

The vibration of a thermoelastic nanobeam due to thermo-electrical effect of graphene nano-strip under Green-Naghdi type-II model

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Abstract. In this paper, a new analysis of thermoelastic, homogeneous, and isotropic nanobeams has been constructed in the context of the Green-Naghdi type-II heat conduction law. The first end of the nanobeam is based on a graphene strip, which contains an electrical current with a small voltage which is a novel application. Under simply supported boundary conditions with fixed side ratios, the nanobeam has been thermally loaded with a heat source due to the thermal effect of the electrical current. On the time variable, the Laplace transform technique was utilized to solve the governing differential equations. The solutions were computed in the Laplace transform's domain. Tzou's approximation technique based on an iteration formula was used to calculate the Laplace transform inversions numerically. The numerical findings for various values of the electrical voltage and electrical resistivity of the graphene nano-strip have been displayed using graphs illustrating distinct scenarios. All the nanobeam's researched functionalities have been reported to be influenced by electrical voltage and electrical resistivity. The electrical voltage and resistivity of a graphene nano-strip might be utilized as a tuner to regulate the nanobeam's vibration and energy.

Keywords: vibration, thermoelasticity, nanobeam, thermo-electrical effect, graphene, electrical voltage, electrical resistivity.

1. Introduction

Thermoelasticity is a kind of heat transfer based on Fourier's law of heat transfer, which consists of two partial differential equations: one for motion and the other for energy conservation [1-4]. Lord and Shulman modified Fourier's rule of heat conduction in the case of an isotropic body by including the relaxation time in the equation [5]. The law of heat conduction (Cattaneo's heat conduction, also known as non-law Fourier's of heat conduction) has been revised to incorporate both the heat flux and its time derivative, and it has been proposed to replace the traditional Fourier's law of heat conduction. The heat equation is hyperbolic in this theory, avoiding the problem of infinite propagation [6-11].

Green and Naghdi (G-N) proposed three different theories of thermoelasticity based on entropy equality rather than entropy inequality [9-11]. In each theory, the vector of the heat flux vector has a distinct constitutive consideration. As a result, Green-Naghdi type-I, Green-Naghdi type-II, and Green-Naghdi type-III were developed. The classical thermoelasticity system is the same as a type-I theory in linearized form. There is no energy dissipation in the type-II theory; instead, it is regarded as a specific instance of type-III, which contains energy dissipation in its definition [12-17].

Because it creates the most vibration, the vibration nanobeam resonator is the most important

of the nano/micro-beam resonators. Alghamdi investigated thermoelastic damping vibration with gaps in a beam resonator using dual-phase-lagging generalised thermoelasticity theory [1]. On the micro/nanoscale, Sharma and Grover explored the transverse vibrations of homogeneous isotropic and thermoelastic thin beam resonators that contained voids [18]. Sun and Saka explored the thermoelastic damping vibration of circular plate resonators using out-of-plane microplates [19]. The vibration and heat transmission mechanisms of nanobeams have been studied by a significant number of writers [20-24]. Eman and Youssef studied the vibration of gold nanobeam due to thermal shock [21]. Kidawa investigated the effects of internal and external damping on transverse vibrations of a beam induced by a moving heat source using Green functions and their characteristics in his research [23]. A rectangular nanobeam with simple support was subjected to a thermal shock that was spread along its span, and Boley investigated the vibrations of the beam [22]. Manolis and Beskos investigated the elastic dynamic response of beam constructions to heat loading by using a computer approach study [24].

In mechanics, the study of viscothermoelastic materials, often known as relaxation characteristics, is becoming increasingly relevant. Biot has published on viscothermoelasticity theory and thermodynamic vibrational principles, among other topics [25, 26]. Drozdov devised a constitutive model for the viscothermoelasticity behaviour of polymers when they are exposed to finite stresses [27]. To study the relaxing effects of volume-features of viscoelastic materials thermoelasticity, Ezzat and El-Karmany used a novel thermo-viscoelasticity model for isotropic media devised by the authors [28]. Carcione et al. devised a numerical technique for wave simulations in elastic media based on the Kelvin-Voigt mechanical model [29]. Grover studied transverse vibrations in micro-scale viscothermoelastic beam resonators [30-32]. A homogeneous thermoelastic thin beam with vacancies on the micro/nano-scale was studied by Sharma and Grover in terms of closed-form equations for transverse vibrations [18]. Grover and Seth studied visothermal elastic micro-scale beam resonators using a dual-phase lagging model [33].

