

Parameter optimization design of rotor dynamic vibration absorber

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Abstract. The dynamic vibration absorber, which is adopted to suppress the unbalanced vibration of rotor, is optimized for the optimal parameters in this paper. This paper proposes a parameter optimization method for dynamic vibration absorbers and seeks parameters of a dynamic vibration absorber with better vibration suppression performance. Firstly, the frequency response function of the dynamic vibration absorber-rotor coupling system is obtained by using the finite element method. Then, basing on the optimal mathematical model, the optimal design variables are solved with the adaptive particle swarm optimization algorithm. Also, an example is used to prove the validity of the optimization design method mentioned in this paper. Further, in order to master the influence of deviation from the optimal value on the suppressing vibration effect, the vibration suppression performance changes of the dynamic vibration absorber whose parameters deviate from the optimal value are analyzed. The results show that: compared with conventional design method, this method is more superior; The dynamic vibration absorber with optimal parameters has better vibration suppression performance; At the same degree deviated from the optimal value, the stiffness has a more remarkable influence on the vibration suppression performance than damping for suppressing the first resonance; For the dynamic vibration absorber which is adopted to suppress the fixed-frequency vibration, the influence of stiffness deviation on the vibration suppression performance appears an obvious interval which is related to working speed.

Keywords: rotor, unbalanced, vibration suppression, dynamic vibration absorber, parameter optimization, adaptive particle swarm optimization algorithm.

Nomenclature

x_A, x_B, y_A, y_B	Translations of shaft element node
$\theta_{xA}, \theta_{xB}, \theta_{yA}, \theta_{yB}$	Rotations of shaft element node
$\mathbf{M}_p, \mathbf{C}_p, \mathbf{G}_p, \mathbf{K}_p$	Mass matrix, damping matrix, gyro matrix, stiffness matrix of the rotor system
$\mathbf{M}, \mathbf{C}, \mathbf{G}, \mathbf{K}$	Mass matrix, damping matrix, gyro matrix, stiffness matrix of the dynamic vibration absorber-rotor coupling system
$\mathbf{F}_p(t), \mathbf{F}(t)$	Unbalanced force vectors of the rotor system and coupling system
$\mathbf{M}_a, \mathbf{C}_a, \mathbf{K}_a$	Mass matrix, damping matrix, stiffness matrix of dynamic vibration absorber system
$\mathbf{C}_{a1}, \mathbf{K}_{a1}$	Damping cross matrix, stiffness cross matrix produced by the coupling the dynamic vibration absorber and the rotor system
m_a, c_a, k_a	Equivalent mass, damping, stiffness of the dynamic vibration absorber in x -direction, y -direction
$\mathbf{x}_p, \dot{\mathbf{x}}_p, \ddot{\mathbf{x}}_p$	Displacement vector, velocity vector and acceleration vector of the rotor system

$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$	Displacement vector, velocity vector and acceleration vector of the coupling system
ω	Rotating speed of the rotor system
X_j	Amplitude of response of node j at ω
$\mathbf{F}_0(\omega)$	Amplitude of unbalanced force
\mathbf{V}_D	Design variable
\mathbf{V}'_D	Normalized design variable
i	Imaginary unit
ω_a	Natural frequency of the dynamic vibration absorber system
ω_p	Critical speed of the rotor system, Natural frequency of equivalent system
γ_a	Ratio between ω_a and ω_p
ζ_a	Damping factor of absorber system
β	Amplitude magnification
γ_{aopt}	Optimum tuning parameter
ζ_{aopt}	Optimum damping ratio
β_{aopt}	Optimum amplitude magnification
ω_f	Working speed of the rotor system
μ	Mass ratio between absorber system and equivalent system
k_s, c_s	Support stiffness and support damping of the rotor system
A_p	Resonant amplitude of the rotor system
A_f	Amplitude of response of the rotor system at ω_f
f_{Fit}	Fitness function
A_{Fit}	Value of fitness function
c_1, c_2	Learning factor
N_{pso}	Population size
m	Particle dimension
k_1, k_2	Coefficient of variation of weight factor

1. Introduction

In modern rotating machinery, most of them work at supercritical or higher speed. Various factors, such as uneven adhesion of dirt, wear or shedding of impeller and blades, might lead to mass unbalances of the system during long-term operation. The mass unbalances will cause vibration of the systems increase. If this problem gets worse, the equipment running stability and safety will be seriously affected.

The usual measure to solve unbalance fault is to return factories for making the dynamic balance [1-3], which not only requiring expensive maintenance, but also caused great economic losses due to production shutdown. In order to solve the problem of downtime maintenance, the automatic balancing technology, which includes passive and active automatic balancing, is developed. In passive automatic balancing technology, spherical structure [4] and pendulum structure [5] are appended in the rotor structure to suppress vibration of the rotor. There are various forms of active automatic balancing mechanism. Early online automatic balancing technology is to online re-balance the systems with unbalanced failure, the main methods of which are as follows: (a) On the correction planes, to increase weight online by spraying [6], or to reduce weight online by using electro corrosion and laser cutting [7], is effective for small-sized rotors, but it has many shortcomings, such as a bad effect on large scale rotor, stress concentration and so on. (b) To redistribute mass, the mass blocks are drove online [8], which requires a greater quality counterweight structure and sustained provision of energy. Late online automatic balancing technology, which mainly takes the form of applying recreation forces on the rotor structure, often uses the following methods: (a) For balancing the system, to apply real-time recreation forces to

the rotor system by using controllable electromagnetic bearings [9-11]. (b) To apply a real-time variable damping force to the system by using controlled magnetorheological fluid damper [12-15]. However, the precipitation problem of magnetorheological fluids affects long-term use. On-line balancing structures require additional control devices and consume a great deal of energy.

In order to find a convenient and economical way to suppress vibration, some scholars have applied the dynamic vibration absorber to the rotor vibration suppression field. Zhang et al. [16] reduced the first order resonance amplitude by 70 % adopting the passive ring dynamic vibration absorbers; Hu et al. [17] effectively reduced the violent vibration when the rotor starts to pass the first critical speed by adopting the rotor dynamic vibration absorber; Yao et al. [18] effectively reduced the lateral vibration of the rotor system by adopting the vibration absorber with negative stiffness property. It is not given definite parameter optimization methods of the dynamic vibration absorbers mentioned in the above references, which means that they do not have optimal vibration suppression performance. In addition, as for the structure, damping elements is not fully considered in the dynamic vibration absorbers mentioned in the above references, which are shown in Fig. 1.

