# **Dynamic characteristics of aero-engine's rotor under large maneuvering flight**

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**Abstract.** In view of the large maneuvering overload of the rotor system in Unmanned Aerial Vehicle (UAV), the dynamic characteristics of rotor system under large maneuvering overload conditions were analyzed in this paper. The finite element model of rotor system under large maneuvering overload was first established by Timoshenko beam element theory and finite element method. The motion differential equations of the rotor system were derived by Lagrange equation, and the additional damping matrix, stiffness matrix and excitation force were obtained. Critical speeds and unbalance response were calculated by characteristic equation method and Newmark-β numerical integration method respectively. The calculation results showed that the additional damping and stiffness leaded the critical speeds of the rotor system to change under large maneuvering flight, and the additional excitation force resulted in a certain static displacement of the whirling orbits.

**Keywords:** large maneuvering overload, beam element theory, finite element method, critical speed, unbalance response.

#### **1. Introduction**

The maneuvering overload of mainstream fighters is generally less than 9 g because of the pilot's tolerance ability. With the continuous improvement of technology, the UAV appears frequently in modern local wars, mainly be responsible for tasks such as intelligence reconnaissance, military strikes, information confrontation, communication relay and logistics support, and its role is becoming more and more significant [1]. Compared with the traditional fighters, the UAV does not need to consider the tolerance ability limit of the human body.

In order to improve the maneuverability greatly, the maneuvering overload of the UAV will far surpass 9 g in the future, and higher requirements are needed for the rotor system in the UAV's engine than in traditional fighters, therefore, it is necessary to study the dynamic characteristics of the rotor system under large overload conditions (10 g-30 g).

A lot of research has been done on dynamic characteristics of rotor system under maneuvering flight by many scholars. Studies showed that maneuvering will add an additional load on the rotor system. As the overload increases, the effect will become more pronounced, and the dynamic characteristics of the rotor system will change [2].

The differential equations of motion under arbitrary maneuvering conditions were derived making use of Lagrange equation, beam element theory and finite element method for the rotor system, and the equations were calculated by characteristic equation method and numerical Newmark- $\beta$  integration method. Then the dynamic characteristics of rotor system were analyzed in this paper.

#### **2. Mathematical model**

According to the small deformation hypothesis, the bending deformation of the rotor is

generally much smaller than the span of the rotor. Under the circumstances, the differential equations of motion of the rotor system can be modeled by the finite element method and Timoshenko beam theory which can take advantage of shear deformation and moment of inertia.

The equation of motion of the rotor system is expressed by Eq. (1) when the maneuvering overload is not considered:

$$
M\ddot{u} + (C + \Omega G)\dot{u} + Ku = F_u + Q,\tag{1}
$$

where  $u$  represents the displacements of generalized degrees of freedom,  $M$  represents the inertia matrix of rotor system,  $C$  represents the damping matrix,  $G$  represents the gyroscopic matrix,  $K$ represents the stiffness matrix, Ω represents the speeds of rotor,  $F_u$  and Q represent unbalance force and external incentives on rotor system respectively.

The coordinate system is established to describe the motion of the rotor system considering the maneuvering overload, as shown in Fig. 1, where  $OXYZ$  represents fixed coordinate system on the ground,  $oxyz$  represents static coordinate system of rotor, and  $\alpha \xi \zeta \eta$  represents rotating coordinate system of the disk.



**Fig. 1.** Coordinate of the rotor system

According to the literatures [3, 4], the differential equation of motion of a disk under arbitrary maneuvering flight condition can be obtained by using the Lagrange equation:

$$
M_d \ddot{q}_d + (C_d + \Omega G_d + C_{dB}) \dot{q}_d + (K_d + K_{dB}) q_d = F_u + F_{dB} + Q_d, \tag{2}
$$

where  $q_d$  is the displacements of generalized degrees of freedom for the disk,  $M_d$ ,  $C_d$ ,  $G_d$  and  $K_d$ represent the mass, damping, gyroscopic and stiffness matrix of the disk respectively.  $Q_d$  is the external incentives on disk,  $C_{dB}$ ,  $K_{dB}$ , and  $F_{dB}$  represent the additional damping matrix, stiffness matrix, and excitation force, the expressions are as follows:

