Equivalent stiffness calculation of composite hat stiffened laminate

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Abstract. The stiffness is the main reason that restricts the large-scale of composite ships, the basis to solve this problem is to solve the stiffness of composite ships' skeleton structure. Based on classical laminates theory, this paper calculated the equivalent in-plane stiffness and bending stiffness of composite hat stiffened laminate using equivalent stiffness principle. The hat stiffened laminate can be simplified and equivalent to orthotropic laminates using this solution idea, which solves the problem that it is difficult to obtain the analytical solutions of deformation, stress and strain of stiffened laminate. The research results can be used to calculate the stiffness of composite stiffened laminate and the local or global stiffness of the hull girder with such a skeleton structure.

Keywords: composite, laminate, hat stiffened, equivalent stiffness.

1. Introduction

The single-shell skeleton structure is a typical form of composite hull, which is composed of composite panels and stiffeners. The stiffener is used to strengthen the stiffness and strength of the panels. Common stiffeners include L-shaped, T-shaped, hat shaped, etc. [1]. Although the hap stiffener is complex in technology, its strength and stiffness performance are more excellent, which is a hot research topic in composite field. For example, Yetman et al. [2-3] have studied the debonding behavior between the hat stiffener and the panel, Mo and Ge [4] have conducted experimental research and structural optimization design on the failure of the hat stiffened plate under axial load, Wan et al. [5] have studied on the buckling and post-buckling behavior of hat-stiffened composite panels under axial compressive load.

However, there are few reports on the stiffness of hat stiffened composite laminate. Hu et al. [6] have carried out research on tensile and bending stiffness of hat stiffened laminated beams, but it is only applicable to beam structures and cannot be extended to plate frame structures. This is because it is very difficult to calculate this structure's stiffness using the classical laminated plate theory [7], and the approximate calculation can only be carried out by combining the equivalent stiffness method. For example, Zhou et al. [8] use the equivalent stiffness method to study the dynamic effects of composite sandwich plates with honeycomb structures, and Sun et al. [9] use the equivalent elastic model to study the elastic properties and lateral stiffness of multi-ribbed walls, and Wang et al. [10] used this method to study the buckling behavior of composite stiffened box structures. These studies all show that the equivalent stiffness is a good method to develop complex composite structures. Qiu and Li [11] calculated the equivalent stiffness of typical elements of composite hat stiffened laminate based on the classical laminated plate theory and the equivalent stiffness method, Wu et al. [12] verified accuracy of this method by experiment and simulation. On this basis, this paper further calculated the equivalent in-plane stiffness and bending stiffness of the entire hat stiffened laminated plate structure.

2. Solution idea of equivalent stiffness on composite hat stiffened laminate

In Fig. 1, the composite hat stiffened laminate with a length of a_p, a width of b_p, an x-axis

along the stiffener direction, and a y-axis perpendicular to the stiffener direction. The cross-section dimension is shown in Fig. 2, the spacing of stiffeners is d_s, the width of plate edge is d_p, and the stiffeners' number is n. In the following formula, subscript e represents stiffened laminated element, and b represents edge panel.

The stiffness is a kind of ability to resist external loads. There is a linear relationship between deformation and load when the structure is stressed within the elastic range, as long as the deformation coordination between its constituent elements is maintained, the forces acting on it can be superimposed. The solution idea of this paper is that divide the hat stiffened laminate into n stiffened plate elements and 2 edge plates, as shown in Fig. 1, calculate their stiffness respectively, and then obtain the stiffness of the stiffened laminate by superposition. The stiffness of the hat stiffened laminated element has been solved in reference [11].
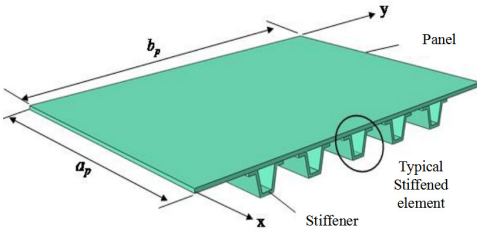