Since the groundbreaking observations of Novoselov et al., graphene has continued to pique the interest of scientists around the world [34]. Single-layer graphene has exciting electrical features, including ballistic transport, a quantum Hall effect at ambient temperature, and a size-dependent band gap, all of which have the potential to allow new types of nanoscale electronic devices and sensors, such as transistors [35].

Green-Naghdi type-II heat conduction law has been used for the first time in this study to analyse thermoelastic, homogeneous, and isotropic nanobeams, which have a graphene strip as the initial end of the nanobeam linked to a low voltage electrical current. The nanobeam will become thermally loaded with a heat source as a result of the electrical current's thermal action on the low-voltage nano-graphene strip. The microbeam was electrically isolated using an electrical insulator with high thermal conductivity.

2. Problem formulation

We consider the flexural deflections are very small for a thin thermoelastic nanobeam of thickness $h(-h/2 \leq z \leq h/2)$, width $b(-b/2 \leq y \leq b/2)$, and length $\ell(0 \leq x \leq \ell)$.

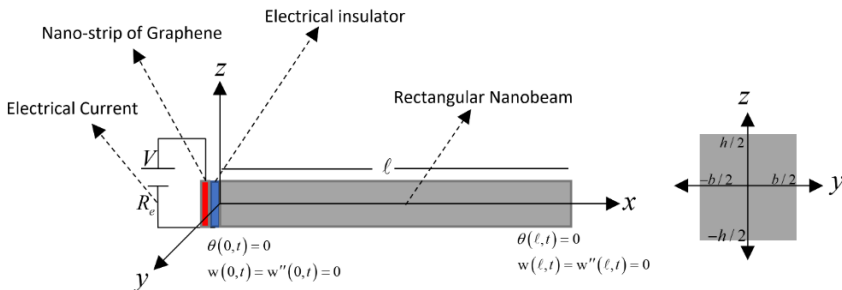


Fig. 1. Rectangular thermoelastic nanobeam based on an electrical insulator and a nano-strip of graphene

The x , y , and z -axes are defined through the longitudinal, width, and thickness directions of the beam as in Fig. 1.

In an equilibrium state, the nanobeam has no damping, unstressed, unstrained, and the reference temperature is T_0 everywhere [6].

Beam bending will use the Euler-Bernoulli equation, which states that any plane cross-section that is perpendicular to the beam's axis (neutral surface) will stay so [30].

Then, the displacement components will take the forms [30, 32, 33, 36]:

$$u(x, y, z, t) = -z \frac{\partial w(x, t)}{\partial x}, \quad v(x, y, z, t) = 0, \quad w(x, y, z, t) = w(x, t). \quad (1)$$

The flexural moment of the cross-section and equation of motion is given by [30, 32, 33, 36]:

$$M(x, t) = \beta M_T(x, t) + (\lambda + 2\mu)I \frac{\partial^2 w(x, t)}{\partial x^2}, \quad (2)$$

where the equation of motion takes the form:

$$\frac{\partial^2 M(x, t)}{\partial x^2} + \rho A \ddot{w}(x, t) = 0. \quad (3)$$

The thermal moment M_T of the nanobeam about the x -axis is given by [30-33, 36, 37]:

$$M_T(x, t) = \int_{-h/2}^{h/2} bT(x, z, t)zdz, \quad (4)$$

where $I = bh^3/12$ gives the moment of inertia of the cross-section around the x -axis, ρ is the density of the material, $A = hb$ is the cross-section area, $\beta = (3\lambda + 2\mu)\alpha_T$, λ and μ are the elastic constant, and α_T is the coefficient of linear thermal expansion.

