135. Free vibration of square plate with temperature effect

Amit Sharma
Amity University Haryana, Gurgaon, India
E-mail: *dba.amitsharma@gmail.com*

Received 10 October 2017; accepted 13 December 2017
DOI https://doi.org/10.21595/jme.2017.19516

Abstract. In modern engineering, researchers/scientists are very keen to know first few modes of vibration because it provides how system/structure behaves under vibration. In this paper, author computed the natural vibration of non homogeneous tapered square plate. Here tapered means that plate's thickness varies linearly along x-axis. Here non homogeneity arises in the plate's material due to simultaneous variation in density (circular variation) and Poisson's ratio (exponential variation). Temperature variation on the plate is viewed bi-parabolic along the axes. Rayleigh-Ritz technique is applied to obtain the frequency equation and vibrational frequency modes under different combination of parameters. The findings of the papers are presented in tabular form.

Keywords: square plate, circular variation, parabolic temperature, free vibration.

1. Introduction

The vibrational phenomena (wanted or unwanted) in mechanical structure or in engineering are very much common. Excess amount of vibrations causes the loss of energy and some time down the performance of system. In order to improve structure design or increase the performance, we have to control the vibration.

In these directions, non homogeneous tapered plates with plays an essential role because of their high tensile strength, durability and elastic behaviour. Since almost all engineering structures (machines, mechanical structures) works under huge temperature, therefore to make trust worthy design, it is also essential to know vibration under temperature effect. A quite significant work has been reported on non homogeneous (linear and exponential variation in density or Poisson's ratio) tapered plate (linear, parabolic and exponential variation in thickness in one or two dimension) with temperature effect (linear of parabolic in one or two dimension). But negligible work is done on simultaneous variation of density and Poisson's ratio (as non homogeneity effect).

Leissa [1] presented plate's vibration of different shapes on different boundary conditions in his excellent monograph. Jain and Soni [2] studied free vibration of rectangular plates with parabolic variation in thickness. The transverse vibration of a rectangular plate with thickness variation in both the directions is presented by Singh and Sexena [3]. Lal and Dhanpati [4] provided the effect of non homogeneity on the vibration of orthotropic rectangular plates of varying thickness resting on Pasternak foundation. Vibrational analysis of non homogeneous orthotropic visco elastic rectangular plate of parabolically varying thickness with temperature effect is given by Singhal and Gupta [5]. Sharma and Sharma [6] presented a mathematical modeling of vibration on parallelogram plate with non homogeneity effect. Sharma et al. [7] studied the vibration of non homogeneous square plate with thermal effect. Sharma et al. [8] presented mathematical model on frequency of rectangular plate with circular variation in Poisson's ratio. Mathematical study of vibration on non homogeneous parallelogram plate with temperature is provided by Sharma et al. [9]. Khanna and Kaur [10] studied the effect of simultaneous variation (exponential variation) in density and Poisson's ratio with linear temperature effect on vibration. Khanna and Singhal [11] gave effect of plate's parameters on vibration of isotropic tapered rectangular plate with different boundary conditions. Kalita, Shivakoti, Ghadai and Haldar [12, 13] presented two interesting companion papers which showed the effect of rotary inertia on dynamic behavior of rectangular plates. Gupta and Sharma [14] studied vibrational frequency of non homogeneous trapezoidal plates with temperature effect.

Sharma et al. [15] analysed the free vibration of tapered square plate with circular variation in Poisson's ratio.

The present paper describes the effect of simultaneous variation in density and Poisson's ratio (as non homogeneity effect) to vibrational frequency. Authors also show the effect of circular variation in density (as a new interesting aspect) and parabolic variation in temperature to the frequency modes.

2. Analysis

The differential equation for plate is:

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2}=\rho h\frac{\partial^2 \phi}{\partial t^2}, \quad (1)$$

where:

$$\begin{aligned}
M_x &= -D_1\left[\frac{\partial^2\phi}{\partial x^2}+\nu\frac{\partial^2\phi}{\partial y^2}\right],\\
M_y &= -D_1\left[\frac{\partial^2\phi}{\partial y^2}+\nu\frac{\partial^2\phi}{\partial x^2}\right],\\
M_{xy} &= -D_1(1-\nu)\frac{\partial^2\phi}{\partial x\,\partial y}.
\end{aligned} \quad (2)$$