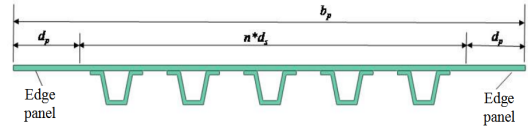**Fig. 1.** Schematic of composite stiffened laminate

Fig. 2. Section schematic of composite laminate with hat stiffeners

3. Calculation of equivalent in-plane stiffness of hat stiffened laminate

In the following formulas, the meaning and calculation formula of $(ET)_{xe}$, $(ET)_{ye}$, μ_{xye} and $(GT)_{xye}$ can refer to reference [11].

3.1. Longitudinal in-plane stiffness

The longitudinal in-plane stiffness $(ET)_x$ of stiffened laminate is equal to the sum of the longitudinal in-plane stiffness of n typical stiffened elements and 2 edge panels, namely:

$$(ET)_x = n(ET)_{xe} + 2(ET)_{xb} = n(ET)_{xe} + 2\,d_p/a_{11p}. \tag{1}$$

3.2. Transverse in-plane stiffness

When calculating the transverse in-plane stiffness of stiffened laminate, which is decomposed into $n + 2$ segments with n typical stiffened elements in the middle and 2 edge panels on both sides. When bearing the axial force T_y per unit length, the longitudinal deformation of the stiffened laminate is the sum of the deformation of each segment, namely:

$$T_y b_p/(ET)_{y'} = \sum T_y b_i/(ET)_{yi} \;\Rightarrow\; (ET)_{y'} = {b_p}\Big/{\sum b_i/(ET)_{yi}}, \tag{2}$$

where, $(ET)_{y'}$ is the transverse in-plane stiffness of the stiffened laminate along the x direction per unit length, b_i is the width of each segment, and $(ET)_{yi}$ is the transverse in-plane stiffness of each segment. The transverse in-plane stiffness of the edge panels is calculated as follows:

$$(ET)_{yb} = E_{yp} t_p = {1}/{a_{22p}}. \tag{3}$$

Then the transverse in-plane stiffness of the stiffened laminate:

$$(ET)_y = a_p(ET)_{y'} = a_pb_p/\left[\frac{2d_p}{(ET)_{yb}} + \frac{nd_s}{(ET)_{ye}}\right]. \tag{4}$$

3.3. In-plane Poisson's ratio

When bearing axial force T_x, the strain of the stiffened laminate along the x direction is ε_x^0, and the strain along the y direction is ε_y^0, then the in-plane Poisson's ratio μ_{xym} is defined as $-\varepsilon_y^0/\varepsilon_x^0$. The deformation of stiffened laminated plate along the y direction is the sum of deformation of each segment, namely:

$$-\mu_{xym}\varepsilon_x^0 b_p = -\sum \mu_{xyi}\varepsilon_x^0 b_i \quad \Rightarrow \quad \mu_{xym} = \frac{1}{b_p}\sum \mu_{xyi} b_i, \tag{5}$$

where, μ_{xyi} is the in-plane Poisson's ratio of each segment, b_i is the width of each segment. The calculation formula of edge panels is as follows:

$$\mu_{xyb} = \mu_{xyp} = -\frac{a_{12p}}{a_{11p}}. \tag{6}$$

Then the in-plane Poisson's ratio of stiffened laminate:

$$\mu_{xym} = \frac{1}{b_p}\left(nd_s\mu_{xye} + 2d_p\mu_{xyb}\right). \tag{7}$$

3.4. In-plane shear stiffness

Assume that the stiffened laminate bears the shear force along the y direction, the shear strain of the stiffened laminate is γ_{xy}, the shear force of each segment is T_{xyi}, and the shear strain of each segment is γ_{xyi}, in order to maintain deformation coordination, the shear strain of each segment should be the same as that of the stiffened laminate, namely:

$$\gamma_{xy} = \gamma_{xyi} \quad \Rightarrow \quad T_{xy}/(GT)_{xy} = T_{xyi}/(GT)_{xyi}, \tag{8}$$

where, $(GT)_{xy}$ is the in-plane shear stiffness of stiffened laminate, and $(GT)_{xyi}$ is the in-plane shear stiffness of each segment.