$$
C_{dB} = \begin{bmatrix} 0 & -2mW_{BZ} & 0 & 0 \\ 2mW_{BZ} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -m(W_{BY}^2 + W_{BZ}^2) & m(W_{BX}W_{BY} - W_{BZ}) & 0 & 0 \end{bmatrix}
$$
(3)

$$
K_{dB} = \begin{bmatrix} -m(w_{BY} + w_{BZ}) & m(w_{BX}w_{BY} - w_{BZ}) & 0 & 0 \\ m(W_{BX}W_{BY} + W_{BZ}) & -m(W_{BX}^2 + W_{BZ}^2) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$
 (4)

$$
F_{dB} = \begin{bmatrix} 0 & 0 & 0 & 0 \ -m(\ddot{X}_B - W_{BZ}\dot{Y}_B + W_{BY}\dot{Z}_B) - mz(W_{BX}W_{BZ} + W_{BY}) \\ -m(\ddot{Y}_B - W_{BZ}\dot{X}_B + W_{BX}\dot{Z}_B) - mz(W_{BY}W_{BZ} + \dot{W}_{BX}) \\ -I_p\Omega W_{BY} - I_d\dot{W}_{BX} \\ I_p\Omega W_{BX} - I_d\dot{W}_{BY} \end{bmatrix},
$$
(5)

where *m* represents the mass of the disk,  $X_B$ ,  $Y_B$ ,  $Z_B$  represent the displacements of the aircraft along the axes of X, Y, Z,  $W_{BX}$ ,  $W_{BY}$ ,  $W_{BZ}$  represent the angular velocity of the aircraft along the axes of X, Y, Z, and z represents the distance of disk and the origin of the coordinate system  $\partial xyz$ ,  $I_P$  is the polar moment of inertia,  $I_d$  is diameter moment of inertia of the disk.

In a similar way, the differential equation of motion of the beam element can be obtained by Lagrange equation:

$$
M_e \ddot{q}_e + (C_e + \Omega G_e + C_{eB}) \dot{q}_e + (K_e + K_{eB}) q_e = F_{eB} + Q_e, \tag{6}
$$

where,  $q_e$  is the displacements of generalized degrees of freedom for the beam element,  $M_e$ ,  $C_e$ ,  $G_e$  and  $K_e$  are the mass, damping, gyroscopic and stiffness matrix of the beam element respectively, and  $Q_e$  is the external incentives on the beam element.  $C_{eB}$ ,  $K_{eB}$ , and  $F_{eB}$  represent the additional damping matrix, stiffness matrix and excitation force respectively.

 $M_e$ ,  $G_e$ ,  $C_{eB}$ ,  $K_{eB}$  and  $F_{eB}$  can be expressed by the physical parameters, shape functions of the beam element and the parameters of maneuvering flight of the aircraft in matrix form, here, the formulas are omitted due to space limitations.

Now the differential equations of motion of the disk and the beam element have been established, and the differential equation of motion of the whole rotor system can be obtained by assembling the motion equations of each shaft segment and disk:

$$
M\ddot{q} + (C + \Omega G + C_B)\dot{q} + (K + K_B)q = F_u + F_B + Q,
$$
\n(7)

where,  $q$  is the displacements of generalized degrees of freedom for the rotor system,  $M, C, G$  and  $K$  are the mass, damping, gyroscopic and stiffness matrix of the rotor system respectively, and  $Q$ is the external incentives on the rotor.  $C_B$ ,  $K_B$ , and  $F_B$  represent the additional damping matrix, stiffness matrix and excitation force respectively.

#### **3. Calculation model**

Calculation model is showed in Fig. 2, the rotor system consists of two supporting and three disks, two of which are simple support disks, and the other one is a cantilever disk. The material of the shaft is 40Cr, and the material of disks is 45 steel, the main dimensions of the rotor are showed in Fig. 3, and other relevant data are showed in Table 1 and Table 2.



**Fig. 3.** The main sizes of rotor









	Horizontal turn	Horizontal rolling
Turning and rolling radius (m)		
Angular velocity (rad/s)	9.9	g g
maneuvering overload $(m/s2)$		

**Table 3.** Parameters of maneuvering flight

Taking maneuvering overload as 30 g as an example to analyze the influence of large maneuvering overload on the dynamic characteristics of rotor system, the parameters under different maneuvering flight are showed in Table 3.

