Influence of bridge geometric nonlinearity on dynamic response of vehicle-bridge coupling system

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Abstract. Based on the theory of vehicle-bridge coupling vibration, the differential equation of vehicle-bridge coupling system is set up according to different conditions. The differential equation of the system is converted into matrix form using mode decomposition method and is solved using MATLAB. The system equation has a non-linear matrix term when the geometric nonlinearity of the bridge is considered. The influence of wheel acceleration on the dynamic response of the bridge is analyzed without simplification under four speeds. The results show that it is acceptable to neglect the influence of wheel acceleration at low speeds, but it has a significant influence which must be considered at high speeds.

Keywords: vehicle-bridge coupling vibration, acceleration, nonlinear vibration.

1. Introduction

With the remarkable and increasing span and vehicle load of bridges, as well as the gradually decreasing mass and stiffness of bridge structures, much attention has been focused on the complexity and diversity of vehicle-bridge coupling vibration.

There are numerous methods of analyzing vehicle-bridge coupling vibration. Xia [1] presented a method of simplifying the bridge into a modal model to analyze the vehicle-bridge-pier system. Based on the finite element method, Chen [2] analyzed the resonance of a rigid-frame bridge under vehicle-bridge coupling vibration. Li [3] conducted in-depth analyses of the vehicle-bridge interaction under stochastic vibration. Shen [4] used an ODE function based on the Runge-Kutta method to compile a secondary development function which can solve the differential equation of the vehicle-bridge coupling system.

The influence of the geometric nonlinearity of the structures on the vehicle-bridge coupling vibration is important, especially for long-span and low-mass bridges. Considering the geometric nonlinearity of the bridge, it can be more realistic to analyze the dynamic response of the structure, which is of great theoretical and practical significance in the design and construction of the bridge structure.

2. Establishment of system equation considering the geometric nonlinearity of the bridge

Fig. 1 shows the simply supported beam subjected to sprung mass, the vehicle system passes through the beam at a uniform speed v, $w(x, t)$ and $Z(t)$ denotes the dynamic deflection of the beam and the mass M_2, respectively. Meanwhile, the displacement of the mass M_1 is consistent with the deflection of the beam at its position expressed as $w(x, t)$.

The main derivation process of the differential equation is as follows. First, the dynamic equilibrium equation of a simply-supported beam can be written as Eq. (1):

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} - \frac{3EA}{2}\frac{\partial^2 w(x,t)}{\partial x^2}\left(\frac{\partial w(x,t)}{\partial x}\right)^2 + c\frac{\partial w(x,t)}{\partial t}$$
$$= \delta(x-vt)\left[(M_1+M_2)g - M_1\frac{d^2 w(x,t)}{dt^2} + k_1[Z(t)-w(x,t)] + c_1\left[\dot{Z}(t) - \frac{dw(x,t)}{dt}\right]\right]. \tag{1}$$
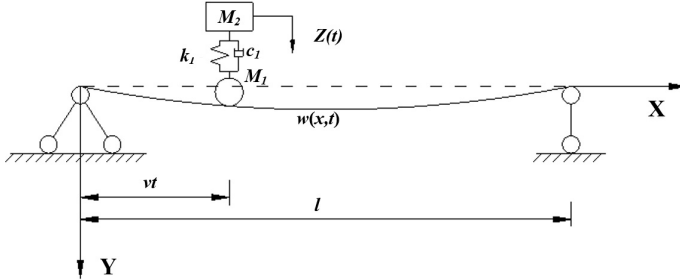

Fig. 1. Simply supported beam subjected to sprung mass model

The dynamic equilibrium equation of M_2 can be written as:

$$M_2 Z(t) + k_1\left[\ddot{Z}(t) - w(x,t)|_{x=vt}\right] + c_1\left[Z(t) - \frac{\partial\dot{w}(x,t)}{\partial t}\Big|_{x=vt}\right] = 0. \tag{2}$$

The external load acting on the beam can be expressed as Eq. (3):

$$P(x,t) = \delta(x-vt)\begin{bmatrix}(M_1+M_2)g - M_1\frac{\partial^2 w(x,t)}{\partial t^2}\\ +k_1[Z(t)-w(x,t)] + c_1\left[\dot{Z}(t) - \frac{\partial w(x,t)}{\partial t}\right]\end{bmatrix}. \tag{3}$$