Substitute Eq. (2) in Eq. (1), we get:

$$\begin{aligned}
&D_1\left(\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\,\partial y^2}+\frac{\partial^4\phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\phi}{\partial y^2}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^3\phi}{\partial x\,\partial y^2}+\frac{\partial\nu}{\partial x}\frac{\partial^2\phi}{\partial y^2}\right)\\
&\quad+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\phi}{\partial y^3}+\frac{\partial^3\phi}{\partial y\,\partial x^2}-\frac{\partial\nu}{\partial x}\frac{\partial^2\phi}{\partial x\,\partial y}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\phi}{\partial x^2}+\nu\frac{\partial^2\phi}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\phi}{\partial y^2}+\nu\frac{\partial^2\phi}{\partial x^2}\right)\\
&\quad+2(1-\nu)\frac{\partial^2 D_1}{\partial x\,\partial y}\frac{\partial^2\phi}{\partial x\,\partial y}+\rho h\frac{\partial^2\phi}{\partial t^2}=0.
\end{aligned} \quad (3)$$

For solution of Eq. (3), we can take:

$$\phi(x,y,t)=\Phi(x,y)*T(t). \quad (4)$$

Using Eq. (4), Eq. (3) becomes:

$$\begin{aligned}
&T\left[D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\,\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\,\partial y^2}+\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial y^2}\right)\right.\\
&\quad+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\,\partial x^2}-\frac{\partial\nu}{\partial x}\frac{\partial^2\Phi}{\partial x\,\partial y}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right)\\
&\quad\left.+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D_1}{\partial x\,\partial y}\frac{\partial^2\Phi}{\partial x\,\partial y}\right]+\rho l\Phi\frac{\partial^2 T}{\partial t^2}=0.
\end{aligned} \quad (5)$$

Separate the variable using variable separable technique, we get:

$$\frac{\left[\begin{array}{c} D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\,\partial y^2}+\frac{\partial \nu}{\partial x}\frac{\partial^2\Phi}{\partial y^2}\right) \\ +2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\,\partial x^2}-\frac{\partial \nu}{\partial x}\frac{\partial^2\Phi}{\partial x\,\partial y}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right) \\ +\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D_1}{\partial x\,\partial y}\frac{\partial^2\Phi}{\partial x\,\partial y}\end{array}\right]}{\rho h\Phi} = -\frac{1}{T}\frac{\partial^2 T}{\partial t^2}=p^2(\text{say}). \tag{6}$$

Taking first and last expression of Eq. (6), we get

$$\begin{aligned}&\left[D_1\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}+\frac{\partial^2\nu}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\,\partial y^2}+\frac{\partial \nu}{\partial x}\frac{\partial^2\Phi}{\partial y^2}\right)\right.\\ &+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial y\,\partial x^2}-\frac{\partial \nu}{\partial x}\frac{\partial^2\Phi}{\partial x\,\partial y}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\Phi}{\partial x^2}+\nu\frac{\partial^2\Phi}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\Phi}{\partial y^2}+\nu\frac{\partial^2\Phi}{\partial x^2}\right)\\ &\left.+2(1-\nu)\frac{\partial^2 D_1}{\partial x\,\partial y}\frac{\partial^2\Phi}{\partial x\,\partial y}\right]-\rho p^2 h\Phi=0.\end{aligned} \tag{7}$$

Eq. (7) represents the differential equation for tapered non homogeneous plate. Here $D_1 = Yh^3/12(1-\nu^2)$ represent flexural rigidity of the plate.

Due to the consideration of non-homogeneity in the material, the expression of density of the plate (circular variation in one dimension) and Poisson's ratio (exponential variation in one dimension) is given by:

$$\rho=\rho_0\left[1+m_1\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right], \tag{8}$$

where m_1, $(0\le m_1<1)$ is non-homogeneity constant:

$$\nu=\nu_0\left[e^{m_2\frac{x}{a}}\right], \tag{9}$$

where m_2, $(0\le m_2<1)$ is another non-homogeneity constant.

Due to consideration of non-uniformity in the plate, the expression for thickness (linear variation in one dimension) is expressed as [8]:

$$h=h_0\left[1+\beta\frac{x}{a}\right], \tag{10}$$

where β, $(0\le\beta\le 1)$is known as tapering parameter.

