Free vibration of circular annular plate with different boundary conditions

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Abstract. This paper deals with the numerical simulation of free vibration analysis of a thin circular annular plate for various boundary conditions at the outer edge and inner edge. Classical plate theory is used to derive the governing differential equation for the transverse deflection of the thin isotropic plate. The finite element method is used to evaluate the first six natural frequencies and mode shapes of the thin uniform circular annular plate with radius ratios (r_1/r_2) for different boundary conditions. These natural frequencies results are compared with those available in the literature. The results are verified with classical plate theory with our Abaqus results and checked with the previous research literature on the topic.

Keywords: circular annular plate, free vibration, numerical simulation.

1. Introduction

Plates are widely used as a structural element and have vast practical applications in many engineering fields such as aerospace, mechanical, civil, nuclear, electronic, automotive, marine and heavy machinery, etc. Various researchers have analyzed the free vibration behavior of circular annular plates of different shapes, sizes, thickness for different boundary conditions. Leissa [1] used the Ritz method to estimate the natural frequencies of the isotropic plate for different boundary conditions. Kim and Dickinson [2] used the Rayleigh-Ritz approximation method for free vibration of a thin plate to extract natural frequencies. Rajalingham et al. [3] used a Rayleigh-Ritz method to analyze the plate characteristics parameter as shape functions and continued his work to formulate a variational reduction expression to analyze frequencies and mode shapes. Liew et al. [4] used the polynomials-Ritz method for the vibration of circular plates by using three-dimensional elasticity solutions. Zhou et al. [5] used the Chebyshev-Ritz method for three-dimensional vibration and mode shapes of the circular plate. Lim et al. [6] used the state-space method to analyze transverse vibration and mode of a thick circular plate. Zhou et al. [7] used the Hamiltonian principle to solve governing equations for free vibration analysis by using the variational principle of mixed energy method. Kumar et al. [8] use a dynamic stiffness method to extract the natural frequency and mode shapes of a thin plate. Piyush et al. [9] used the Rayleigh-Ritz method to compute the natural frequencies of the thin plate.

2. Basic formulation

Consider a homogeneous, isotropic circular annular plate in cylindrical coordinates (r, θ, z) with uniform thickness h as shown in Fig. 1.

Classical plate theory is used to derive the governing differential equation for transverse vibration in the polar coordinate system is defined as:

$$D\nabla^4 W(r, \theta, t) + \rho h w(r, \theta) = 0, \quad \nabla^4 = \nabla^2 \nabla^2, \tag{1}$$

where Laplacian operator: $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$, ρ is the mass density, $D = Eh^3/[12(1-v^2)]$ is the flexural rigidity and v is Poisson's ratio.

Transverse deflection of natural vibrations for thin circular plate is assumed to be:

$$W(r,\theta,t) = w(r,\theta)e^{i\omega t}, \tag{2}$$

where ω is the natural frequency and $w(r,\theta)$ is natural mode.

Substituting Eqs. (2) in (1), we get:

$$\nabla^4 W - \gamma^4 W = 0, \quad \text{or} \quad (\nabla^2 + \gamma^2)(\nabla^2 + \gamma^2)W = 0, \tag{3}$$

where:

$$\gamma^4 = \frac{\rho h \omega^2}{D}. \tag{4}$$

The general finite element equation for the transverse deflection of thin plate is given by:

$$[M]\{\ddot{q}\} + [K]\{q\} = 0, \tag{5}$$

where $[M]$ is the mass matrix, $[K]$ is stiffness matrix, $\{\ddot{q}\}$ is the nodal acceleration vector, and $\{q\}$ is nodal displacement vector.

The non-dimension natural frequency parameter ϖ are calculated as:

$$\varpi = 2\pi\omega\left[\sqrt{\rho h/D}\right]r_1^2, \tag{6}$$

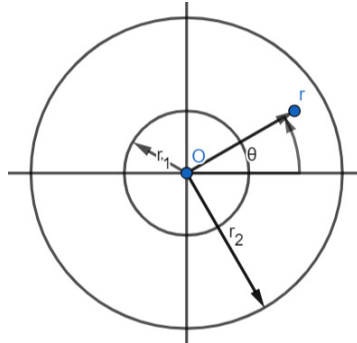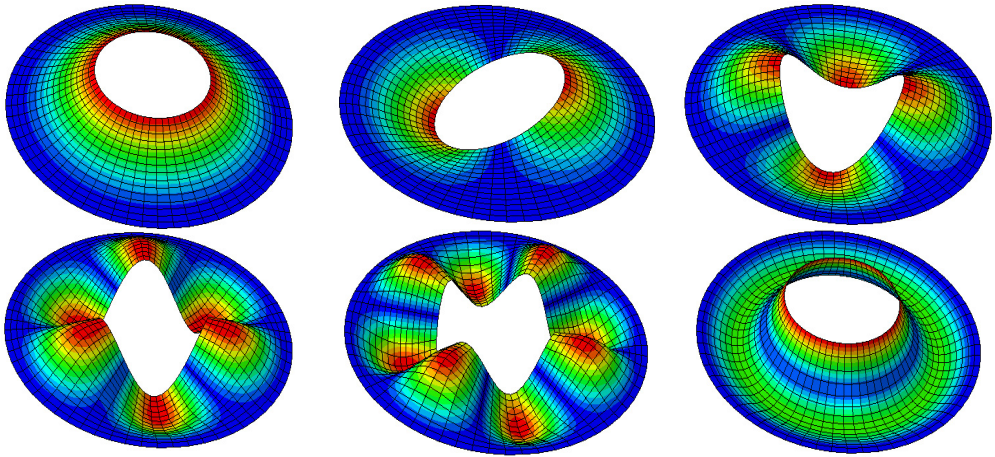where ω is the natural frequency in Hz.