Thus, the equation of motion which gives thermally induced lateral vibrations of the nanobeam takes the form [30]:

$$\beta \frac{\partial^2 M_T(x, t)}{\partial x^2} + (\lambda + 2\mu)I \frac{\partial^4 w(x, t)}{\partial x^4} + \rho A \ddot{w}(x, t) = 0. \quad (5)$$

The Green-Naghdi type-II heat conduction equation takes the form [30]:

$$\frac{\partial^2 T(x, z, t)}{\partial x^2} + \frac{\partial^2 T(x, z, t)}{\partial z^2} = \frac{\rho C_v}{K^*} \dot{T}(x, z, t) + \frac{\beta T_0}{K^*} \ddot{e}(x, z, t) - \frac{Q(x, z, t)}{K^*}, \quad (6)$$

where $K^* = (\lambda + 2\mu)C_v/4$ which is called the Green-Naghdi theorem parameter, C_v is the specific heat under constant strain, and Q is the external heat source.

The cubical dilatation (volumetric strain) has the form:

$$e(x, z, t) = \frac{\partial u(x, z, t)}{\partial x} + \frac{\partial v(x, z, t)}{\partial y} + \frac{\partial w(x, z, t)}{\partial z}. \quad (7)$$

Thus, from the Eq. (1) we have:

$$e(x, z, t) = -z \frac{\partial^2 w(x, t)}{\partial x^2}. \quad (8)$$

Then, we obtain:

$$\sigma_{xx}(x, z, t) = (\lambda + 2\mu)e(x, z, t) - \beta(T(x, z, t) - T_0). \quad (9)$$

The upper and lower surfaces of the beam do not have heat transfer, so $\left. \frac{\partial T(x, z, t)}{\partial z} \right|_{z=\pm \frac{h}{2}} = 0$.

Hence, for a nanobeam, we can postulate the temperature depends on a “sin(pz)” function through the thickness direction of the beam, where $p = \pi/h$, which gives [38]:

$$\theta(x, z, t) = T(x, z, t) - T_0 = \vartheta(x, t)\sin(pz), \quad (10)$$

and

$$Q(x, z, t) = q(x, t)\sin(pz), \quad (11)$$

where $\theta(x, z, t)$ is devoted to the temperature increment.

Hence, from the Eqs. (4), (5), and (10), we obtain:

$$\frac{12\beta}{h^3(\lambda + 2\mu)} \frac{\partial^2 \vartheta(x, t)}{\partial x^2} \int_{-h/2}^{h/2} \sin(pz)z dz + \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{12\rho}{h^2(\lambda + 2\mu)} \ddot{w}(x, t) = 0. \quad (12)$$

By using Eqs. (6) and (10), we get:

$$\left(\frac{\partial^2 \vartheta(x, t)}{\partial x^2} - p^2 \vartheta(x, t) \right) \sin(pz) = \varepsilon \ddot{\vartheta}(x, t) \sin(pz) - \frac{T_0 \beta}{K^*} z \frac{\partial^2 \ddot{w}(x, t)}{\partial x^2} - \frac{q(x, t)}{K^*} \sin(pz), \quad (13)$$

where $\varepsilon = \rho C_v / K^*$ which gives the thermal viscosity parameter of the material.

After executing the integrations, the Eq. (12) has the form:

$$\frac{24\beta}{h \pi^2(\lambda + 2\mu)} \frac{\partial^2 \vartheta(x, t)}{\partial x^2} + \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{12\rho}{h^2(\lambda + 2\mu)} \ddot{w}(x, t) = 0. \quad (14)$$

After re-write the Eq. (13), we obtain:

$$\frac{\partial^2 \vartheta(x, t)}{\partial x^2} - p^2 \vartheta(x, t) = \varepsilon \ddot{\vartheta}(x, t) - \frac{T_0 \beta}{K^*} \frac{z}{\sin(pz)} \frac{\partial^2 \ddot{w}(x, t)}{\partial x^2} - \frac{q(x, t)}{K^*}. \quad (15)$$

The Eq. (9) takes the form:

$$\sigma_{xx} = (\lambda + 2\mu)e - \beta \vartheta \sin(pz). \quad (16)$$

The following non-dimensional variables will be used [19]:

$$\begin{aligned} (x', w', h', \ell', z') &= \varepsilon c_0(x, w, h, \ell, z), & (t', \tau') &= \varepsilon c_0^2(t, \tau), \\ \sigma' &= \frac{\sigma}{\lambda + 2\mu}, & \vartheta' &= \frac{\vartheta}{T_0}, & q' &= \frac{q}{T_0 K^* \varepsilon^2 c_0^3}, & c_0^2 &= \frac{\lambda + 2\mu}{\rho}. \end{aligned} \quad (17)$$

