# Two span bridge under moving load

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*Received 26 February 2019; accepted 20 March 2019*
*DOI https://doi.org/10.21595/vp.2019.20607*

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**Abstract.** The dynamic effect of the moving load on the bridge construction is the subject of the solution in this article. The bridge is modeled as two span continuous beam with two degrees of freedom. The assumption describing the dynamic deflection curve and the assumption describing the load distribution on individual lumped masses are adopted at the creation of bridge computational model. The plane computational model of heavy vehicle with five degrees of freedom is adopted. The problem is described by ordinary differential equations which are solved numerically by using MATLAB. The results are presented by graphical and numerical form.

**Keywords:** moving load, bridges, dynamics, numerical solution, computational models.

## 1. Introduction

The problems of vehicle bridge interaction belongs to the oldest solved problems of structural dynamics. The works of the civil engineer R. Willis [1] and mathematician G. G. Stoks [2] in which they tried to clarify the breakdown of Chester Rail Bridge in England in 1847 are not only considered the first attempts to solve the problems of vehicle runway interaction but also the first works in the field of structural dynamics. Slovak and Czech Republic are world-known by the high level of bridge engineering and theoretical approach to the solution of dynamical problems of bridges. Basic knowledge concerning of the dynamic investigation of highway and railway bridges was published in monographs [3, 4]. Numerical methods offer an effective tool for the solution of this problem. Current state of computers enables to solve all the problems in real time. The results obtained from numerical analyses are used in the process of the design of optimal parameters of bridges with respect to its lifetime and reliability.

## 2. Computational vehicle and bridge model

This contribution works with half planar computational model of the lorry Tatra 815, Fig. 1. Equations of motion for 5 unknown functions $r_1(t) \div r_5(t)$ describing the vehicle vibration are derived as ordinary differential Eq. (1), [5]:

$$\begin{aligned}
\ddot{r}_1(t) &= -\frac{\left\{k_1 \cdot d_1(t) + b_1 \cdot \dot{d}_1(t) + k_2 \cdot d_2(t) + b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dfrac{\dot{d}_2(t)}{\dot{d}_c}\right\}}{m_1}, \\
\ddot{r}_2(t) &= -\frac{\left\{-a \cdot k_1 \cdot d_1(t) - a \cdot b_1 \cdot \dot{d}_1(t) + b \cdot k_2 \cdot d_2(t) + b \cdot b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dfrac{\dot{d}_2(t)}{\dot{d}_c}\right\}}{I_{y1}}, \\
\ddot{r}_3(t) &= -\frac{\left\{-k_1 \cdot d_1(t) - b_1 \cdot \dot{d}_1(t) + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t)\right\}}{m_2},
\end{aligned} \tag{1}$$

$$\ddot{r}_4(t) = -\frac{\left\{ \begin{array}{l} -k_2 \cdot d_2(t) - b_2 \cdot \dot{d}_2(t) - f_2 \cdot \dfrac{\dot{d}_2(t)}{\dot{d}_c} + k_4 \\ \cdot\, d_4(t) + b_4 \cdot \dot{d}_4(t) + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t) \end{array} \right\}}{m_3},$$
$$\ddot{r}_5(t) = -\frac{\left\{-c \cdot k_4 \cdot d_4(t) - c \cdot b_4 \cdot \dot{d}_4(t) + c \cdot k_5 \cdot d_5(t) + c \cdot b_5 \cdot \dot{d}_5(t)\right\}}{I_{y3}}.$$

The contact forces $F_{int,i}$ ($i = 6, 7, 8$) belong to individual contact points [5] are expressed as:

$$\begin{aligned} F_{int,6}(t) &= -G_6 + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t), \\ F_{int,7}(t) &= -G_7 + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t), \\ F_{int,8}(t) &= -G_8 + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t). \end{aligned} \tag{2}$$

In the above equations $d$ represents the deformation of joining elements, $k$, $b$, $m$ are stiffness, damping, mass characteristics and $G$ is gravity force. The dot above the symbol marks the derivative with respect to time $t$.