Fig. 1. Schematic diagram and coordinate system of annular circular plate

3. Results and discussions

In this section, the first six non-dimensional natural frequencies and mode shapes of the circular annular plate are estimated by using the finite element method. Here, we calculated different eigenvalues for different boundary conditions with different radii ratio (r_1/r_2). Different combinations of boundary conditions are applied to compute the natural frequencies and mode shapes of the circular annular plate. 2820 elements and 5922 nodes are used to estimate the natural frequencies and mode shape function of thin circular annular plates after convergence study. The present natural frequencies results are compared with those available in the literature.

Tables 1-4 shows that the first six non-dimensional natural frequencies values for the circular annular plate. These present results are nearly the same as Leissa [1] and Zhou [8] under different

boundary conditions.

Fig. 2. Natural modes of a clamped annular plate with a free inner boundary, $r_1/r_2 = 0.4$

Table 5-7 presents the effect of the radii ratio on the non-dimensional frequency parameter of thin plates. It is observed from these tables that as the radii ratio increases non-dimensional frequency parameter increases.

Table 1. Comparison of the non-dimensional natural frequency parameter with Leissa [1] and Zhou [8] for clamped outer and free inner boundary ($v = 1/3$, $r_1/r_2 = 0.4$)

Results	Mode number					
	1	2	3	4	5	6
Leissa [1]	13.54	19.80	31.34	–	–	–
Zhou [8]	13.500	19.389	31.338	46.855	65.984	66.924
Present	12.871	18.497	29.901	44.70	62.982	63.935

Table 2. Comparison of the non-dimensional natural frequency parameter with Leissa [1] and Zhou [8] for free outer and clamped inner boundary ($v = 1/3$, $r_1/r_2 = 0.4$)

Results	Mode number					
	1	2	3	4	5	6
Leissa [1]	9.096	10.37	–	–	–	–
Zhou [8]	9.0719	9.1294	10.366	14.726	22.530	33.455
Present	9.0257	9.0868	10.3267	14.6805	22.5682	33.4769

Table 3. Comparison of the non-dimensional natural frequency parameter with Leissa [1] and Zhou [8] for free outer and free inner boundary ($v = 1/3$, $r_1/r_2 = 0.4$)

Results	Mode number					
	1	2	3	4	5	6
Leissa [1]	–	4.567	–	–	–	–
Zhou [8]	4.5325	8.5510	11.765	17.043	21.262	31.356
Present	4.5197	8.5070	11.7368	16.9765	21.233	31.2565

Table 4. Comparison of the non-dimensional natural frequency parameter with Leissa [1] and Zhou [8] for clamped outer and clamped inner boundary ($v = 1/3$, $r_1/r_2 = 0.4$)

Results	Mode number					
	1	2	3	4	5	6
Leissa [1]	62.33	62.92	66.406	–	–	–
Zhou [8]	61.872	62.966	66.672	73.630	84.594	99.904
Present	62.0056	63.1121	66.7350	73.6148	84.5044	99.7988

Table 5. Non-dimensional frequency parameter $\omega r_2^2 \sqrt{\rho h/D}$ for the annular circular plate with clamped outer and free inner edge ($\nu = 1/3$)

n	r_1/r_2			
	0.2	0.4	0.6	0.8
0	10.2922	12.871	25.3966	92.3617
1	20.4306	18.497	28.3519	94.2334
2	33.6865	29.001	36.2745	98.3097
3	50.4963	44.70	47.9049	105.6264
4	69.6342	62.982	62.8458	115.7173
5	91.0597	83.751	79.4797	128.4994

Table 6. Non-dimensional frequency parameter $\omega r_2^2 \sqrt{\rho h/D}$ for the annular circular plate with free outer and clamped inner edge ($\nu = 1/3$)

n	r_1/r_2			
	0.2	0.4	0.6	0.8
0	4.795	9.0257	20.5213	84.5418
1	5.190	9.0868	20.8923	85.1491
2	6.322	10.3267	22.3356	87.0250
3	12.367	14.6805	25.6329	90.3110
4	21.502	22.5682	31.5306	95.2026
5	32.438	33.4769	40.3467	101.941

Table 7. Non-dimensional frequency parameter $\omega r_2^2 \sqrt{\rho h/D}$ for the annular circular plate with clamped outer and clamped inner edge ($\nu = 1/3$)

n	r_1/r_2			
	0.2	0.4	0.6	0.8
0	4.795	9.0257	141.4272	593.2696
1	5.190	9.0868	144.8754	593.8935
2	6.322	10.3267	149.467	595.8484
3	12.367	14.6805	156.3015	599.0929
4	21.502	22.5682	165.7894	603.7515
5	32.438	33.4769	178.2596	609.866

4. Conclusions

In this paper, numerical analysis for free vibration analysis of a thin annular solid plate is carried out using the finite element method for different boundary conditions at the inner and outer radius. It is found that those natural frequency results are quite close to those reported in previous works of literature. The novelty of this paper is the effect of the radii ratio on natural frequency is discussed and found that with increasing radii ratio, natural frequency increases and another novelty is by using shell element modeling as per Abaqus convention the dimensionless frequency parameter as found in the literature are completely validated.

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