Based on the above problems, in this paper, the dynamic vibration absorber, which is adopted to suppress the first resonance and fixed-frequency vibration, was optimized for the optimal parameters, and the vibration suppression performance of the absorber was studied when the parameter deviates from the optimal value. Dynamic vibration absorbers have mass, spring and damping unit. For solving the optimal parameters of the dynamic vibration absorbers, this paper presents an optimization methods based on the finite element theory and the adaptive particle swarm optimization algorithm. Firstly, the frequency response function of the dynamic vibration absorber-rotor coupling system is obtained by using the finite element method. Then, by taking stiffness and damping as the design variables, the optimal mathematical model is established based on the frequency response function. And the optimization problem is solved with the adaptive particle swarm optimization algorithm. In order to prove the effectiveness of the optimization method, the optimal parameters are compared with the optimal parameters solved by the common optimization methods. The influence of parameters deviation from the optimal value on vibration suppression performance is analyzed to further master the effect of parameter variation on the absorbers.

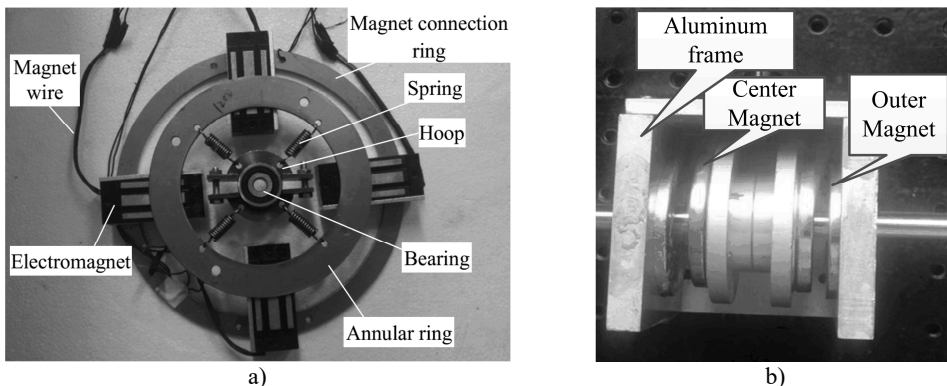


Fig. 1. For the translation mechanism and the rotating body of the dynamic vibration absorber

2. Kinetic equation of the dynamic vibration absorber-rotor coupling system and programming for optimization problems

2.1. Kinetic equation of the dynamic vibration absorber-rotor coupling system

(1) Kinetic equation of the rotor system without dynamic vibration absorber.

The rotor system is a single span-single disk structure which is shown in Fig. 2. Linear

symmetric supports are adopted at the both ends of the rotation axle. There is mass eccentricity for disk.

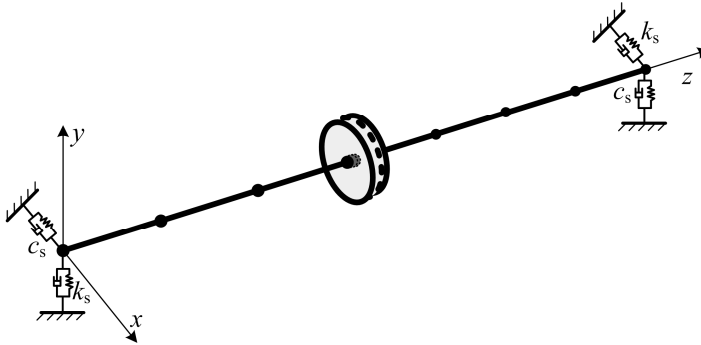


Fig. 2. System diagram

The rotor system without dynamic vibration absorber is dispersed into q nodes and $q - 1$ flexible shaft section elements by finite element method. The shaft section element, as showed in Fig. 3, takes the Euler Bernoulli beam model.

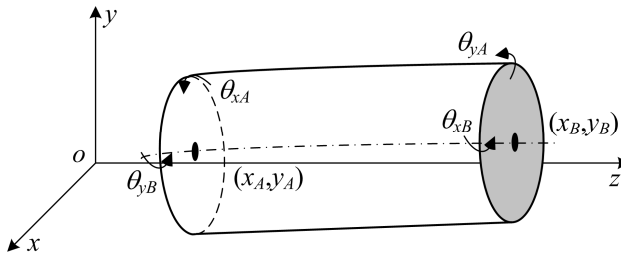


Fig. 3. Euler Bernoulli beam model

Generalized coordinate of each shaft section element is as follows:

$$[x_A \quad y_A \quad \theta_{xA} \quad \theta_{yA} \quad x_B \quad y_B \quad \theta_{xB} \quad \theta_{yB}]^T. \quad (1)$$

Considering the gyroscopic moment and moment of inertia, the kinetic equation of the rotor system can be formulated as follows based on the Lagrange equation:

$$\mathbf{M}_p \ddot{\mathbf{x}}_p + (\mathbf{C}_p + \mathbf{G}_p) \dot{\mathbf{x}}_p + \mathbf{K}_p \mathbf{x}_p = \mathbf{F}_p(t). \quad (2)$$

(2) Dynamic model simplification of rotor dynamic vibration absorber.

Based on the structure of vibration absorbers reported in the references, the structure of the vibration absorber designed in this paper is shown in Fig. 4(a). Because the rotor system is mainly affected by unbalanced forces, the rotor dynamic vibration absorber is decomposed into two independent, single degree of freedom, mass - damping - spring systems, are shown in the dotted boxes in Fig. 4.

(3) Dynamic modeling of dynamic vibration absorber-rotor coupling system.

Introducing the simplified vibration absorber in the rotor system, the kinetic equation of the dynamic vibration absorber -rotor coupling system can be expressed as:

$$\mathbf{M} \ddot{\mathbf{x}} + (\mathbf{C} + \mathbf{G}) \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{F}(t), \quad (3)$$

where:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_a \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_p + \mathbf{C}_{a1} & \mathbf{C}_{pa} \\ \mathbf{C}_{pa}^T & \mathbf{C}_a \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}_p + \mathbf{K}_{a1} & \mathbf{K}_{pa} \\ \mathbf{K}_{pa}^T & \mathbf{K}_a \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{F}(t) = \begin{bmatrix} \mathbf{F}_p(t) \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{M}_a = \begin{bmatrix} m_a & 0 \\ 0 & m_a \end{bmatrix}, \quad \mathbf{C}_a = \begin{bmatrix} c_a & 0 \\ 0 & c_a \end{bmatrix},$$

$$\mathbf{K}_a = \begin{bmatrix} k_a & 0 \\ 0 & k_a \end{bmatrix}, \quad \mathbf{C}_{pa} = \begin{bmatrix} 0 & \dots & 0 & -c_a & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & -c_a & 0 & 0 & \dots & 0 \end{bmatrix}^T,$$

$$\mathbf{K}_{pa} = \begin{bmatrix} 0 & \dots & 0 & -k_a & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & -k_a & 0 & 0 & \dots & 0 \end{bmatrix}^T.$$