Then, we have:

$$\frac{\partial^4 w(x, t)}{\partial x^4} + \varepsilon_1 \ddot{w}(x, t) + \varepsilon_2 \frac{\partial^2 \vartheta(x, t)}{\partial x^2} = 0, \quad (18)$$

$$\frac{\partial^2 \vartheta(x, t)}{\partial x^2} - \varepsilon_3 \vartheta(x, t) = \varepsilon_4 \ddot{\vartheta}(x, t) - \varepsilon_5 \frac{\partial^2 \dot{w}(x, t)}{\partial x^2} - \varepsilon_6 q(x, t), \quad (19)$$

$$\sigma_{xx} = e - \varepsilon_7 \theta, \quad (20)$$

where:

$$\varepsilon_1 = \frac{12}{h^2}, \quad \varepsilon_2 = \frac{24\beta T_0}{h\pi^2(\lambda + 2\mu)}, \quad \varepsilon_3 = p^2, \quad \varepsilon_4 = \varepsilon c_0^2,$$

$$\varepsilon_5 = \frac{c_0^2 \beta z}{K^* \sin(pz)}, \quad \varepsilon_6 = c_0, \quad \varepsilon_7 = \frac{\beta T_0}{(\lambda + 2\mu)}.$$

(The primes have been dropped for convenience).

Laplace transform will be applied with the following definition:

$$\bar{f}(x, s) = \int_0^{\infty} f(x, t) e^{-st} dt, \quad (21)$$

where the inverse of the Laplace transform takes the form:

$$L^{-1}(\bar{f}(s)) = f(t) \approx \frac{e^{vt}}{t} \left[\frac{1}{2} \bar{f}(v) + \operatorname{Re} \sum_{n=1}^N (-1)^n \bar{f}\left(v + \frac{in\pi}{t}\right) \right], \quad (22)$$

where Re means the real part, while i is the imaginary number unit, for faster convergence, many experiments confirmed that the value of the parameter “ v ” could satisfy the relation $vt \approx 4.7$ [39].

Then, the Eqs. (18)-(20) will take the following forms:

$$\frac{d^4 \bar{w}}{dx^4} + \varepsilon_1 s^2 \bar{w} + \varepsilon_2 \frac{d^2 \bar{\vartheta}}{dx^2} = 0, \quad (23)$$

$$\frac{d^2 \bar{\vartheta}}{dx^2} - \varepsilon_3 \bar{\vartheta} = \varepsilon_4 s^2 \bar{\vartheta} - \varepsilon_5 s^2 \frac{d^2 \bar{w}}{dx^2} - \varepsilon_6 \bar{q}, \quad (24)$$

$$\bar{\sigma}_{xx} = \bar{e} - \varepsilon_7 \bar{\theta}, \quad (25)$$

$$\bar{e} = -z \frac{d^2 \bar{w}}{dx^2}. \quad (26)$$

Consider that the beam is subjected to a particular heating source that comes from the thermal effect due to connection with a simple circuit with electrical resistance R_e (Ω) and electric voltage V (V). Then, Joule’s equation of electrical heating is given by:

$$q = \frac{V^2}{R_e} t. \quad (27)$$

Then, after using Laplace transform, we have:

$$\bar{q} = \frac{V^2}{R_e s^2}, \quad (28)$$

which gives:

$$\frac{d^2 \bar{\vartheta}}{dx^2} - \varepsilon_3 \bar{\vartheta} = \varepsilon_4 s^2 \bar{\vartheta} - \varepsilon_5 s^2 \frac{d^2 \bar{w}}{dx^2} - \frac{\varepsilon_6 V^2}{s^2 R_e}. \quad (29)$$

We re-write the Eqs (23) and (24) obtain the forms:

$$(D^4 + \varepsilon_1 s^2)\bar{w} + \varepsilon_2 D^2 \bar{\vartheta} = 0, \tag{30}$$

and

$$-\varepsilon_5 s^2 D^2 \bar{w} + (D^2 - (\varepsilon_3 + \varepsilon_4 s^2))\bar{\vartheta} = -\frac{\varepsilon_6 V^2}{s^2 R_e}, \tag{31}$$

where $D^r = d^r/dx^r$.