According to the mode decomposition method, the right end of Eq. (3) can be transformed into:

$$P_n(t) = P_{n1}(t) + P_{n2}(t),$$
$$P_{n1}(t) = \frac{2}{ml}\left[(M_1+M_2)g\sin\frac{n\pi vt}{l} - M_1\sum_{i=1}^{\infty}\ddot{q}_i(t)\sin\frac{i\pi vt}{l}\sin\frac{n\pi vt}{l}\right],$$
$$P_{n2}(t) = \frac{2}{ml}[k_1 Z(t) + c_1\dot{Z}(t)]\sin\frac{n\pi vt}{l} - \frac{2}{ml}\sum_{i=1}^{\infty}[k_1 q_i(t) + c_1\dot{q}_i(t)]\sin\frac{i\pi vt}{l}\sin\frac{n\pi vt}{l}. \tag{4}$$

The left end of the Eq. (1) can be transformed into:

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) + \frac{3A}{8I}\omega_n^2 q_n^3(t). \tag{5}$$

Therefore, Eq. (3) can be expressed as Eq. (6):

$$\left[\ddot{q}_n(t) + \frac{2M_1}{ml}\sum_{i=1}^{\infty}\ddot{q}_i(t)\sin\frac{i\pi vt}{l}\sin\frac{n\pi vt}{l}\right]$$
$$+ \left[2\xi_n\omega_n\dot{q}_n(t) + \frac{2c_1}{ml}\sum_{i=1}^{\infty}\dot{q}_i(t)\sin\frac{i\pi vt}{l}\sin\frac{n\pi vt}{l}\right] \tag{6}$$

$$+\frac{3A}{8I}\omega_n{}^2 q_n{}^3(t) + \left[\omega_n{}^2 q_n(t) + \frac{2k_1}{ml}\sum_{i=1}^{\infty} q_i(t)\sin\frac{i\pi vt}{l}\sin\frac{n\pi vt}{l}\right]$$

$$-\frac{2}{ml}\sin\frac{n\pi vt}{l} = \frac{2}{ml}g\sin\frac{n\pi vt}{l}.$$

Similarly, using the mode decomposition method, the Eq. (2) can be written as:

$$M_2\ddot{Z}(t) + c_1\dot{Z}(t) + k_1 Z(t) - c_1\sum_{i=1}^{\infty}\dot{q}_i(t)\sin\frac{i\pi vt}{l} - k_1\sum_{i=1}^{\infty} q_i\sin\frac{i\pi vt}{l} = 0. \tag{7}$$

By combining Eq. (6) and Eq. (7), the system dynamic equilibrium equations of the vehicle coupling with the simply-supported beam are obtained. The Nth order matrix expression of the system motion equation is:

$$M\{\ddot{X}\} + C\{\dot{X}\} + K\{X\} = \{F\}, \tag{8}$$

where $\{X\} = [q_1, q_2, \cdots q_N, Z]^T$ represents the generalized displacement vector, $\{F\} = [\rho_F\varphi_1, \rho_F\varphi_2, \cdots \rho_F\varphi_N, 0]^T$ represents the generalized force vector, M represents the generalized mass matrix, C represents the generalized damping matrix and K represents the generalized stiffness matrix, H is used to describe the nonlinear matrix. This can be described by the following matrices:

$$M = \begin{bmatrix} 1+\rho_M\varphi_{11} & \rho_M\varphi_{12} & \cdots & \rho_M\varphi_{1N} & 0 \\ \rho_M\varphi_{21} & \rho_M\varphi_{22} & & \rho_M\varphi_{2N} & 0 \\ \vdots & & \ddots & \vdots & 0 \\ \rho_M\varphi_{N1} & \rho_M\varphi_{N2} & \cdots & \rho_M\varphi_{NN} & 0 \\ 0 & 0 & & 0 & M_2 \end{bmatrix}, \quad H = \frac{3A}{8I}\begin{bmatrix} \omega_1{}^2 & 0 & \cdots & 0 & 0 \\ 0 & \omega_2{}^2 & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \omega_N{}^2 & 0 \\ 0 & 0 & & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 2\xi_1\omega_1+\rho_c\varphi_{11} & \rho_c\varphi_{12} & \cdots & \rho_c\varphi_{1N} & -\rho_c\varphi_1 \\ \rho_c\varphi_{21} & 2\xi_2\omega_2+\rho_c\varphi_{22} & & \rho_c\varphi_{2N} & -\rho_c\varphi_2 \\ \vdots & & \ddots & \vdots & \\ \rho_c\varphi_{N1} & \rho_c\varphi_{N2} & \cdots & 2\xi_N\omega_N+\rho_c\varphi_{NN} & -\rho_c\varphi_N \\ -c_1\varphi_1 & -c_1\varphi_2 & & -c_1\varphi_N & c_1 \end{bmatrix},$$