In addition, $T_{xy} = \sum T_{xyi}$, the stiffness of stiffened laminate can be obtained:

$$(GT)_{xy} = \sum (GT)_{xyi} = n(GT)_{xye} + 2(GT)_{xyb} = n(GT)_{xye} + \frac{2d_p}{a_{66p}}. \tag{9}$$

4. Calculation of equivalent bending stiffness of composite stiffened laminate

In the following formulas, the meaning and calculation formula of $(EI)_{xe}$, $(EI)_{ye}$, μ_{xye} and $(GJ)_{xye}$ can refer to reference [11].

4.1. Longitudinal bending stiffness

The longitudinal bending stiffness of stiffened laminate is equal to the sum of that of n stiffened elements and 2 the edge panels, namely:

$$(EI)_x = n(EI)_{xe} + 2(EI)_{xb} = n(EI)_{xe} + 2d_p \left[1 + 12\left(\frac{t_p/2 - z_0}{t_p}\right)^2\right]\Big/ d_{11p}. \tag{10}$$

4.2. Transverse bending stiffness

When calculating the transverse bending stiffness of stiffened laminate, which is decomposed into $n + 2$ segments. Under the bending moment, the corners φ at both ends of the stiffened laminate should be the sum of the corners φ_i of each segment in y direction, namely:

$$\varphi = \sum \varphi_i \Rightarrow \frac{T_y b_p}{(EI)_{y'}} = \sum \frac{T_y b_i}{(EI)_{yi}} \Rightarrow (EI)_{y'} = b_p\Big/\sum \frac{b_i}{(EI)_{yi}}, \tag{11}$$

where, $(EI)_{y'}$ is the transverse bending stiffness per unit width of the stiffened laminate along the x direction, and $(EI)_{yi}$ is the transverse bending stiffness per unit width of each segment along the x direction. The calculation formula of the transverse in-plane stiffness of the edge panels is as follows:

$$(EI)_{yb} = E_{yp} t_p^3/12 = 1/d_{22p}. \tag{12}$$

Then the transverse bending stiffness of the stiffened laminate:

$$(EI)_y = a_p(EI)_{y'} = a_p b_p\Big/\left[\frac{2d_p}{(ET)_{yb}} + \frac{nd_s}{(ET)_{ye}}\right]. \tag{13}$$

4.3. Bending Poisson's ratio

When bearing axial force T_x, assume that the curvature of the bending deformation of stiffened laminate in the x direction is κ_x. The curvature of bending deformation in y direction is κ_y. Then the bending Poisson's ratio of stiffened laminate μ_{xyb} is defined as $-\kappa_y/\kappa_x$. The bending angle φ at both ends of the stiffened laminate along the y direction is the sum of the angles φ_i of each segment, namely:

$$\varphi = \sum \varphi_i \Rightarrow -\mu_{xyb}\kappa_x b_p = -\sum \mu_{xyi}\kappa_x b_i \Rightarrow \mu_{xyb} = \frac{1}{b_p}\sum \mu_{xyi} b_i, \tag{14}$$

where, μ_{xyi} is the bending Poisson's ratio of each segment and b_i is the width of each segment. The bending Poisson's ratio formula of the edge panels is as follows:

$$\mu_{xyb} = \mu_{xyp} = -d_{12p}/d_{11p}. \tag{15}$$

Then the bending Poisson's ratio of stiffened laminate:

$$\mu_{xyb} = \frac{1}{b_p}\left(nd_s\mu_{xye} + 2d_p\mu_{xyb}\right). \tag{16}$$