By eliminating the Eqs. (30) and (31), we obtain:

$$\{D^6 - LD^4 + MD^2 - N\}\bar{w} = 0, \tag{32}$$

and

$$\{D^6 - LD^4 + MD^2 - N\}\bar{\vartheta} = -\psi, \tag{33}$$

where $L = \varepsilon_3 + \varepsilon_4 s^2 - \varepsilon_5 \varepsilon_2 s^2$, $M = \varepsilon_1 s^2$, $N = \varepsilon_1 s^2 (\varepsilon_3 + \varepsilon_4 s^2)$, and $\psi = \varepsilon_1 \varepsilon_6 V^2 / R_e$.

The general solution of the Eq. (32) as follows:

$$\bar{w} = \sum_{j=1}^3 A_j \sinh(k_j(\ell - x)). \tag{34}$$

The general solution of the Eq. (33) as follows:

$$\bar{\vartheta} = \frac{\psi}{N} + \sum_{j=1}^3 B_j \sinh(k_j(\ell - x)). \tag{35}$$

To get the relation between the parameters A_j and B_j , we use the relation in Eq. (30), which gives:

$$(K_j^4 + \varepsilon_1 s^2)A_j + \varepsilon_2 K_j^2 B_j = 0, \quad j = 1, 2, 3. \tag{36}$$

Then we have:

$$\bar{\vartheta} = \frac{\psi}{N} - \frac{1}{\varepsilon_2} \sum_{j=1}^3 \frac{(K_j^4 + \varepsilon_1 s^2)}{K_j^2} A_j \sinh(k_j(\ell - x)). \tag{37}$$

The boundary conditions:

$$\sum_{j=1}^3 A_j \sinh(k_j \ell) = 0, \tag{38}$$

$$\sum_{j=1}^3 k_j^2 A_j \sinh(k_j \ell) = 0, \tag{39}$$

$$\sum_{j=1}^3 \frac{(K_j^4 + \varepsilon_1 s^2)}{K_j^2} A_j \sinh(k_j \ell) = \frac{\varepsilon_2 \psi}{N}. \tag{40}$$

By solving the Eqs. (38)-(40), we get the parameters A_1, A_2, A_3 as follows:

$$A_1 = \frac{\varepsilon_2(\varepsilon_3 + \varepsilon_4 s^2)\psi}{(k_1^2 - k_2^2)(k_1^2 - k_3^2)\sinh(k_1 \ell)}, \quad A_2 = \frac{\varepsilon_2(\varepsilon_3 + \varepsilon_4 s^2)\psi}{(k_2^2 - k_1^2)(k_2^2 - k_3^2)\sinh(k_2 \ell)},$$

$$A_3 = \frac{\varepsilon_2(\varepsilon_3 + \varepsilon_4 s^2)\psi}{(k_3^2 - k_1^2)(k_3^2 - k_2^2)\sinh(k_3 \ell)}.$$

That completes the solution of the Laplace transform domain.

The lateral deflection function is as follows:

$$\bar{w}(x, s) = \varepsilon_2(\varepsilon_3 + \varepsilon_4 s^2)\psi \left[\frac{\sinh(k_1(\ell - x))}{(k_1^2 - k_2^2)(k_1^2 - k_3^2)\sinh(k_1 \ell)} + \frac{\sinh(k_2(\ell - x))}{(k_2^2 - k_1^2)(k_2^2 - k_3^2)\sinh(k_2 \ell)} + \frac{\sinh(k_3(\ell - x))}{(k_3^2 - k_1^2)(k_3^2 - k_2^2)\sinh(k_3 \ell)} \right], \quad (41)$$

and the temperature increment function is as follows:

$$\bar{\theta}(x, s) = \frac{\psi \sin(pz)}{\varepsilon_1 s^2(\varepsilon_3 + \varepsilon_4 s^2)} - (\varepsilon_3 + \varepsilon_4 s^2)\psi \sin(pz) \left[\frac{(K_1^4 + \varepsilon_1 s^2)\sinh(k_1(\ell - x))}{K_1^2(k_1^2 - k_2^2)(k_1^2 - k_3^2)\sinh(k_1 \ell)} + \frac{(K_2^4 + \varepsilon_1 s^2)\sinh(k_2(\ell - x))}{K_2^2(k_2^2 - k_1^2)(k_2^2 - k_3^2)\sinh(k_2 \ell)} + \frac{(K_3^4 + \varepsilon_1 s^2)\sinh(k_3(\ell - x))}{K_3^2(k_3^2 - k_1^2)(k_3^2 - k_2^2)\sinh(k_3 \ell)} \right]. \quad (42)$$