$$K = \begin{bmatrix} \omega_1{}^2+\rho_k\varphi_{11} & \rho_k\varphi_{12} & \cdots & \rho_k\varphi_{1N} & -\rho_k\varphi_1 \\ \rho_k\varphi_{21} & \omega_2{}^2+\rho_k\varphi_{22} & & \rho_k\varphi_{2N} & -\rho_k\varphi_2 \\ \vdots & & \ddots & \vdots & \\ \rho_k\varphi_{N1} & \rho_k\varphi_{N2} & \cdots & \omega_N{}^2+\rho_N\varphi_{NN} & -\rho_k\varphi_N \\ -k_1\varphi_1 & -k_1\varphi_2 & & -k_1\varphi_N & k_1 \end{bmatrix},$$

where $\rho_M = 2M_1/ml$, $\rho_C = 2C_1/ml$, $\rho_k = 2k_1/ml$ and $\varphi_n = \sin n\pi vt/l$.

It can be seen from the above derivations that in cases where the geometric nonlinearity of the bridge is taken into consideration, the system equation has a non-linear matrix term. It can be found that the non-linear matrix is related only to the nature of the bridge itself, and not to the parameters of the vehicle.

3. Establishment of system equation without simplifying wheel acceleration

The external load acting on the bridge is deduced by Eq. (6). In fact, the overall analysis of the vehicle shows that the external loads on the bridge can be expressed as Eq. (9):

$$P(x,t) = \delta(x - vt)\left[(M_1 + M_2)g - M_1\frac{d^2w(x,t)}{dt^2} - M_2\ddot{Z}(t)\right], \tag{9}$$

$$\frac{d^2w(x,t)}{dt^2} = (i)\frac{\partial^2w(x,t)}{\partial t^2} + (ii)2\frac{\partial^2w(x,t)}{\partial x\partial t}v + (iii)\frac{\partial^2w(x,t)}{\partial x^2}v^2, \tag{10}$$

where (i) represents the vertical acceleration of the vibration of the bridge at the position of the wheel, regardless of the change in the position of the load; (ii) represents the vertical acceleration of the bridge caused by the movement of the load; (iii) represents the vertical acceleration of the bridge caused by the curvature during structural vibration. The external loads on the bridge can be described by Eq. (11), which is calculated by substituting Eq. (10) into Eq. (9):

$$P(x,t) = \delta(x - vt)[(M_1 + M_2)g - M_1\left[\frac{\partial^2w(x,t)}{\partial t^2} + 2\frac{\partial^2w(x,t)}{\partial x\partial t}v + \frac{\partial^2w(x,t)}{\partial x^2}v^2\right] - M_2\ddot{Z}(t)]. \tag{11}$$

According to the mode decomposition method, the right end of Eq. (6) can be obtained and converted into the following equations:

$$P_n(t) = P_{n1}(t) + P_{n2}(t) + P_{n3}(t), \tag{12}$$

$$P_{n1}(t) = \frac{2}{ml}\left[(M_1 + M_2)g\sin\frac{n\pi vt}{l} - M_1\sum_{i=1}^{\infty}\ddot{q}_i(t)\sin\frac{i\pi vt}{l}\sin\frac{n\pi vt}{l}\right], \tag{13}$$

$$P_{n2}(t) = -\frac{2M_1}{ml}2v\frac{n\pi v}{l}\cos\frac{n\pi vt}{l}c_1\dot{q}_i(t) - \left[-\frac{2M_1}{ml}v^2\left(\frac{n\pi v}{l}\right)^2\sin\frac{n\pi vt}{l}k_1q_i(t)\right], \tag{14}$$

$$P_{n3}(t) = \frac{2M_2}{ml}\ddot{Z}(t)\sin\frac{n\pi vt}{l}. \tag{15}$$

The system dynamic equilibrium equations of a simply-supported beam, wheel and spring (damper) mass system can be obtained by combining Eq. (5), Eq. (13), Eq. (14) and Eq. (15).