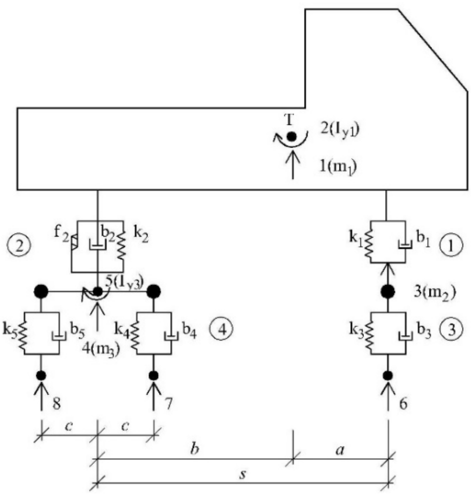


**Fig. 1.** Planar vehicle computational model

For the bridge the beam computational model with two degrees of freedom was adopted. The shape of deflection curve $v(x, t)$ in time moment $t$ and load distribution function $\phi(x)$ were adopted in the computation as sine function, Fig. 2.

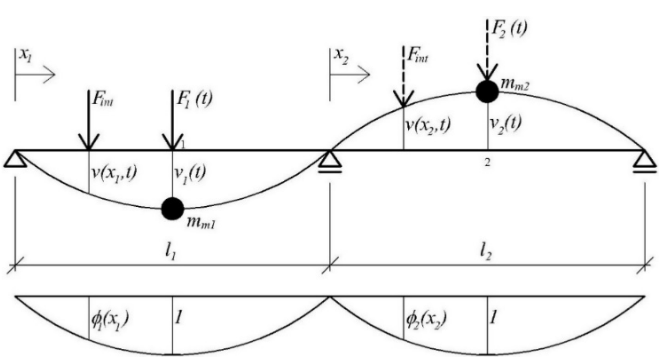


**Fig. 2.** Beam computational model of a bridge with two degrees of freedom and load distribution function

Than by the [6]:

$$v(x,t) = \phi(x) \cdot v(t) + h(x), \quad v_x(t) = \phi_x(t) \cdot v(t) + h(t), \tag{3}$$

where:

$$\phi(x) = \sin\left(\pi \cdot \frac{x}{l}\right), \quad \phi_x(t) = \sin\left(\pi \cdot e \cdot \frac{t}{l}\right) = \sin(\omega \cdot t), \quad \omega = \pi \cdot \frac{e}{l}, \tag{4}$$

$$F(t) = F_{int}(t) \cdot \phi_x(t). \tag{5}$$

Equations of motion can be written as:

$$[m]_D \cdot \{\ddot{v}(t)\} + 2 \cdot \omega_b \cdot [m]_D \cdot \{\dot{v}(t)\} + [k] \cdot \{v(t)\} = \{F(t)\}. \tag{6}$$

In the above expressions $t$ is time coordinate, $x$ length coordinate, $d(t)$ deformation of joining elements, $h(x)$, $h(t)$ road roughness, $e$ speed of vehicle motion in [m/s], $\omega$ angular frequency, $\omega_b$ damping angular frequency. Derivations of functions with respect to time are denoted by dot above the symbol.

## 3. Numerical solution

For the numerical simulation of vehicle motion along the bridge structure the computer program in the program language MATLAB was created. The following input data for vehicle TATRA 815 and for the bridge were used during computation.

Vehicle input data and initial conditions: $k_1 = 287433$ N/m, $k_2 = 1522512$ N/m, $k_3 = 2550600$ N/m, $k_4 = k_5 = 5022720$ N/m, $b_1 = 19228$ kg/s, $b_2 = 260197$ kg/s, $b_3 = 2746$ kg/s, $b_4 = b_5 = 5494$ kg/s, $m_1 = 22950$ kg, $m_2 = 910$ kg, $m_3 = 2140$ kg, $I_{y1} = 62298$ kg·m$^2$, $I_{y3} = 932$ kg·m$^2$, $a = 3.135$ m, $b = 1.075$ m, $s = 4.210$ m, $c = 0.660$ m, $r_1(0) = -0.02$ m, $\dot{r}_1(0) = 0.0$ m/s, $r_2(0) = 0.00$ rad, $\dot{r}_2(0) = 0.0$ rad/s, $r_3(0) = -0.002$ m, $\dot{r}_3(0) = 0.0$ m/s, $r_4(0) = -0.003$ m, $\dot{r}_4(0) = 0.0$ m/s, $r_5(0) = 0.00$ rad, $\dot{r}_5(0) = 0.0$ rad/s.