From Eqs. (26) and (41), we obtain the deformation in the form:

$$e(x, z, s) = -z\varepsilon_2(\varepsilon_3 + \varepsilon_4 s^2)\psi \left[\frac{k_1^2 \sinh(k_1(\ell - x))}{(k_1^2 - k_2^2)(k_1^2 - k_3^2)\sinh(k_1 \ell)} + \frac{k_2^2 \sinh(k_2(\ell - x))}{(k_2^2 - k_1^2)(k_2^2 - k_3^2)\sinh(k_2 \ell)} + \frac{k_3^2 \sinh(k_3(\ell - x))}{(k_3^2 - k_1^2)(k_3^2 - k_2^2)\sinh(k_3 \ell)} \right]. \quad (43)$$

The strain-energy density function through the nanobeam is given by [40]:

$$\varpi(x, z, t) = \sum_{i,j}^3 \frac{1}{2} \sigma_{ij}(x, z, t) e_{ij}(x, z, t) = \frac{1}{2} \sigma(x, z, t) e(x, z, t). \quad (44)$$

3. Numerical results

Copper is taken as the thermoelastic material, so the following values of the different physical constants have been used [32]: $\alpha_T = 1.78 \cdot 10^{-5} \text{ K}^{-1}$, $\rho = 8954 \text{ kg m}^{-3}$, $T_0 = 300 \text{ K}$, $C_v = 383.1 \text{ J kg}^{-1} \text{ K}^{-1}$, $\lambda = 77.6 \times 10^9 \text{ N m}^{-2}$, $\mu = 38.6 \times 10^9 \text{ N m}^{-2}$.

The resistivity of the graphene in the nanoscale has values $R_e = 500 \text{ } \Omega$ [35].

The aspect ratios of the nanobeam are fixed as $\ell/h = 8$ and $b = h/2$. We will take the range of the nanobeam length $\ell(1 - 100) \times 10^{-12} \text{ m}$, and the original time t , and the relaxation time τ_0 of order 10^{-12} sec and 10^{-14} sec , respectively.

The figures were prepared by using the dimensionless variables for nanobeam length $\ell = 1.0$, $\theta_0 = 1.0$, $z = h/4$, and $t = 1.0$.

The numerical results of the problem have been illustrated in two groups of figures; the first group shows the distributions of the vibration (lateral deflection), cubical deformation, temperature increment, stress, and strain-energy density when the value of the electrical resistance of the nano-strip of graphene is constant and takes the value $R_e = 500 \text{ } \Omega$ and for three different

values of the electrical voltage $V = (1.0, 1.1, 1.2)$ V, while the second group represents the distributions of the same functions when the value of the electrical voltage of nano-strip of graphene is constant and takes the value $V = 1.0$ V and for three different values of the electrical resistance $R_e = (500, 550, 600)$ Ω .

Figs. 2-6 show that the value of the electrical voltage has considerable impacts on the distributions of the temperature increment, vibration (lateral deflection), cubical deformation, stress, and strain energy density. The increase in the value of the electrical voltage leads to an increase in the temperature increment and the absolute values of the vibration (lateral deflection), cubical deformation, stress, and strain energy density.