## **4. Results and discussions**

## **4.1. Critical speeds calculation**

The critical speeds of the rotor system can be calculated by the characteristic equation method [5] when Eq. (1) and Eq. (7) are converted to homogeneous equations. The first three orders critical speeds of the rotor regardless of maneuvering conditions are showed in Table 4. The calculation results of the critical speeds under maneuvering conditions are showed in Table 5.

According to the equation of motion of the rotor system under large overload conditions, maneuvering will provide additional stiffness, damping, excitation on the rotor system, the additional stiffness is very small compared with the inherent stiffness of the system. The additional damping directly relates the maneuvering flight of horizontal rolling. When the rolling angular velocity is sufficiently large, the additional damping can reach the magnitude of the rotor system's inherent damping.

Therefore, when the aircraft makes a horizontal turn action, only a small additional stiffness is generated, and the critical speeds vary small, which can be neglected. While the aircraft performs the horizontal rolling motion, additional damping is generated which can cause a larger changes of the critical speeds. For the calculation model in this paper, the additional damping has the greatest impact on the first-order critical speed, which varies by about 2.45 % compared with the condition when the maneuvering overload is not considered.

Order	Critical speed (rpm)
First order	4040
Second order	6566
Third order	15319

**Table 4.** The first three critical speeds regardless of maneuvering conditions

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Maneuver	Order	Critical speed (rpm)	Contrast with straight flight (%)	
Horizontal turn $(30 g)$	First order	4039	0.025	
	Second order	6566		
	Third order	15319		
Horizontal rolling $(30 g)$	First order	3941	2.45	
	Second order	6542	0.37	
	Third order	15223	0.63	

**Table 5.** Effect of maneuvering flight on critical speed

## **4.2. Unbalanced response analysis:**

The steady response of the rotor system was solved by the Newmark- $\beta$  numerical integration method. The response results are plotted as three-dimensional spectrums, as shown in Fig. 4 to Fig. 6.









a) Disk 1 along x-axis b) Disk 1 along y-axis c) Disk 3 along x-axis d) Disk 3 along y-axis **Fig. 4.** Three-dimensional spectrogram regardless of maneuvering conditions









a) Disk 1 along x-axis b) Disk 1 along y-axis c) Disk 3 along x-axis d) Disk 3 along y-axis **Fig. 5.** Three-dimensional spectrogram of horizontal turning









a) Disk 1 along  $x$ -axis b) Disk 1 along  $y$ -axis c) Disk 3 along  $x$ -axis d) Disk 3 along  $y$ -axis

**Fig. 6.** Three-dimensional spectrogram of horizontal rolling c) Disk 3 along  $x$ -axis



**Fig. 7.** Comparison of orbit in various maneuvering actions

According to the three-dimensional spectrogram, the additional centrifugal force and gyroscopic moment cause the rotor to generate a zero-frequency displacement component. When neglected. When the aircraft performs the horizontal rolling, the critical speeds make a certain change, and this verifies the previous conclusion.

Due to the additional centrifugal force and gyroscopic moment generated by the maneuvering flight action, the whirling orbit of the rotor has made a certain static displacement, the comparing whirling orbits in each case are showed in Fig. 7. The horizontal turn action generates a static displacement along both  $x$  and  $y$  directions, while the horizontal rolling action generates a static displacement along x direction. In addition, the horizontal rolling action causes a variation of the critical speeds, which can be seen from the variation of the whirling radius.

### **5. Conclusions**

The motion differential equations of the rotor system under arbitrary maneuvering conditions were first derived by beam element theory, finite element method and Lagrange equation, and the critical speeds and unbalance response were calculated by characteristic equation method and Newmark- $\beta$  numerical integration method in this paper. The simulation results showed that the maneuvering flight of the aircraft will have an important impact on the dynamic characteristics of the rotor system. Additional damping and stiffness generated under large maneuvering overload will change the critical speeds of the rotor system. The additional damping makes a greater influence than the additional stiffness on critical speeds. In addition, the maneuvering flight will generate additional excitation force on the rotor system, including centrifugal force and gyroscopic moment. The additional excitation force will cause a certain static displacement of the whirling orbits, which will increase the risk of rub between the rotor and the stator. Therefore, the design of clearance between rotor and stator in aero-engine under maneuvering flight conditions need to be focused.

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