It is also taken into account that, temperature variation on the plate along axes is bi parabolic. Therefore, the expression for temperature is expressed as [8]:

$$\tau=\tau_0\left[1-\left(\frac{x}{a}\right)^2\right]\left[1-\left(\frac{y}{a}\right)^2\right], \tag{11}$$

where τ and τ_0 are known as temperature above the mentioned temperature at any point on the plate and at the origin i.e., $x=y=0$. For engineering material modulus of elasticity is:

$$Y=Y_0(1-\gamma\tau), \tag{12}$$

where Y_0 is Young's modulus at $\tau = 0$ and γ is known as slope of variation.

Substitute Eq. (11) in Eq. (12), we have:

$$Y = Y_0\left[1 - \alpha\left\{1 - \left(\frac{x}{a}\right)^2\right\}\left\{1 - \left(\frac{y}{a}\right)^2\right\}\right], \tag{13}$$

where $\alpha = \gamma\tau_0$, $(0 \le \alpha \le 1)$ is known as temperature gradient.

On using Eqs. (9), (10) and Eq. (13), flexural rigidity of the plate becomes:

$$D_1 = \frac{Y_0 {h_0}^3\left[1 - \alpha\left\{1 - \left(\frac{x}{a}\right)^2\right\}\left\{1 - \left(\frac{y}{a}\right)^2\right\}\right]\left[1 + \beta\frac{x}{a}\right]^3}{12\left(1 - {\nu_0}^2 e^{2m_2\frac{x}{a}}\right)}. \tag{14}$$

Here we computed vibrational frequency on clamped plate. Therefore, boundary conditions of the plate are:

$$\Phi = \frac{\partial\Phi}{\partial x} = 0, \text{ at } x = 0, a \text{ and } \Phi = \frac{\partial\Phi}{\partial y} = 0, \text{ at } y = 0, a. \tag{15}$$

The two term deflection function that satisfies Eq. (15) could be:

$$\Phi(x, y) = \left[\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{a}\right)\right]^2\left[C_1 + C_2\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{a}\right)\right]. \tag{16}$$

3. Solution for frequency equation

We use Rayleigh-Ritz technique (i.e., maximum strain energy V_s must equal to maximum kinetic energy T_s) to solve the frequency equation, therefore we have:

$$\delta(V_s - T_s) = 0. \tag{17}$$

Here expression for V_s and T_s is given by:

$$V_s = \frac{1}{2}\int_0^a\int_0^a D_1\left\{\left(\frac{\partial^2\Phi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\Phi}{\partial y^2}\right)^2 + 2\nu\,\frac{\partial^2\Phi}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2} + 2(1 - \nu)\left(\frac{\partial^2\Phi}{\partial x\,\partial y}\right)^2\right\}dydx, \tag{18}$$

$$T_s = \frac{1}{2}p^2\int_0^a\int_0^a \rho\, h\, \Phi^2\, dydx. \tag{19}$$

Using $X = x/a$ and $Y = y/a$ as two non dimensional variable in Eqs. (18) and (19), we get:

$$\begin{aligned} {V_s}^* = \frac{Y_0 {h_0}^3}{24a^2}\int_0^1\int_0^1 \Bigg[&\frac{[1 - \alpha\{1 - X^2\}\{1 - Y^2\}][1 + \beta X]^3}{(1 - {\nu_0}^2 e^{2m_2 X})} \\ &\cdot\left\{\left(\frac{\partial^2\Phi}{\partial X^2}\right)^2 + \left(\frac{\partial^2\Phi}{\partial Y^2}\right)^2 + 2\nu_0 e^{m2X}\left(\frac{\partial^2\Phi}{\partial X^2}\frac{\partial^2\Phi}{\partial Y^2}\right) + 2(1 - \nu_0 e^{m2X})\left(\frac{\partial^2\Phi}{\partial X\,\partial Y}\right)^2\right\}\Bigg] dYdX, \end{aligned} \tag{20}$$

$$ {T_s}^* = \frac{1}{2}a^2p^2\rho_0 h_0\int_0^1\int_0^1 \left[1 + m_1\left(1 - \sqrt{1 - X^2}\right)\right]\,[1 + \beta X]\,\Phi^2\,dYdX. \tag{21}$$

By using Eqs. (20), (21), Eqs. (17) becomes:

$$\delta({V_s}^* - \lambda^2 {T_s}^*) = 0, \tag{22}$$

where $\lambda^2 = 12\rho_0 a^4 p^2 / Y_0 {h_0}^2$ is the frequency parameter. Eq. (22) has two unknown constants C_1 and C_2 because of deflection $\Phi(x, y)$. These can be evaluated by:

$$\frac{\partial}{\partial C_n}({V_s}^* - \lambda^2 {T_s}^*) = 0, \qquad n = 1,2. \tag{23}$$

After simplifying Eq. (23), we get a homogeneous system of equation as:

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{24}$$

where d_{11}, $d_{12} = d_{21}$ and d_{22} involves parametric constant and frequency parameter. In order to get frequency equation, the determinant of coefficient matrix of Eq. (24) must vanish. Therefore, we have:

$$\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix} = 0. \tag{25}$$

On solving Eq. (25) we get two frequency modes as λ_1 (first mode) and λ_2 (second mode).