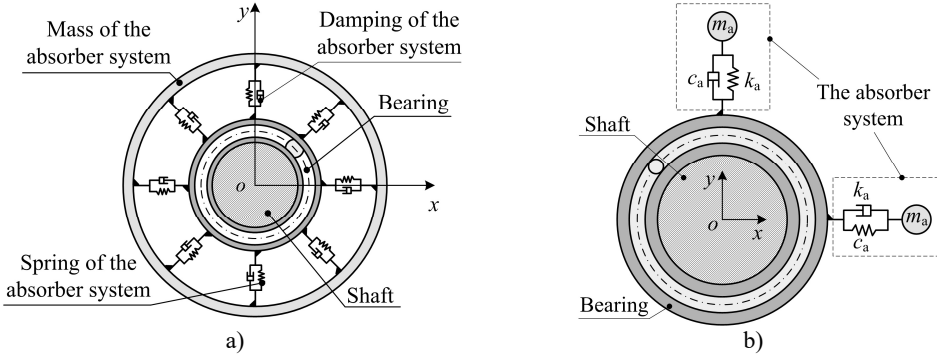


Fig. 4. Model simplification of rotor dynamic vibration absorber

Let $\mathbf{F}(t) = \mathbf{F}_0(\omega)e^{i\omega t}$, $\mathbf{x} = \mathbf{X}e^{i\omega t}$, then Eq. (3) can be transferred as:

$$-\omega^2 \mathbf{M} \mathbf{X} + i\omega (\mathbf{C} + \mathbf{G}_p) \mathbf{X} + \mathbf{K} \mathbf{X} = \mathbf{F}_0(\omega). \quad (4)$$

Then, the response amplitude of the system can be expressed as:

$$\mathbf{X} = \mathbf{H}^{-1} \mathbf{F}_0(\omega), \quad (5)$$

where:

$$\mathbf{H} = -\omega^2 \mathbf{M} + i\omega (\mathbf{C} + \mathbf{G}_p) + \mathbf{K}.$$

The frequency response function of the j th node in the system can be expressed as:

$$X_j = \mathbf{H}_{r_j}^{-1} \mathbf{F}_0(\omega), \quad (6)$$

where, let $\mathbf{H}_{r_j}^{-1}$ denote the j th row of \mathbf{H}^{-1} .

In what follows, we refer to the rotor system without dynamic vibration absorber as the original system, the system composed of dynamic vibration absorber and rotor system as coupling system, the rotor system in coupling system as the main system.

2.2. Programming for optimization problems

(1) Optimization mathematical model.

There are two types of dynamic vibration absorber, which are used to suppress vibration of the rotor system affected by unbalanced forces: One is used to suppress the first resonance of the rotor system during startup process, to reduce the response amplitude of first resonances; and another

used to suppress the fixed-frequency vibration generated while the rotor running at speed ω_f to reduce vibration response amplitude. But one thing they have in common is that the optimization design methods they used, which can obtain the minimum response amplitude of the main system by finding optimal stiffness and damping of absorber. Based on this aspect, the design variable and objective function of optimization design is the same for both types, that is:

- The objective function is $f(\mathbf{V}_D) = X_j = \mathbf{H}_{r_j}^{-1} \mathbf{F}_0(\omega)$.
- The design variable is $\mathbf{V}_D = [k_a, c_a]^T$.

The aim to attach the dynamic vibration absorber for suppressing the first resonance is not to minimize the amplitude of the main system while it running at speed ω_p , but to minimize the first resonance amplitude during its startup process. But aim to attach the dynamic vibration absorber for suppressing the fixed-frequency vibration is to minimize response amplitude of the main system while it running at speed ω_f . So, for the optimization design of two types of dynamic vibration absorber, there are some differences in the extremal problem of the objective function and constraint condition, which are as follows:

- a) The dynamic vibration absorber for suppressing the first resonance of the rotor system:

The objective function is $\min(\max(f(\mathbf{V}_D)))$;

The constraint condition is $0 \leq k_a \leq 4m_a\omega_p^2, 0 \leq c_a \leq 2\sqrt{m_a k_a}, 0.8\omega_p \leq \omega \leq 1.2\omega_p$.

- b) The dynamic vibration absorber for suppressing the fixed-frequency vibration:

The objective function is $\min(f(\mathbf{V}_D))$;

The constraint condition is $0 \leq k_a \leq 4m_a\omega_f, 0 \leq c_a \leq 2\sqrt{m_a k_a}, \omega = \omega_f$.

By means of the parameters substitution, the optimization design of two types of dynamic vibration absorber can be described as follows:

The dynamic vibration absorber for suppressing the first resonance of the rotor system

In the range of $0 \leq \gamma_a \leq 2, 0 \leq \zeta_a \leq 1$ and $0.8 \leq \lambda \leq 1.2$, to find the best design variable $\mathbf{V}'_{D_{opt}} = [\gamma_{aopt} \ \zeta_{aopt}]^T$ that satisfies the function of:

$$\min(\max(\beta(\mathbf{V}'_D))), \tag{7}$$

where, the frequency ratio of the dynamic vibration absorber is $\gamma_a = \omega_a/\omega_p$. The damp ratio of the dynamic vibration absorber is $\zeta_a = c_a/2\sqrt{m_a k_a}, \beta(\mathbf{V}'_D) = f(\mathbf{V}_D)/A_p$.

- a) The dynamic vibration absorber for suppressing the fixed-frequency vibration

In the range of $0 \leq \gamma_a \leq 2, 0 \leq \zeta_a \leq 1$, and $\lambda_f = 1$, to find the best design variable $\mathbf{V}'_{D_{opt}} = [\gamma_{aopt} \ \zeta_{aopt}]^T$ that satisfies the function of:

$$\min(\beta(\mathbf{V}'_D)), \tag{8}$$

where, the frequency ratio of the dynamic vibration absorber is $\gamma_a = \omega_a/\omega_f$. The damp ratio of the dynamic vibration absorber is $\zeta_a = c/2\sqrt{m_a k_a}, \beta(\mathbf{V}'_D) = f(\mathbf{V}_D)/A_f$.

(2) Optimization method.

Because the discrete system has a large number of degrees of freedom, it is difficult to find a definite optimal solution formula, although the frequency response function of the coupling system can obtain the semi-numerically semi-analytical response solution. In this case, can only adopt a numerical optimization method to find optimal design variable that satisfies the Eq. (7) or Eq. (8). The adaptive particle swarm optimization algorithm method is adopted to solve the optimization mathematical model in this paper, as showed in the following steps.