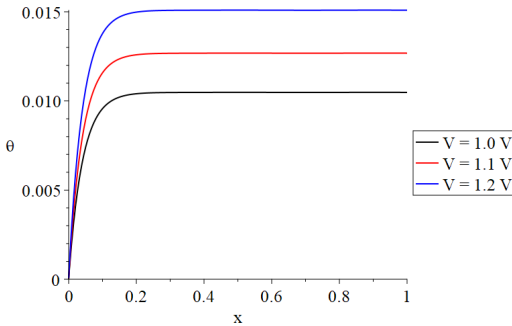


Fig. 2. The temperature increment distribution with variance voltage and constant resistance

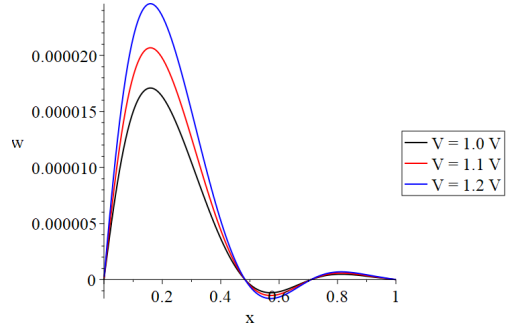


Fig. 3. The vibration distribution with variance voltage and constant resistance

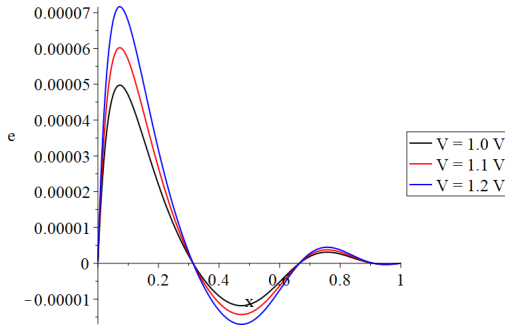


Fig. 4. The strain distribution with variance voltage and constant resistance

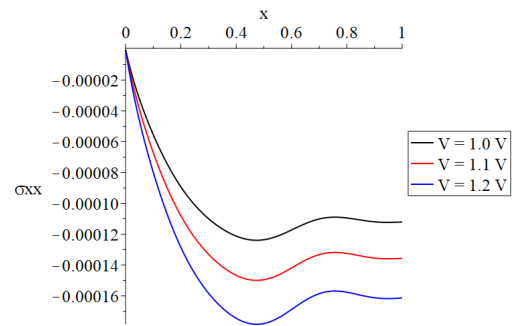


Fig. 5. The stress distribution with variance voltage and constant resistance

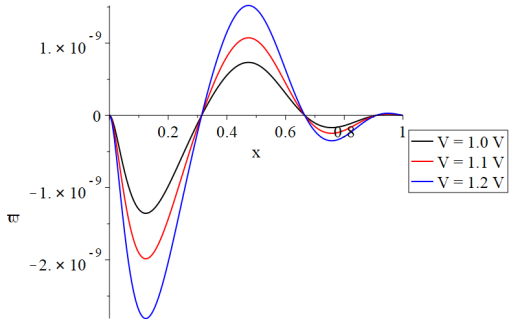


Fig. 6. The strain-energy density distribution with variance voltage and constant resistance

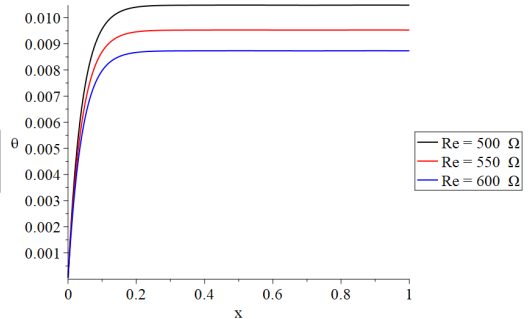


Fig. 7. The temperature increment distribution with variance resistance and constant voltage

In Fig. 2, in the three curves, we find that the value of the temperature increment increases

sharply up in a distance of not more than 0.15 due to the thermo-electrical effect at this end and then the curves go to the state of stability to the end but with different values based on the voltage value. In Figs. 3, 4 and 6, each curve has three peak points where the absolute value of the peak point increases when the value of the voltage increases. Along the x -axis, the values of the peak points decrease when the distance x increases. In Fig. 5, each curve of the stress curves has two peak points, and its absolute value increases when the voltage value increases.