The Nth-order matrix expression can be expressed as follows:

$$\begin{aligned}
M &= \begin{bmatrix}
1 + \rho_m\varphi_1\varphi_1 & \rho_m\varphi_1\varphi_2 & \cdots & \rho_m\varphi_1\varphi_N & \rho_M\varphi_1 \\
\rho_m\varphi_2\varphi_1 & 1 + \rho_m\varphi_2\varphi_2 & \cdots & \rho_M\varphi_{2N}\varphi_1 & \rho_M\varphi_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\rho_m\varphi_N\varphi_1 & \rho_m\varphi_N\varphi_2 & \cdots & 1 + \rho_m\varphi_N\varphi_N & \rho_M\varphi_N \\
0 & 0 & \cdots & 0 & M_2
\end{bmatrix}, \\
H &= \frac{3A}{8I}\begin{bmatrix}
\omega_1{}^2 & 0 & \cdots & 0 & 0 \\
0 & \omega_2{}^2 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & \omega_N{}^2 & 0 \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix}, \\
C &= \begin{bmatrix}
2\xi_1\omega_1 + \rho_m\varphi_1\dot{\varphi}_1 & \rho_m\varphi_1\dot{\varphi}_1 & \cdots & \rho_m\varphi_1\dot{\varphi}_1 & 0 \\
\rho_m\varphi_2\dot{\varphi}_1 & 2\xi_2\omega_{21} + \rho_m\varphi_2\dot{\varphi}_2 & \cdots & \rho_m\varphi_1\dot{\varphi}_1 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\rho_m\varphi_N\dot{\varphi}_1 & \rho_m\varphi_1\dot{\varphi}_1 & \cdots & 2\xi_N\omega_N + \rho_m\varphi_N\dot{\varphi}_N & 0 \\
-c_1\varphi_1 & -c_1\varphi_2 & \cdots & -c_1\varphi_N & c_1
\end{bmatrix}, \\
K &= \begin{bmatrix}
\omega_1{}^2 + \rho_m v^2\varphi_1\ddot{\varphi}_1 & \rho_m v^2\varphi_1\ddot{\varphi}_2 & \cdots & \rho_m v^2\varphi_1\ddot{\varphi}_N & 0 \\
\rho_m v^2\varphi_2\ddot{\varphi}_1 & \omega_1{}^2 + \rho_m v^2\varphi_2\ddot{\varphi}_2 & \cdots & \rho_m v^2\varphi_1\ddot{\varphi}_N & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\rho_m v^2\varphi_1\ddot{\varphi}_1 & \rho_m v^2\varphi_1\ddot{\varphi}_1 & \cdots & \omega_1{}^2 + \rho_m v^2\varphi_N\ddot{\varphi}_N & 0 \\
-k_1\varphi_1 & -k_1\varphi_2 & \cdots & -k_1\varphi_N & k_1
\end{bmatrix}, \\
\{F\} &= [\rho_F\varphi_1, \rho_F\varphi_2, \cdots, \rho_F\varphi_N, 0]^T,
\end{aligned} \tag{16}$$

where:

$$\rho_m = \frac{2M_1}{ml}, \quad \rho_M = \frac{2M_2}{ml}, \quad \rho_F = \frac{2(M_1 + M_2)g}{ml}, \quad \varphi_n = \sin\frac{n\pi vt}{l},$$

$$\dot{\varphi}_n = \frac{n\pi v}{l}\cos\frac{n\pi vt}{l}, \quad \ddot{\varphi}_n = -\left(\frac{n\pi v}{l}\right)^2 \sin\frac{n\pi vt}{l}.$$

4. Effect of velocity on dynamic response of bridge without simplifying wheel acceleration

For the dynamic response of the wheel mass coupled acceleration of a high-speed vehicle on the bridge, it is necessary to analyze the influence of the last two terms of Eq. (12) (*ii*) and (*iii*) on the dynamic response of the vehicle-bridge coupling system [1, 5].

In this section, the influence of wheel acceleration on the bridge's dynamic response is analyzed without simplification under four speeds: 20 m/s, 25 m/s, 30 m/s and 35 m/s. The calculating model of Eq. (12) is defined as Model A, and the model neglecting (*ii*) and (*iii*) is defined as Model B. Fig. 2 shows the vertical displacement response of the two models at different speeds. The maximum displacement of the structure and the maximum displacement ratio of the two models are shown in Table 1.