(a) Single degree of freedom system, which can be equivalent to the original system, is solved by the characteristic vector method [19]. And then the absorber which is applicable to this system is designed in this step, the parameter of which is $\mathbf{V}'_{D_{opt}}$; (b) By using an adaptive particle swarm

optimization algorithm method, the optimal value of $\mathbf{V}'_{D_{opt}}$ is searched. Initialize each particle of the initial particle group randomly in the region that $[0.5, 1.5] \times \mathbf{V}'_{D_{0opt}}$; The fitness functions for optimizing the two dynamic vibration absorbers is respectively defined as $f_{Fit} = \max(\beta(\mathbf{V}'_D))$ and $f_{Fit} = \beta(\mathbf{V}'_D)$.

For details of adaptive particle swarm optimization algorithm method, refer to [20] listed in references.

3. Numerical simulations

3.1. Parameters and modal analysis of the original system

The original system is dispersed into 10 nodes. And the specific parameters of each shaft section are shown in Table 1. The node n_5 is in the center of the disc, mass eccentricity moment of which is $me = 1 \times 10^{-5}$ kgm. Both node n_1 and n_{10} are in a support position, their support stiffness is $k_s = 1 \times 10^8$ N/m, their damping is $c_s = 7 \times 10^5$ Ns/m.

Table 1. Parameters of shaft

Parameters	L_i / mm	d_i / mm	Parameters	L_i / mm	d_i / mm
l_1	60	10	l_6	50	10
l_2	60	10	l_7	50	10
l_3	50	10	l_8	50	10
l_4	10	80	l_9	50	10
l_5	10	80	—	—	—

Performing modal analysis on the original system, the results are shown in Table 2 and Fig. 5 respectively, to obtain the first two critical speeds and the first vibration mode.

Observing the vibration shape diagram shows that the maximum amplitude node of the original system is n_5 , when the first resonance occurs. So, we act n_5 as an objective vibration suppression point, and its amplitude as an evaluation object for vibration suppression performance of dynamic vibration absorbers.

Table 2. Main second order critical speed of the original system

Order	The first order	The second order
Critical speed / Hz	52.7	389.5

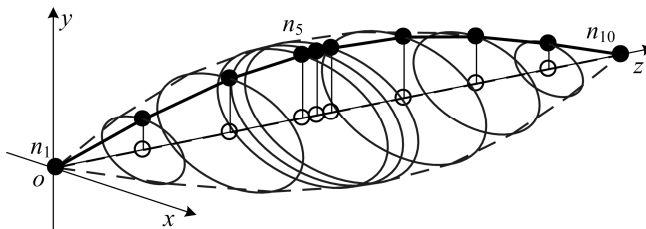


Fig. 5. First order form of original system

3.2. APSO convergence and accuracy of the algorithm

The APSO algorithm is used to optimize the parameters of the dynamic vibration absorber with mass ratio of $\mu = 0.15$, and compared with PSO and CFPSO algorithm. The basic parameters of the APSO algorithm are: the particle dimension is $m = 2$, which is the frequency ratio γ_a and damping ratio ζ_a of the absorber; the population size is $N_{ps0} = 100$; the number of iterations is 50; the learning factor $c_1 = 2.05$, $c_2 = 2.05$; the coefficient of variation $k_1 = 1.5$ and $k_2 = 2$.

After several experiments, the optimal convergence curve in the optimal case is shown in

Fig. 6. As can be seen from the figure, the APSO algorithm has a faster convergence rate than the PSO [21] algorithm and the CFPSO [22] algorithm. Also, A_{Fit} is smaller than the other two algorithms. This also proves that the APSO algorithm is reliable.

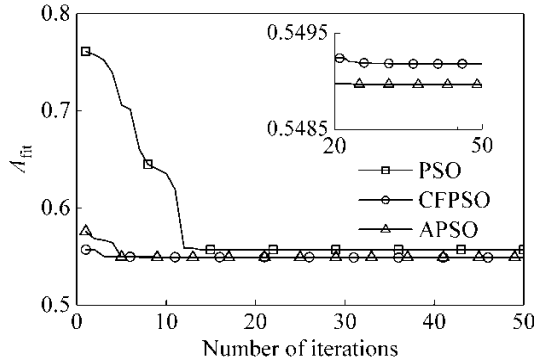


Fig. 6. Optimized convergence curves for PSO, CFPSO and APSO Algorithms

3.3. Optimization design of the dynamic vibration for suppressing the first resonance

(1) Comparison with conventional optimal design method.

With suppressing a certain order resonance of the continuum system, often, we calculate the equivalent mass and equivalent stiffness of this order mode, and then simplify this system as a single degree of freedom system. And then attach the single degree of freedom dynamic vibration absorber whose natural frequency is equal or close to the natural frequency of the main system on it. According to whether damping is existed in the main system and the vibration absorber system, two degrees of freedom coupling system are classified into three categories; each one corresponds to a common parameter optimization method. In this section, the three methods are respectively used in the optimization design of dynamic vibration absorber parameters.

The parameters of the single degree of freedom system which is equivalent to the first order mode of the original system are obtained by the characteristic vector method, that is, $m_p = 1.1073\text{kg}$, $k_p = 1.115 \times 10^5 \text{ N/m}$ and $c_p = 25.3 \text{ Ns/m}$. Here, the mass ratio of the dynamic vibration absorber to the main system is expressed as $\mu = m_a/m_p$. Under the condition that $\mu = 0.15$, the dynamic vibration absorber, which is attached at position of node n_7 , is optimized.

Approach 1: It is assumed that the main system is a single degree of freedom mass-spring system. Then the dynamic vibration absorber which given the code name DVA1 is designed as a single degree of freedom mass-spring system. In this case, it only needs to meet $\omega_a = \omega_p$.

