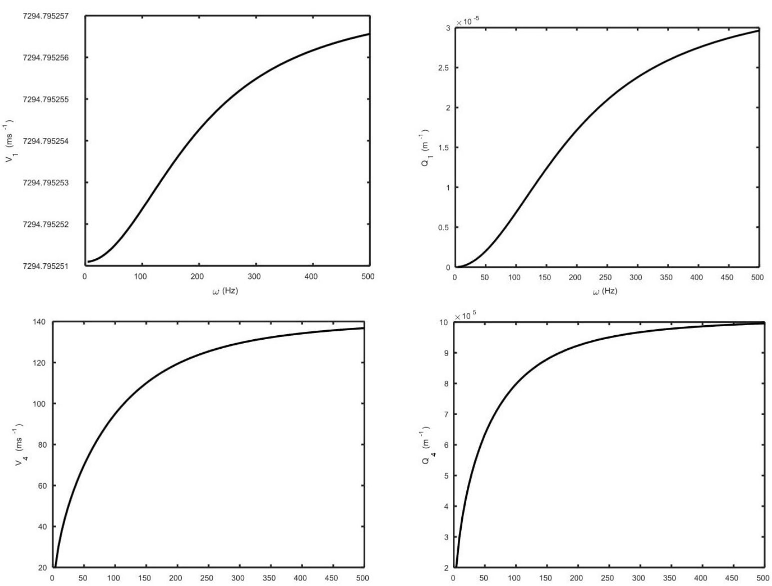

Fig. 8. The wave speeds and attenuation coefficients of the P_1 and P_4 waves against frequency ω (in absence of hyperstress and diffusion)

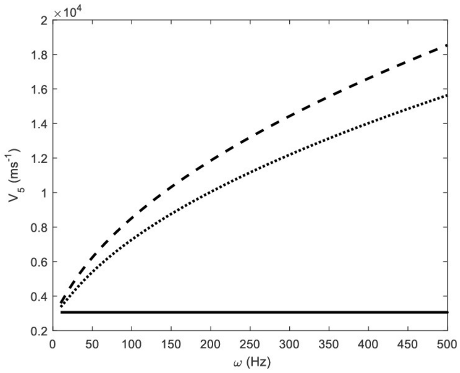

Fig. 9. The wave speed of the SV wave against frequency ω, when coefficient of hyperstress tensor ν_1 = 0(solid line), ν = 0.005 (dotted line) and ν = 0.05 (dashed line)

6. Conclusions

A problem on propagation of plane waves is considered in an isotropic, linear and homogeneous strain-gradient thermoelastic diffusive medium. The governing equations are formulated in context of Aoudai et al. [30] and Lord and Shulman [2] theories and are specialized in two-dimension. Two velocity equations for plane waves are derived, which suggest the propagation of four coupled longitudinal waves and a shear vertical wave. The numerical computations in MATLAB for a specific material (Magnesium) ensure the existence of these waves. The graphical illustrations show that the wave speeds and attenuation coefficients of these waves are affected by the circular frequency, measure constants of diffusion and thermo-diffusion, thermal and diffusion relaxation times and the coefficient of hyperstress tensor. Following some important observations are made from the present numerical results:

1) The speeds and attenuation coefficients of coupled longitudinal waves increase nonlinearly as the frequency increases.

2) For a given value of frequency, the P_1 and P_3 waves travel faster as the measure constant a increases and the P_1, P_2 and P_3 waves travel faster as the measure constant b increases.

3) The effect of thermal relaxation is observed on the speed and attenuation coefficient of P_4 wave only. The speed of P_4 wave becomes slow as the value of τ_0 increases at a given frequency. The effect of diffusion relaxation time is observed on the speeds of P_1 and P_3 waves and on attenuation coefficient of P_2 and P_3 waves. For a given frequency, the speed of P_1 becomes faster as the value of τ_1 increases, whereas the speed of P_3 waves becomes slower.

4) The effect of hyperstress is observed on the speeds and attenuations of P_1 and P_2 waves. For a given frequency, the speeds of P_1 and P_2 becomes faster as the value of ν increases.

5) The effects of measure constants of diffusion and thermo-diffusion, thermal and diffusion relaxation times and the coefficients of hyperstress tensor becomes more considerable as the frequency increases.

6) In absence of hyperstress or diffusion parameters, the number of coupled longitudinal waves will reduce to three. In absence of hyperstress coefficients, the P_2 wave will not propagate and the P_3 wave will not propagate in the absence of diffusion.

7) In presence of hyperstress, the speed of SV wave increases nonlinearly against the frequency. For a given frequency, the the SV wave travels faster as the value of ν increases. This effect of hyperstress on the SV wave is observed more considerable as the frequency increases.

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Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Author contributions

Baljeet Singh has formulated the governing equations for the material under study. Himanshu Singla solved these equations for plane waves. Himanshu Singla also illustrated the numerical results as per numerical plan suggested by Baljeet Singh. Himanshu Singla wrote the whole manuscript and author Baljeet Singh finalized the manuscript after giving inputs in each section of the manuscript.

Conflict of interest

The authors declare that they have no potential conflict of interest.

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