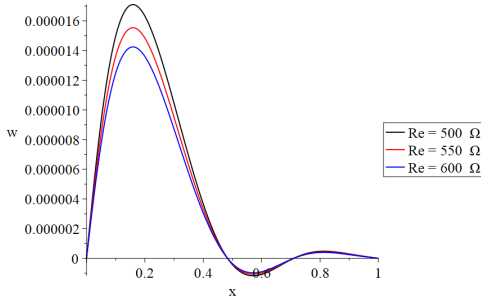


Fig. 8. The vibration distribution with variance resistance and constant voltage

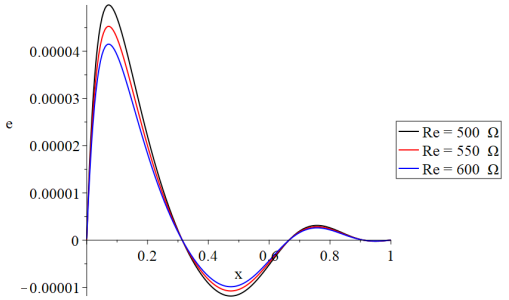


Fig. 9. The strain distribution with variance resistance and constant voltage

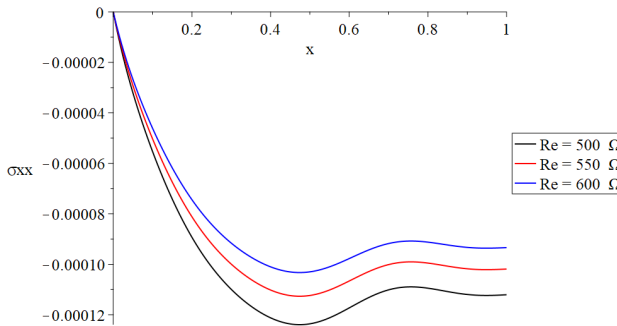


Fig. 10. The stress distribution with variance resistance and constant voltage

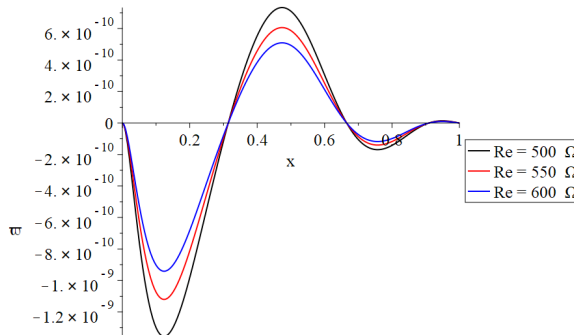


Fig. 11. The strain-energy density distribution with variance resistance and constant voltage

The distributions of temperature increment, vibration (lateral deflection), cubical deformation, stress, and strain energy density are all significantly affected by the electrical resistance value, as shown in Figs. 7-11. The temperature increment and absolute values of the vibration (lateral deflection), cubical deformation, stress, and strain energy density decrease with an increase in the electrical resistance.

In Fig. 7, we find that the value of temperature increment increases sharply up in a distance of not more than 0.15 due to the thermo-electrical effect at this end and then the curves go to the state

of stability to the end but with different values based on the electrical resistance value. In Figs. 8, 9, and 11, each curve has three peak points where the value of each peak point decreases when the value of the electrical resistance increases. Along the x -axis, the values of the peak points decrease when the distance x increases. In Fig. 10, each curve of the stress curves has two peak points, and its absolute value decreases when the electrical resistance increases.

As a result, we can claim that the electrical resistance may be utilized to control both the vibration and the energy generated along the microbeam.

To ensure that the findings are reliable, we compared them to those reported in [36, 37].

4. Conclusions

Green-Naghdi type-II heat conduction model has been applied to analyze thermoelastic microbeams in this study. The nano-strip of graphene at the microbeam's beginning end is connected by a low-voltage electrical current. The extended thermoelasticity theory Lord-Shulman model was used. The microbeam has been thermally loaded because of the electrical current operating on a low-voltage nano-strip of graphene.

Electrical voltage and electrical resistance have been discovered to play a substantial role in all the functions tested.

Gradually increasing electrical voltage raises thermal and mechanical stress and strain energy density as well as the lateral deflection, stress and strain energy density values. Temperature increment, absolute values of vibration (lateral deflection), cubical deformation, tension, and strain energy density increase with increasing electrical resistance.

The graphene microstrip might be used as a tuner to regulate the vibration and energy of the microbeam by controlling the voltage and resistance of the microstrip.

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