Approach 2: It is assumed that the main system is a single degree of freedom mass-spring system. Then the dynamic vibration absorber which given the code name DVA2 is designed as a single degree of freedom mass-damping-spring system. Research by Den Hartog [23] and Brock [24] shows that the optimal vibration suppression performance can be achieved when the parameters of this type of dynamic vibration absorber satisfies Eq. (9):

$$\begin{cases} \gamma_{aopt} = \frac{\omega_a}{\omega_p} = \frac{1}{(1 + \mu)}, \\ \zeta_{aopt} = \sqrt{\frac{3\mu}{8(1 + \mu)^3}}. \end{cases} \quad (9)$$

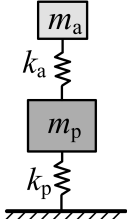
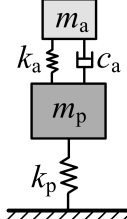
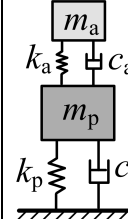
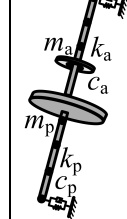
Approach 3: It is assumed that the main system is a single degree of freedom mass-damping-spring system. Then the dynamic vibration absorber which given the code name

DVA3 is designed as a single degree of freedom mass-damping-spring system. Based on the optimum design formula induced by Seto et al. [25], the optimal vibration suppression performance can be achieved when the parameters of this type of dynamic vibration absorber satisfies Eq. (10):

$$\begin{cases} \gamma_{\text{aopt}} = \frac{1}{1 + \mu} - (0.241 + 1.74\mu - 2.6\mu^2)Z - (1 - 1.9\mu + \mu^2)Z^2, \\ \zeta_{\text{aopt}} = \sqrt{\frac{3\mu}{8(1 + \mu)}} + (0.13 + 0.12\mu + 0.4\mu^2)Z - (0.01 + 0.9\mu + 3\mu^2)Z^2. \end{cases} \quad (10)$$

Based on the above three approaches, single degree of freedom dynamic vibration absorber is designed, and the optimum parameters of which are shown in Table 3.

Table 3. Parameters of four types of optimal dynamic vibration absorbers

Types		DVA1	DVA2	DVA3	DVA4
System schematic					
V'_{Dopt}	γ_{aopt}	0.9993	0.868	0.8645	0.9911
	ζ_{aopt}	0	0.2216	0.2576	0.2976
V_{Dopt}	$k_{\text{aopt}} / (\text{N/m})$	1.67×10^4	1.26×10^4	1.25×10^4	1.643×10^4
	$c_{\text{aopt}} / (\text{N} \cdot \text{s/m})$	0	19.43	22.5	29.8

Three different amplitude -frequency curves of the main system which is respectively attached DVA1, DVA2 and DVA3 are shown in Fig. 7. The definition of amplitude magnification β , given by the ratio of resonance amplitude of the main system which is attached dynamic vibration absorber and original system, is used to represent the amplitude of the main system. For the main system which is attached DVA1, there are two new formants, which appear in the first order critical speed region of the original system.

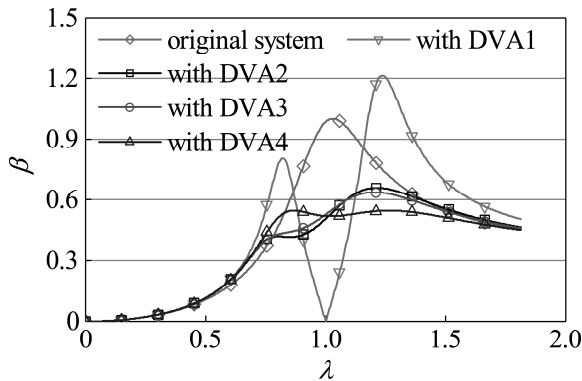


Fig. 7. Amplitude frequency response of the main system with three types of absorbers

Although the first peak is smaller than the resonance peak of the original system, the second peak far greater than the original peak. So, its vibration suppression effect of suppressing vibration

is far from ideal. For the main system which is respectively attached DVA2 and DVA3, the two new resonance peaks of both are smaller than the resonance peak of the original system, and the second peak is far greater than the first. Based on the experience, one can conclude that none of the three types of dynamic vibration absorbers is optimal. Dynamic vibration absorber, which is given the code name DVA4, is designed by means of the method mentioned in this study. For the main system which is attached DVA4, the two new resonance peaks are far smaller than the resonance peak of the original system, and the two are equal. This indicates that DVA4 is the best dynamic vibration absorber and reflects the accuracy and superiority of the parameter optimization method in this paper.

Dynamic vibration absorbers with different mass ratio are optimized by the design methods of DVA2, DVA3 and the optimization method in this paper respectively. The results are shown in Figure 8. As seen in the diagram, with any given mass ratio, compared with other two optimization methods, the dynamic vibration absorber designed by the optimization method in this paper has a higher frequency ratio and damping ratio. From the optimum amplitude magnification perspective, for any given mass ratio, the amplitude magnification of the main system which is attached DVA4 is smaller than that of the main system which is respectively attached DVA2 and DVA3. This fact shows that the dynamic vibration absorber designed by the method of this paper has a better vibration suppression performance, which further proves the effectiveness and superiority of this method.

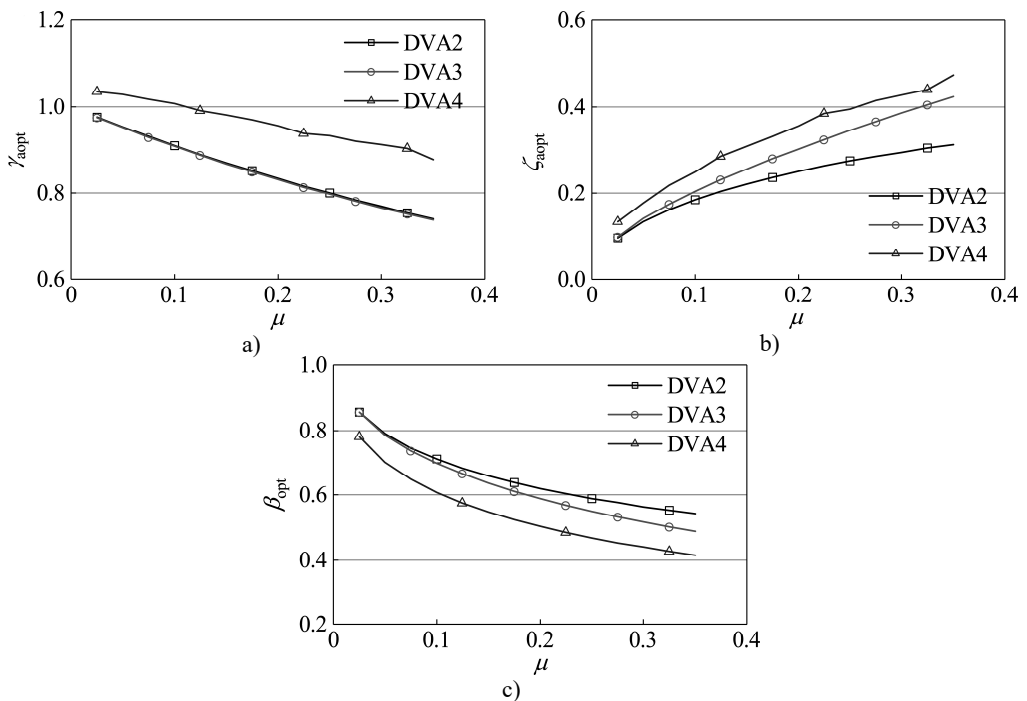


Fig. 8. Optimal parameters of dynamic absorber with different mass ratios of primary system:
 a) optimal frequency ratio, b) optimal damping ratio, c) amplitude magnification

(2) Vibration suppression performance analysis for dynamic vibration absorber when its parameters deviate from the optimal value.