4. Results and discussions

Vibrational frequency modes for non homogeneous and tapered square plate are calculated for different values of temperature gradient (α), non homogeneity constants (m_1, m_2) and tapering parameter (β). All the calculations are presented in the form of tables.

Table 1 provides the frequency modes corresponding to non homogeneity (Poisson's ratio) keeping density parameter to be zero, for three different values of temperature and thickness parameter of the plate. From the Table 1, we conclude that frequency increases when non homogeneity varies from 0 to 1 for all the three values. On the other hand, when combined value of thickness and temperature increases from 0 to 0.8, frequency also increases.

Table 1. Frequency vs non homogeneity corresponding to Poisson's ratio

m_2	$m_1 = 0$					
	$\beta = \alpha = 0.0$		$\beta = \alpha = 0.4$		$\beta = \alpha = 0.8$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	38.32	149.97	42.53	166.66	46.13	182.22
0.2	38.89	152.24	43.24	169.65	46.92	185.91
0.4	39.67	155.34	44.23	173.86	48.06	191.26
0.6	40.74	159.66	45.62	179.95	49.69	199.17
0.8	42.23	165.84	47.62	188.91	52.06	211.08
1.0	44.33	174.83	50.46	202.29	55.47	229.15

Table 2 shows the frequency of vibration corresponding to non homogeneity (density parameter) by keeping Poisson's ratio to be zero for all the three values of temperature and thickness as in Table 1. It is interesting to see that vibrational frequency decreases (with small rate of decrement) for all the three values due to circular variation. When, the combined value of temperature and thickness of plate increases from 0 to 0.8, frequency modes increases (with small rate of decrement as compared to table 1) due to circular variation in density.

Table 3 gives the frequency modes corresponding to temperature gradient (by keeping density parameter to be zero) for three values of non homogeneity and tapering parameter. Frequency modes decrease as temperature on the plate increases from 0 to 0.8. When the combined value of non homogeneity (Poisson's ratio) and thickness of the plate increases from 0.2 to 0.8, vibrational frequency increases.

Table 2. Frequency vs non homogeneity corresponding to density

m_1	$m_2 = 0$					
	$\beta = \alpha = 0.0$		$\beta = \alpha = 0.4$		$\beta = \alpha = 0.8$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	38.32	149.97	42.53	166.66	46.13	182.22
0.2	37.75	147.42	41.88	163.66	45.44	178.80
0.4	37.20	144.99	41.26	160.81	44.73	175.56
0.6	36.67	142.69	40.66	158.12	44.08	172.50
0.8	36.17	140.49	40.09	155.56	43.45	169.60
1.0	35.69	138.40	39.55	153.13	42.85	166.85

Table 3. Frequency vs thermal gradient

α	$m_1 = 0$					
	$\beta = m_2 = 0.2$		$\beta = m_2 = 0.4$		$\beta = m_2 = 0.8$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	42.97	168.25	48.36	189.63	62.42	247.53
0.2	41.08	160.91	46.35	181.92	60.05	238.92
0.4	39.09	153.22	44.23	173.86	57.56	230.01
0.6	36.98	145.13	41.98	165.43	54.91	220.74
0.8	34.71	136.57	39.57	156.54	52.06	211.08

Table 4 also provides the frequency modes corresponding to temperature gradient (by keeping Poisson's ratio to be zero) for the three values of non homogeneity (density parameter) and thickness of the plate. Like Table 3, frequency mode decreases when the temperature on the plate increases from 0 to 0.8. But here, frequency for both modes is lesser as compared to Table 3 because of circular variation in density. Also, the frequency mode increases when the combined value of non homogeneity (density) and thickness of the plate increases from 0.2 to 0.8. Here also rate of increment is smaller as compared to Table 3 due to circular variation in density parameter.