Bridge input data and initial conditions: $\mu = 19680.0$ kg/m, $I = 1.60622$ m$^4$, $E = 3.85e10$ N/m$^2$, $\omega_b = 0.1$ rad/s, $m_{m1} = 285360.0$ kg, $l_1 = 29.0$ m, $m_{m2} = 285360.0$ kg, $l_2 = 29.0$ m, $v_1(0) = 0.0$ m, $\dot{v}_1(0) = 0.0$ m/s, $v_2(0) = 0.0$ m, $\dot{v}_2(0) = 0.0$ m/s.

The results of the numerical simulation are the time course of the vehicle and bridge vibration in graphical or numerical form. For the vehicle the time courses of the displacement components of characteristic points, corresponding the individual degrees of freedom, can be displayed. For the bridge the time courses of vertical mid span deflections of the individual bridge spans can be displayed. It is possible to change vehicle and bridge parameters, initial conditions and vehicle speed. The road profile can be smooth or a random variable.

As an example of possible outputs the time courses of bridge and vehicle vibration while the lorry Tatra 815 passes the bridge at the speed 70 km/h along smooth road surface are presented. Fig. 3 and 4 show the time courses of vertical deflections in the middle of the 1st and 2nd spans.

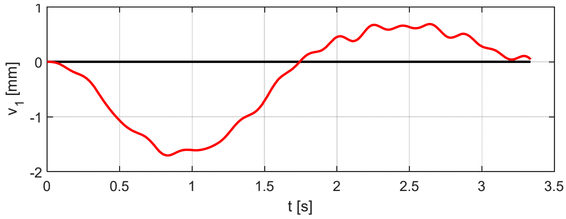


**Fig. 3.** Vertical bridge deflection in the middle of the 1st span. Vehicle speed $V = 70$ km/h

In Fig. 5 the time course of vertical deflection of the vehicle sprung mass gravitational center is displayed. In Fig. 6 the time course of rotation of the vehicle sprung mass is displayed.

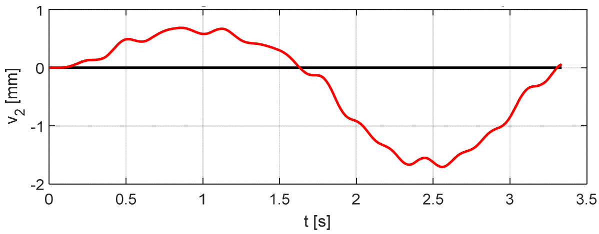


**Fig. 4.** Vertical bridge deflection in the middle of the 2nd span. Vehicle speed $V = 70$ km/h

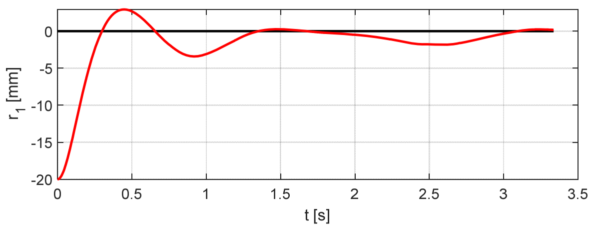


**Fig. 5.** Vertical deflection of the vehicle sprung mass gravitational center. Vehicle speed $V = 70$ km/h

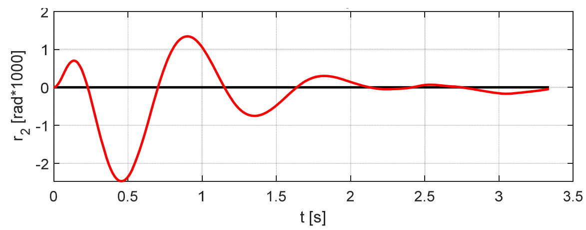


**Fig. 6.** Rotation of the vehicle sprung mass. Vehicle speed $V = 70$ km/h

## 4. Conclusions

Numerical modeling of the problems of vehicle and bridge interaction is an effective tool for the solution of real tasks of engineering practice. It allows numerically to obtain results that were previously obtained only by experimental test on the finished structure. Quality of obtained results is dependent on the quality of input data. Current state of computers enables the numerical processing of solved problems in real time. From the practical point of view the influence of various vehicle and bridge parameters is interested. The results obtained from numerical analyses are used in the process of design of optimal bridge parameters with respect to lifetime and reliability of the bridge structure.

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