When the stiffness or the damping deviates from the optimal value (optimal deviation is that the stiffness or the damping is η_v times of optimal value), vibration suppression performance of dynamic vibration absorber will be, undoubtedly, reduced. The performance changes as illustrated in Fig. 9. The mass ratio μ vary between 0.01-0.35. The deviation interval from the optimal

stiffness, or damping, vary between 0.001-4 times of optimal value. The meaning of the vertical coordinate in the diagram is η_a (or η_b): η_a (η_b) = (maximum amplitude of the main system when the stiffness (damping) occurs optimal deviation – optimal amplitude)/(resonance amplitude of the original system – optimal amplitude). The greater the value is, the greater performance degradation will occur. If this value is greater than 1, it is indicated that, the dynamic vibration absorber aggravates the vibration instead of suppressing. When the stiffness deviates from the optimal value, the dynamic vibration absorber with any mass ratio can suppress vibration; At the same deviation degree, performance degradation is more severe for the dynamic vibration absorber with the small mass ratio than large mass ratio.

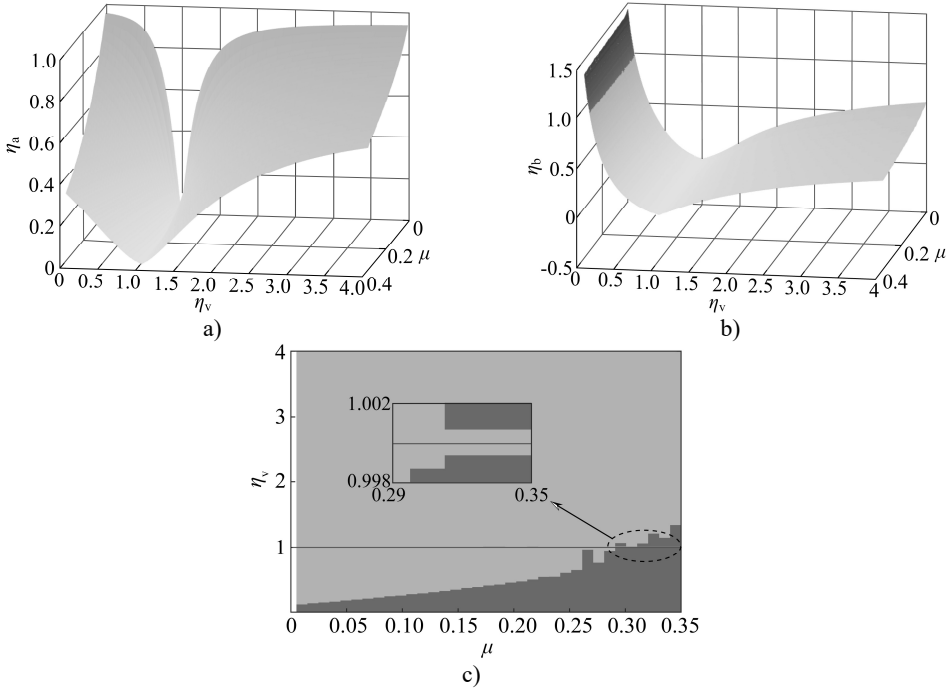


Fig. 9. Impact of the deviation from the optimal parameter on vibration suppression performance:
 a) when the stiffness deviate from the optimal value, b) when the damping deviate from the optimal value,
 c) when the stiffness and damping deviate from the optimal value in the same proportion,
 the sensitivity contrast of vibration suppression performance to the both

When the damping deviates from the optimal value, not all dynamic vibration absorber can suppress vibration. For the dynamic vibration absorber with any mass ratio, the vibration will become stronger when the damping value is below $0.1c_{opt}$, which are shown in the dark area of Fig. 9; At the same deviation degree, the dynamic vibration absorber with different mass ratio possesses essentially the same degree of vibration suppression performance degradation.

The meaning of Fig. 9(c) is: When the stiffness and damping deviate from the optimal value in the same proportion, for the dynamic vibration absorber with any mass ratio, the sensitivity of its vibration suppression performance to the both deviations. In both colors, the deep color means that the amplitude of the main system when the damping deviates from the optimal value is greater than that when the stiffness deviate from the optimal value; the opposite is the case for light color. With any given mass ratio, the sensitivity of the vibration suppression performance to deviation from optimal stiffness is higher than that to deviation from optimal damping; when the stiffness (damping) is less than optimal value, with the increase of mass ratio, the vibration suppression performance of dynamic vibration absorber become more sensitive to changes of damping.

3.4. Optimum design of dynamic vibration absorber for suppressing the fixed-frequency vibration

Assuming that the amplitude closed to the dangerous threshold of parking while the system running at a fixed speed, in which situation the dynamic vibration absorber needs to be used to suppress vibration and reduce amplitude. This section discusses the parameters optimization design of dynamic vibration absorber for suppressing the fixed-frequency vibration, and changes of vibration suppression performance when its parameters deviate from the optimal value.

Here, select a speed between the first and second critical speed of the original system, 200 Hz, as the working speed. Select $\mu = 0.15$ as the mass ratio of dynamic vibration absorber. The parameters of optimal dynamic vibration absorber DVA5 are obtained by the optimization method in this paper, which are as follows: $\gamma_{aopt} = 1.0152$, $\zeta_{aopt} = 0$, that is, $k_{aopt} = 2.665 \times 10^5$ N/m, $c_{aopt} = 0$ Nm/s. The time domain response of the coupling system which is attached DVA5 is shown in Fig.10. When DV5 is attached, the amplitude of the main system falls to 40 % of the original system. Compared with the main system which is attached DVA6 whose parameters are $k_a = m_a(2\pi\omega)^2$ and $c_a = 0$, the amplitude of the main system which is attached DVA5 fall by about 45 %. These reflect the effectiveness and superiority of this method in suppressing fixed-frequency vibration.