Table 4. Frequency vs thermal gradient

α	$m_2 = 0$					
	$m_1 = \beta = 0.2$		$m_1 = \beta = 0.4$		$m_1 = \beta = 0.8$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	41.66	162.66	45.13	175.74	52.23	202.28
0.2	39.82	155.51	43.25	168.44	50.22	194.61
0.4	37.89	148.02	41.26	160.81	48.10	186.64
0.6	35.84	140.13	39.15	152.81	45.86	178.32
0.8	33.64	131.78	36.90	144.38	43.45	169.60

5. Conclusions

From the present study, authors drag the attentions of the readers on two important aspects. Firstly, effect of simultaneous variation of density and Poisson's ratio (as non homogeneity) to vibrational frequency. Secondly, effect of circular variation (as a new interesting aspects) in density to vibration. Circular variation in density provides lesser frequency or lesser rate of increment/decrement in frequency modes as shown in Table 3 and Table 4. When Poisson ratio varies exponentially the frequency modes increases. But when density parameter varies circularly, the frequency mode decreases as shown in Table 1 and Table 2.

References

[1] **Leissa A. W.** Vibration of Plates. NASS-SP, 1969.

[2] **Jain R. K., Soni S. R.** Free vibration of rectangular plates of parabolically varying thickness. Indian Journal of Pure and Applied Mathematics, Vol. 4, 1973, p. 267-277.

[3] **Singh B., Sexena V.** Transverse vibration of a rectangular plate with bidirectional thickness variation. Journal of Sound and Vibration, Vol. 198, Issue 1, 1996, p. 51-65.

[4] **Lal R., Dhanpati** Effect of non-homogeneity on the vibration of orthotropic rectangular plates of varying thickness resting on pasternak foundation. Journal of Vibration and Acoustic, Vol. 131, 2009, p. 1-9.

[5] **Singhal P., Gupta A. K.** Vibrational analysis of non-homogeneous orthotropic visco-elastic rectangular plate of parabolically varying thickness with thermal effect. International Journal of Acoustics and Vibration, Vol. 21, Issue 2, 2016, p. 178-183.

[6] **Sharma A., Sharma A. K.** Mathematical modeling of vibration on parallelogram plate with non homogeneity effect. Romanian Journal of Acoustics and Vibration, Vol. 13, Issue 1, 2016, p. 53-57.

[7] **Sharma A., Sharma A. K., Raghav A. K., Kumar V.** Vibrational study of square plate with thermal effect and circular variation in density. Romanian Journal of Acoustics and Vibration, Vol. 13, Issue 2, 2016, p. 146-152.

[8] **Sharma A., Raghav A. K., Sharma A. K., Kumar V.** A modeling on frequency of rectangular plate. International Journal of Control Theory and Applications, Vol. 9, Issue 20, 2016, p. 277-282.

[9] **Sharma A., Raghav A. K., Kumar V.** Mathematical study of vibration on non homogeneous parallelogram plate with thermal gradient. International Journal of Mathematical Sciences, Vol. 36, Issue 1, 2016, p. 1801-1809.

[10] **Khanna A., Kaur N.** Effect of thermal gradient on vibration of non-uniform rectangular plate. Journal of The Institution of Engineers (India): Series C, Issue 2, 2016, p. 141-148.

[11] **Khanna A., Singhal A.** Effect of plate's parameters on vibration of isotropic tapered rectangular plate with different boundary conditions. Journal of Low Frequency Noise, Vibration and Active Control, Vol. 35, Issue 2, 2016, p. 139-151.

[12] **Kalita K., Shivakoti I., Ghadai R. K., Haldar S**. Rotary inertial effect in isotropic plate I: uniform thickness. Romanian Journal of Acoustics and Vibration, Vol. 13, Issue 2, 2016, p. 68-74.

[13] **Kalita K., Shivakoti I., Ghadai R. K., Haldar S**. Rotary inertial effect in isotropic plate II: taper thickness. Romanian Journal of Acoustics and Vibration, Vol. 13, Issue 2, 2016, p. 75-80.

[14] **Gupta A. K., Sharma P.** Vibration study of non homogeneous trapezoidal plates of variable thickness under thermal gradient. Journal of Vibration and Control, Vol. 22, Issue 5, 2016, p. 1369-1379.

[15] **Sharma A., Kumar V., Raghav A. K.** Vibrational frequency of tapered square plate. Romanian Journal of Acoustics and Vibration, Vol. 14, Issue 1, 2017, p. 21-27.