Vibration-based damage growth monitoring in beam-like structures

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Abstract. Damage growth monitoring plays an important role in providing early warning of structural failure. The existing methods for damage growth monitoring are mainly local inspection methods, such as acoustic emission. These methods need a priori knowledge of accessible damage vicinity, which may not be realized in practice. Hence, vibration-based global approach is adopted to overcome these difficulties. Natural frequency, as a global modal parameter, can be measured easily and is used for vibration-based damage growth monitoring in this study. A concept of damage-induced relative natural frequency change (RNFC) curve is defined first and its relation with mode shape is then derived analytically, giving a good way to approximate RNFC curves. For monitoring damage growth, a damage growth indicator is proposed based on RNFCs between two damaged stages of a beam. The effectiveness of the indicator for damage growth monitoring is proved by both numerical and experimental cases in beam-like structures.

Keywords: relative natural frequency change curve, mode shape, damage growth monitoring, beam-like structures.

1. Introduction

Structural health monitoring (SHM) is a promising research field for maintaining structural health. SHM methods mainly focus on damage diagnosis including damage localization and severity evaluation [1, 2]. In comparison, less attention has been paid to damage growth monitoring, which plays an important role in providing early warning of structural failure.

The relevant literature regarding damage growth monitoring is summarized here. Bakuckas et al. [3] studied the use of acoustic emission (AE) to monitor damage growth in laminates. Daniel et al. [4] adapted AE technique for fatigue damage growth monitoring in metals. Maillet et al. [5] combined acousto-ultrasonics and AE for damage evolution monitoring in minicomposites. Chen et al. [6] used acoustography for real-time monitoring of fatigue damage growth in composites. Kumar et al. [7] used rotating frequency changes as an indicator to monitor damage growth in turbine blades. There are some other techniques used for damage growth monitoring, including thermography [8, 9], guided waves [10, 11], and digital image correlation [12, 13]. It is seen that the existing methods for damage growth monitoring are mainly local inspection methods. Such methods need a priori knowledge of accessible damage vicinity, which is not practical in many applications.

Natural frequency, as a global modal parameter, has been widely used for damage detection [14-17] based on natural frequency changes between intact and damaged stages of structures. In this paper, a simple indicator based on relative natural frequency changes (RNFCs) is formulated for damage growth monitoring from any reference state to the current state in beam-like structures. The proposed method holds the promise of developing a real-time monitoring technique.

2. Theoretical analysis

2.1. Damage-induced RNFC curves

The fundamental frequency of a beam can be obtained based on Rayleigh's method [18]. The maximum values of kinetic energy T_{max} and potential energy U_{max} of a beam are expressed as:

$$T_{max} = \frac{1}{2}\omega^2 \int_0^1 \bar{m}\phi^2(\zeta)d\zeta, \tag{1}$$

$$U_{max} = \frac{1}{2}\int_0^1 EI(\phi''(\zeta))^2 d\zeta, \tag{2}$$

where ω is the fundamental frequency, $\bar{m}$ is the linear density, ϕ is the fundamental mode shape, ζ is the dimensionless coordinate, E is the modulus of elasticity, and I is the moment of inertia. By equating T_{max} and U_{max}, the fundamental frequency is derived as:

$$\omega^2 = \frac{\int_0^1 EI(\phi''(\zeta))^2 \, d\zeta}{\int_0^1 \bar{m}\,\phi^2(\zeta)d\zeta}. \tag{3}$$

Damage is modeled by stiffness reduction at the position of damage. When a decrement in E occurs at ζ_i (the coordinate of damage i), the corresponding frequency of the damaged beam can be calculated as:

$$(\omega^d(\zeta_i))^2 \approx \frac{\int_0^1 (E - \delta E(\zeta_i))I(\phi''(\zeta))^2 \, d\zeta}{\int_0^1 \bar{m}\,\phi^2(\zeta)d\zeta}, \tag{4}$$

where $\delta E(\zeta_i)$ represents an increment in E at ζ_i. The following relation is then derived from Eq. (3) and (4):

$$\frac{(\omega^d(\zeta_i))^2}{\omega^2} = 1 - \frac{\delta E(\zeta_i)I(\phi''(\zeta_i))^2 d\zeta}{\int_0^1 EI(\phi''(\zeta))^2 \, d\zeta}. \tag{5}$$

The RNFC, denoted by $\Delta\omega(\zeta_i)$, is approximated from Eq. (5) as:

$$\Delta\omega(\zeta_i) \approx \frac{\delta E(\zeta_i)I(\phi''(\zeta_i))^2 d\zeta}{2\int_0^1 EI(\phi''(\zeta))^2 \, d\zeta}. \tag{6}$$

It is seen that the RNFC can be represented as a function of damage location and severity. $\Delta\omega(\zeta_i)$ is rescaled by min-max normalization:

$$\Delta\bar{\omega}(\zeta_i) = \frac{\Delta\omega(\zeta_i) - \min_i(\Delta\omega(\zeta_i))}{\max_i(\Delta\omega(\zeta_i)) - \min_i(\Delta\omega(\zeta_i))}. \tag{7}$$