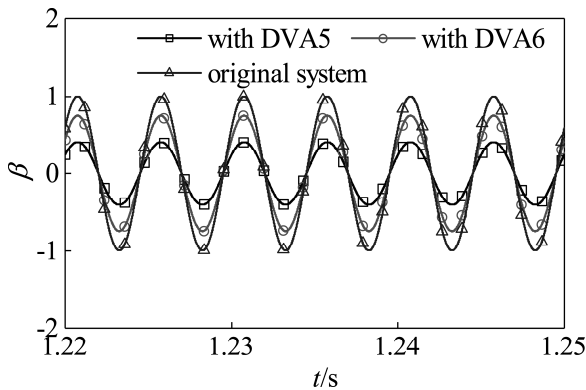


Fig. 10. Time domain response of the main system with DVA5 and DVA6

(1) Parameter optimization for dynamic vibration absorber with different mass ratio.

When the vibration frequency meets $\omega = 200$ Hz, the optimization design is studied with dynamic vibration absorber whose mass ratio vary between 0.01-0.35. The parameters and vibration suppression performance of optimal dynamic vibration absorber is shown in Fig. 11. The optimal frequency ratio increases gradually with the rising of the mass ratio; The optimal damping ratios are zero, which indicates that the dynamic vibration absorber without damping elements has better the fixed-frequency vibration suppression performance than that with. With exceptions that the mass ratio is 0.02 and 0.03, the amplitude magnification decreases with the increase of mass ratio.

(2) Relation between the optimal parameters of dynamic vibration absorber and rotational speed.

When the rotor run at different working speeds, the parameters and vibration suppression performance of the optimal dynamic vibration absorber is shown in Fig. 12. Here, select $\mu = 0.15$ as the mass ratio of dynamic vibration absorber. As shown in the chart, the optimal frequency ratio is increased gradually with the rising of working speed; the optimal damping ratios under any speeds are zero; the amplitude magnification increases gradually with increasing speed, which indicate that optimal vibration suppression performance reduce gradually with increasing speed.

At different speeds, the ratio of the amplitude magnification of the main system which is

attached optimal dynamic vibration absorber and the amplitude magnification of the main system which is attached DAV6 is shown in Fig. 13. It shows the relationship in vibration suppression performance between the two kinds of absorbers. A smaller value of η_1 means that the optimal dynamic vibration absorber possesses the higher vibration suppression performance than DVA6. As seen in the diagram, vibration suppression performances of optimal vibration absorber and DVA6 are not constant proportionality under any speed, the vibration suppression performance of the optimal dynamic vibration absorber gradually approaches DVA6 as the speed increases. However, under the speed reaches 200 Hz, one can also find out that the amplitude magnification of the optimal dynamic vibration absorber is only 55 % of that of DVA6, which indicate that the amplitude of the main system which is attached the optimal dynamic vibration absorber is only 55 % of that of the main system which is attached DVA6. Therefore, when designed the dynamic vibration absorber for suppressing the fixed-frequency vibration, the parameters can't be simply set to $k_a = m_a(2\pi\omega_f)^2$.

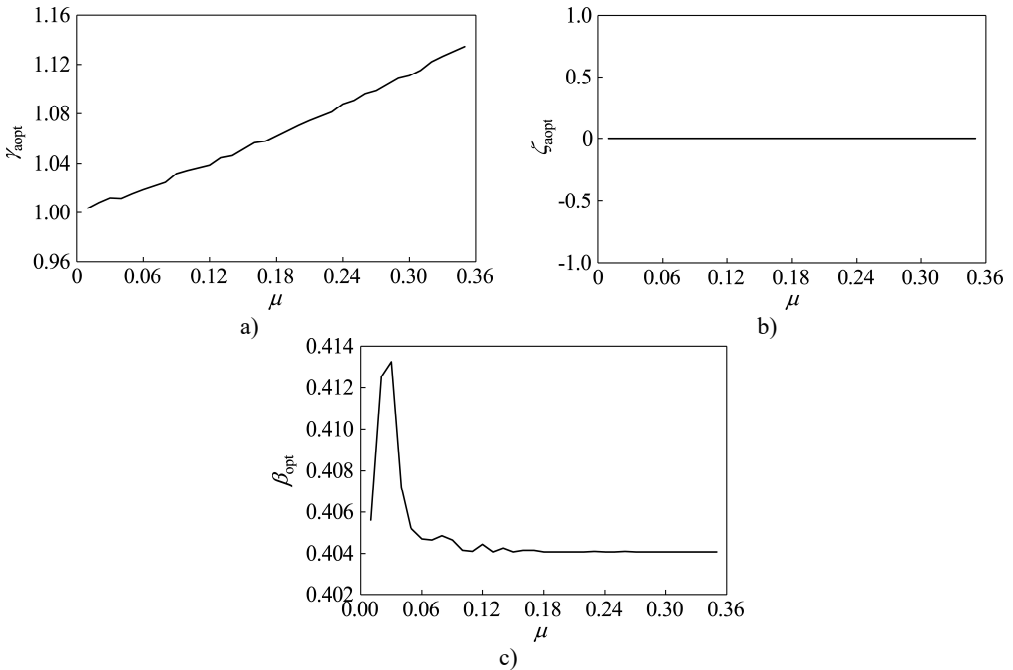


Fig. 11. Optimal parameters of the optimal absorber with different mass ratios while the main system running at $\omega = 200$ Hz: a) optimal frequency ratio, b) optimal damping ratio, c) optimal amplitude magnification

(3) Changes of vibration suppression performance when the stiffness deviates from the optimal value.

The dynamic vibration absorber with mass ratio of 0.05, 0.15 and 0.3 is chosen, and their vibration suppression performance is analyzed when the stiffness deviates from the optimal value. The result is shown in Fig. 14. The meaning of z axes in this diagram is: $\eta_a = (\text{amplitude of the main system when the stiffness (damping) deviates from the optimal value optimal amplitude}) / (\text{the amplitude of the original system optimal amplitude})$. The bigger value of η_a means that the bigger performance degradation. The dynamic vibration absorber worsens vibration of main system if $\eta_a > 1$.

In this diagram, at upper there is three dimensional graph whose projection on xoy plane is below the diagram. The deeper the color, the bigger the value of η_a , namely the bigger the performance degradation. The area close to black is used to represent the range of parameters that

aggravate vibration. When the stiffness deviates from the optimal value, vibration absorbers with three kinds of mass ratio has the vibration suppression performance in most parameters range. Parameters that aggravate vibration show an obvious interval, such as: When $\omega < \omega_p$, $k_a > k_{aopt}$, the dynamic vibration absorber aggravate vibration of the main system; When $\omega > \omega_p$, $k_a < k_{aopt}$, the dynamic vibration absorber aggravate vibration. This regularity will provide reference for parameters selection of semi-active and active dynamic vibration absorber.