Table 1. The dynamic responses of the bridge structure

v velocity(m/s)	D_{Amax}	D_{Bmax}	D_{Amax}/D_{Bmax}
20	79.1	74.2	1.0660
25	86.3	74.4	1.1599
30	99.1	73.1	1.3557
35	123.6	70.5	1.7532
D_{Amax} and D_{Bmax} represent the maximum displacement in the span of Model A and Model B			

a) $v = 20$ m/s   b) $v = 25$ m/s

c) $v = 30$ m/s   d) $v = 35$ m/s
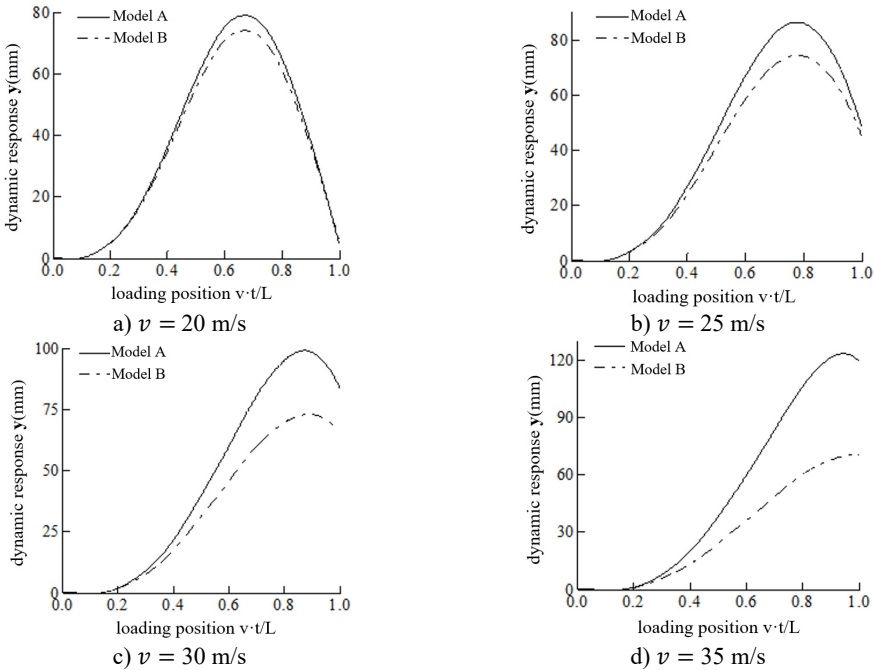
Fig. 2. Comparisons of dynamic response of the mid-span at different speeds

It can be observed that D_{Amax} increases continuously with the increasing velocity. However, D_{Bmax} changes slightly with the increase in velocity, varying between 70 mm and 75 mm. With

the increase in velocity, the position of the maximum mid-span displacement gradually recedes for both Model A and Model B. When the speed is 35 m/s, the maximum mid-span displacement of Model A and Model B occurs when the vehicle is about to leave the bridge.

When the speed is $V \leq 20$ m/s, Model A and Model B exhibit little difference in their mid-span displacement response. However, when the speed is $V \geq 30$ m/s, the ratio D_{Amax}/D_{Bmax} is 1.3557, and model A should be adopted. When the speed is 20 m/s $< V <$ 30 m/s, the appropriate model can be selected according to the calculation accuracy requirements.

5. Conclusions

In this paper, the effect of the geometric nonlinearity of bridges on the dynamic response of the vehicle-bridge coupling system is taken into account, and the following conclusions are obtained:

(1) The system equation has a non-linear matrix term when the geometric nonlinearity of the bridge is considered. The non-linear matrix is related only to the nature of the bridge structures, and the parameters of the vehicles have no influence on the matrix.

(2) The value of the non-linear matrix will increase as the inertia moment decreases when the other conditions remain unchanged.

(3) The influence of wheel acceleration on the bridge dynamic response is analyzed without simplification under four speeds. It is acceptable to neglect the influence of wheel acceleration at $V \leq 20$ m/s, while it is necessary to consider the significant influence of the wheel acceleration at $V \geq 30$ m/s.

Acknowledgements

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