$\Delta\bar{\omega}(\zeta_i)$ is defined as RNFC curve and denoted as $g(\zeta_i)$. The RNFC can be written as:

$$\Delta\omega(\zeta_i) = s \cdot g(\zeta_i). \tag{8}$$

Natural frequencies of a beam at damage stages I and II with severity s_1 and s_2 are denoted as $\omega^{d_1}(\zeta_i)$ and $\omega^{d_2}(\zeta_i)$, respectively. The following equations can be obtained according to Eq. (8):

$$\Delta\omega^{d_1}(\zeta_i) = s_1 \cdot g(\zeta_i), \tag{9}$$

$$\Delta\omega^{d_2}(\zeta_i) = s_2 \cdot g(\zeta_i), \tag{10}$$

$$\Delta\omega^{d_1 d_2}(\zeta_i) = \Delta\omega^{d_2}(\zeta_i) - \Delta\omega^{d_1}(\zeta_i) = (s_2 - s_1) \cdot g(\zeta_i). \tag{11}$$

This means that the relation in Eq. (8) remains true when the reference frequency is from a damage state of a beam.

From Eq. (6) and (7), g is obtained by min-max normalization of $(\phi'')^2$. Similarly, the Rayleigh's quotient is applied to higher modes of vibration and then each mode of g is obtained in the same way. The mode shapes of a SS beam are written as [19]:

$$\phi_j(\zeta) = C\sin\lambda_j\zeta, \tag{12}$$

where j is the number of mode, $\lambda_j = j\pi$. The first four modes of g are plotted in Fig. 1.
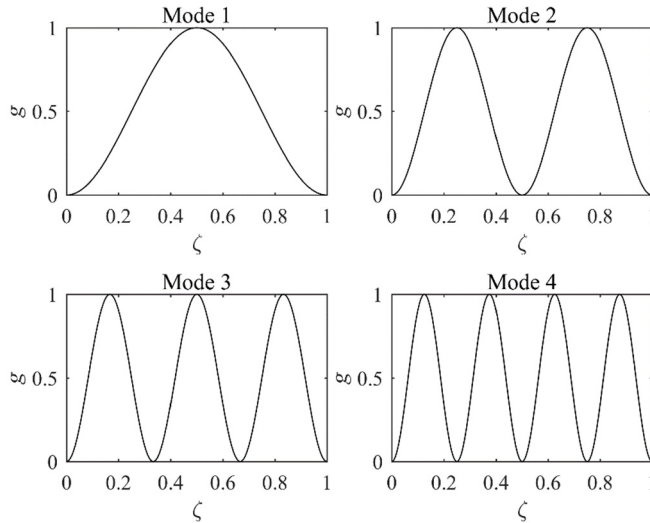

Fig. 1. First four modes of g for the SS beam

2.2. Damage indicator

Natural frequencies of a beam at damage stages I and II are given by $\omega_j^{d_1}$ and $\omega_j^{d_2}$, respectively. Proposed indicator is created step by step [20].

Calculation of the RNFCs:

$$\Delta\omega_j = \frac{\omega_j^{d_1} - \omega_j^{d_2}}{\omega_j^{d_1}}. \tag{13}$$

Min-max normalization of $\Delta\omega_j$:

$$\Delta\bar{\omega}_j = \frac{\Delta\omega_j - \min(\Delta\omega_j)}{\max(\Delta\omega_j) - \min(\Delta\omega_j)}. \tag{14}$$

If $\max(\Delta\omega_j)$ is equal to $\min(\Delta\omega_j)$, let $\Delta\bar{\omega}_j = 0$.

Definition of damage growth position function (DGPF):

$$DGPF_{i,j} = 1 - |g_j(\zeta_i) - \Delta\bar{\omega}_j|. \tag{15}$$

Calculation of the Bayesian probability by fusing m DGPFs according to Bayes formula [21]:

$$P_i = \frac{\prod_{j=1}^{m} DGPF_{i,j}}{\sum_{k=1}^{n}\left(\prod_{j=1}^{m} DGPF_{k,j}\right)}. \tag{16}$$

Improvement of Bayesian probability:

$$Q_i = \sqrt{P_i P_{n+1-i}}. \tag{17}$$

Z-score normalization:

$$Z - score = \frac{Q - \bar{Q}}{\sigma}, \tag{18}$$

where $\bar{Q}$ is the mean of Q and σ is its standard deviation.