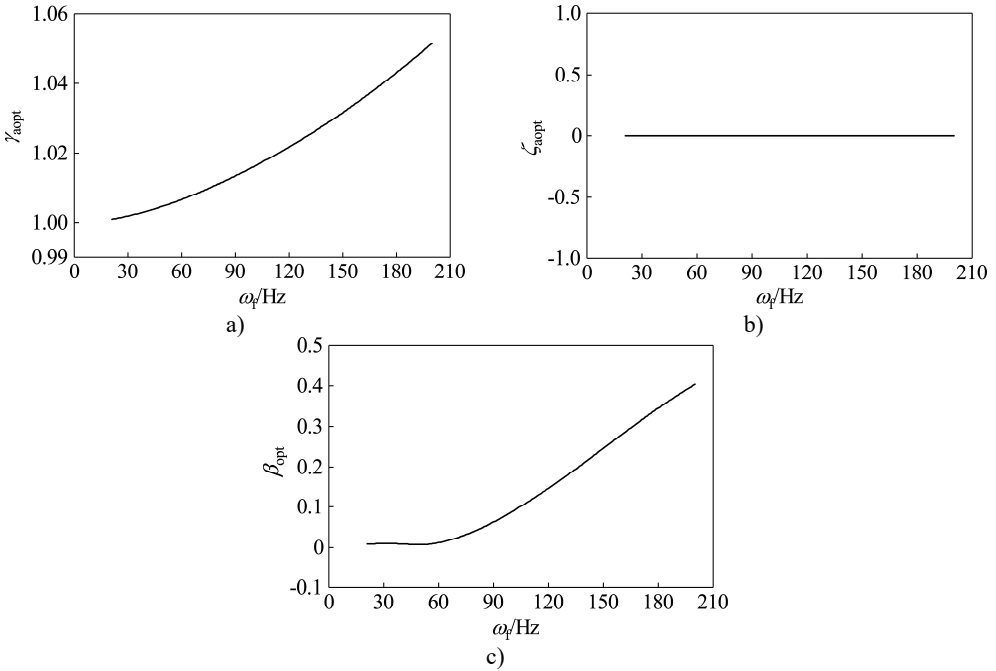


Fig. 12. Parameters of the optimal absorber with the mass ratio that $\mu = 0.15$ while the main system running at different speeds: a) optimal frequency ratio, b) optimal damping ratio, c) optimal amplitude magnification

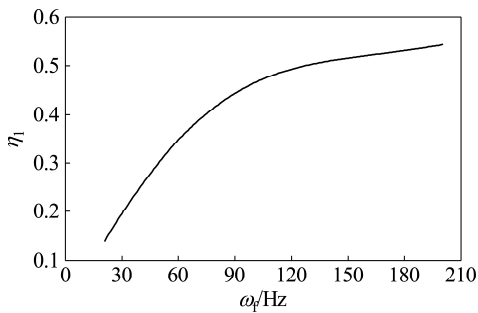


Fig. 13. Comparison of the vibration suppression performance of the optimal absorber and DVA6 while the main system running at different speeds

Zi Liang Liu designed research, performed research, analyzed data and wrote the manuscript. Qin Zhang analyzed data and was involved in writing the manuscript. Yan Bo Cao and Qi Xu discussed the results and were involved in revising the manuscript. Hong Liang Yao and Bang Chun Wen contributed to refining the ideas and revising the manuscript.

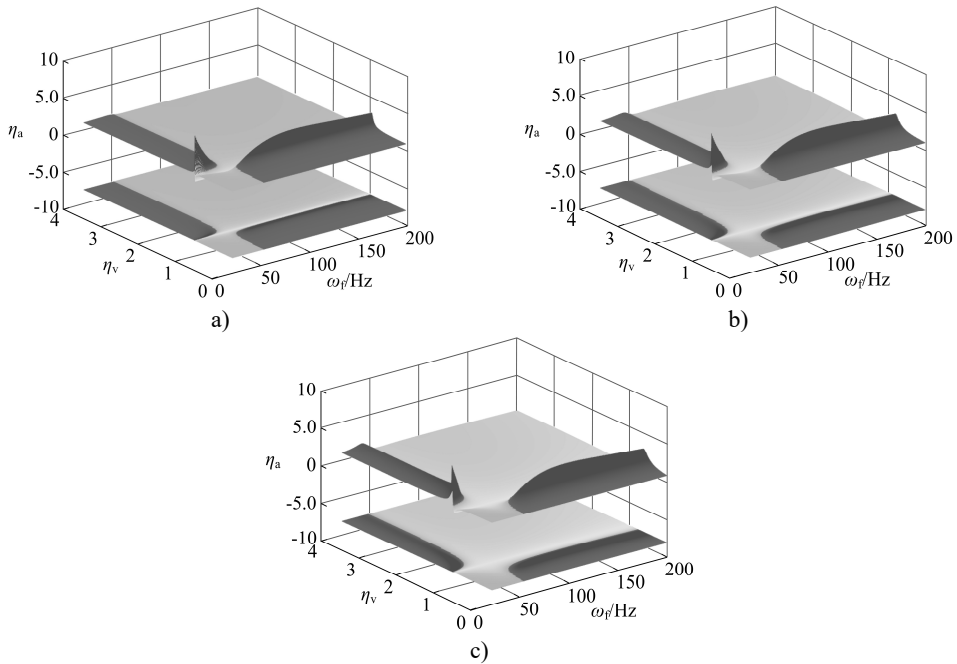


Fig. 14. Changes in vibration suppression performance of the dynamic vibration absorber when the stiffness deviate from the optimal value: a) $\mu = 0.05$, b) $\mu = 0.15$, c) $\mu = 0.3$

4. Conclusions

In this study, the frequency response function of the dynamic vibration absorber-rotor coupling system is obtained by using the finite element method. Then, basing on the optimal mathematical model, the optimal parameters of the dynamic vibration absorber are successfully solved with the adaptive particle swarm optimization algorithm. By comparing the optimal dynamic absorber with the absorber designed by the common optimization design methods, the validity and superiority of the optimization method are verified.

For the dynamic absorber which is adopted to suppress the first resonance, the stiffness has a greater influence on the vibration suppression performance than damping in most cases. For the dynamic vibration absorber which is adopted to suppress the fixed-frequency vibration, its performance changes, relative to the parameter deviation degree from the optimal value, appear an obvious interval whose range is in specific relation to the working speed. These rules provide a reference for selecting parameter of the semi-active and active dynamic vibration absorbers.

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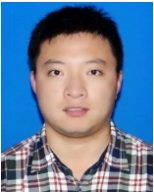
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