4.4. Torsional stiffness

When bearing torque T_{xy}, the torsion ratio of stiffened laminate is φ', the torque of each segment is T_{xyi}, and the torsion ratio is φ_i'. The torque of the stiffened laminate is the sum of the torque of each segment, and the torque ratio is equal to that of each segment, namely:

$$\varphi' = \varphi'_i \Rightarrow T_{xy}/(GJ)_{xy} = T_{xyi}/(GJ)_{xyi} \Rightarrow \sum T_{xyi}/(GJ)_{xy} = T_{xyi}/(GJ)_{xyi}. \tag{17}$$

Simplify the above formula to obtain the torsional stiffness of stiffened laminate:

$$(GJ)_{xy} = \sum (GJ)_{xyi} = n(GJ)_{xye} + 2(GJ)_{xyb} = n(GJ)_{xye} + 8\, d_p/d_{66p}. \tag{18}$$

5. Calculation of equivalent stiffness coefficient

After the calculation of equivalent stiffness, composite stiffened laminate can be equivalent to orthotropic composite laminate. The stiffness coefficient can be calculated according to the relationship between the mid-plane strain, curvature, internal force and internal moment of the composite laminate.

5.1. Calculation of in-plane stiffness coefficient

The relationship between in-plane strain and in-plane internal force of orthotropic composite laminate:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}, \tag{19}$$

where, $\varepsilon_{x,y}^0$ is the in-plane strain of the laminate, $N_{x,y}$ is the in-plane internal force, a_{ij} is the in-plane flexibility coefficient.

When the composite stiffened laminate bears in-plane internal force N_x, the following formula can be obtained according to the Eq. (19):

$$\varepsilon_x^0 = a_{11}N_x \Rightarrow P_x/(ET)_x = a_{11}N_x \Rightarrow N_x b_p/(ET)_x = a_{11}N_x, \quad \varepsilon_y^0 = a_{12}N_x. \tag{20}$$

Simplify the above equation to obtain:

$$a_{11} = b_p/(ET)_x, \quad \mu_{xym} = -\varepsilon_y^0/\varepsilon_x^0 = -a_{12}/a_{11}. \tag{21}$$

In the same way, we can get:

$$a_{22} = a_p/(ET)_y, \quad a_{66} = b_p/(GT)_{xy}. \tag{22}$$

In-plane stiffness matrix of composite stiffened laminate is the transpose of in-plane flexibility matrix.

5.2. Calculation of bending stiffness coefficient

The relationship between curvature and internal moment of orthotropic laminate:

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & 0 \\ d_{12} & d_{22} & 0 \\ 0 & 0 & d_{66} \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}, \tag{23}$$

where, $\kappa_{x,y}$ is the curvature of the composite laminate, $M_{x,y}$ is the internal moment, d_{ij} is the bending flexibility coefficient.

In the same way as section 5.1, we can get:

$$d_{11} = \frac{b_p}{(EI)_x}, \quad \frac{d_{12}}{d_{11}} = \frac{\kappa_y}{\kappa_x} = -\mu_{xyb}, \quad d_{22} = \frac{a_p}{(EI)_y}, \quad d_{66} = \frac{2b_p}{(GJ)_{xy}}. \tag{24}$$

Bending stiffness matrix of composite stiffened laminate is the transpose of bending flexibility matrix.

6. Conclusions

The research on the stiffness characteristics of composite stiffened laminate is the basis of that of composite hull. Based on the classical laminated plate theory, the equivalent in-plane stiffness and bending stiffness of composite hat stiffened laminate are obtained using the stiffness equivalence method.

Using the solution idea in this paper, the arbitrary composite stiffened laminate can be transformed into orthotropic laminate. Under the condition of the specific boundary conditions and loads, the deformation, internal force and bending moment of orthotropic laminated plates can be solved first, and then the deformation, stress and strain of each element of stiffened laminate can be further solved according to the classical laminated plate theory. The equivalent stiffness of stiffened laminate calculated by this method refers to the average distributed stiffness. Therefore, the more intensive the number of stiffeners is, the more accurate the calculation results will be.

The research results in this paper can be used to calculate the stiffness of composite stiffened laminate, and the local or global stiffness of the hull girder with such a skeleton structure.

Acknowledgements

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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