Formulation of damage growth indicator (DGI):

$$DGI = \begin{cases} Z - score, & Z - score \geq 0 \\ 0, & Z - score < 0 \end{cases}. \tag{19}$$

3. Numerical verification

A finite element model consisting of 200 elements is developed to simulate a SS beam. The geometrical parameters of the specimen are $1000 \times 30 \times 2$ mm^3. The material properties are as follows: modulus of elasticity 69 GPa, and density 2720 kg/m^3. Damage is first induced via a 5 % reduction in the modulus of elasticity of the 30th element. Then the damage severity is increased to 10 %. The first 5 natural frequencies of the specimen at two damage stages are calculated as shown in Table 1.
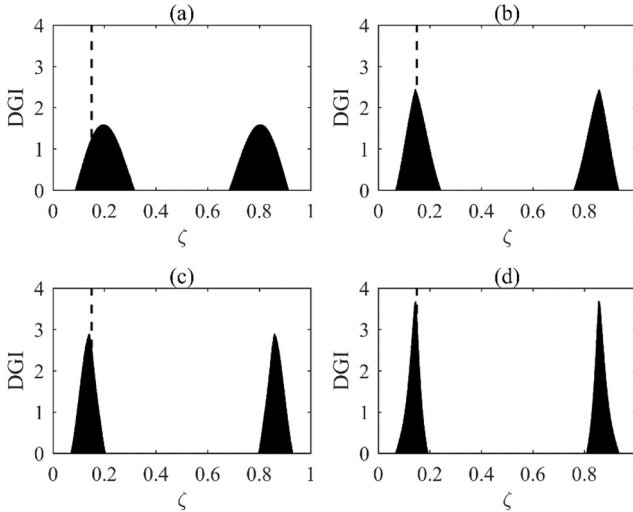

Fig. 2. DGIs obtained by fusing first: a) 2, b) 3, c) 4, and d) 5 DGPFs

The DGIs are calculated by fusing the first 2, 3, 4, and 5 DGPFs (Fig. 2). The real damage growth position is marked in dashed lines. It is clear from Fig. 2 that the monitoring precision and resolution maintain an increasing trend with the increase in the number of fused DGPFs. It should be noted that this method always shows damage growth positions symmetrically because of the symmetry in the structure [14, 20].

Table 1. Natural frequencies of the damaged beam at two stages (Hz)

Severity	1	2	3	4	5
5 %	4.567	18.267	41.097	73.060	114.162
10 %	4.567	18.264	41.086	73.040	114.144

4. Experimental validation

A test is conducted on a SS aluminum beam. The dimensions of the beam are 1000×30×3 mm³. Three damage stages (I, II, and III) are created. In Stage I, a through-thickness crack is cut to 3 mm in depth at the location 53 mm ($\xi = 0.053$) from the left end. In Stages II and III, the crack is extended to 6 and 12 mm in depth successively. The first 7 natural frequencies (Table 2) are measured using a scanning laser vibrometer (Fig. 3).

Stage I is set as the baseline to monitor damage growth from Stage I to Stages II and III. The DGIs are obtained by fusing the first 5 DGPFs and the results are presented in Fig. 4. It is seen from Fig. 4 that the DGIs peak at the dashed lines, thereby pinpointing the actual damage growth locations. It is noteworthy that the proposed method successfully locates damage growth near the boundary, where there are challenges to detecting such damage using some other methods, such as mode shape-based method, because of the interference of the boundary.
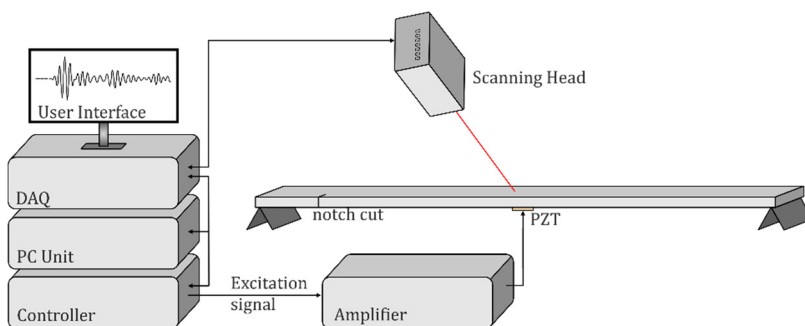

Fig. 3. Experimental setup

Table 2. Natural frequencies of the beam at three damage stages (Hz)

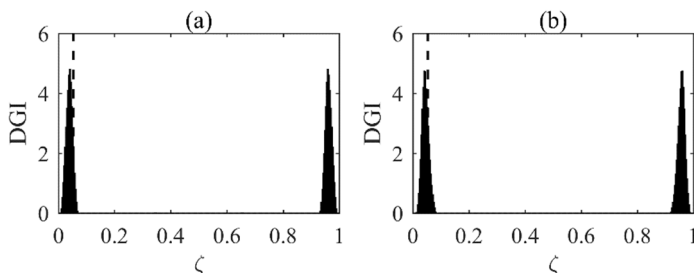| Stage	1	2	3	4	5	6	7
I	9.375	29.063	70.156	120.469	184.003	257.813	343.594
II	9.375	29.063	70.156	120.313	183.594	257.813	341.563
III	9.375	29.063	70.156	120.313	183.750	257.030	341.560

Fig. 4. DGIs obtained by fusing first 5 DGPFs: a) stages I to II; b) stages I to III

5. Conclusions

Damage-induced RNFC curves in beam-like structures are derived by theoretical analysis. For monitoring damage growth, an indicator titled DGI is proposed. The effectiveness of the indicator for monitoring damage growth is confirmed by both numerical and experimental cases. The

method can be used for monitoring damage growth in an efficient and cost-effective manner since natural frequencies can be conveniently measured at one accessible point of the structure.

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