Mathematical model of elastic ribbed shell dynamics interaction with viscous liquid under vibration

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Abstract. The mechanical model of the system, formed by two surfaces of the coaxial cylindrical shells interacting with viscous incompressible liquid between them under vibration, is considered. The outer shell is geometrically irregular, and inner one is an absolutely rigid cylinder. The mathematical model of this system, consisting of differential equations in partial derivatives for describing dynamics of viscous incompressible liquid and an elastic ribbed shell together with boundary conditions is constructed. The expressions for amplitude frequency characteristics of outer geometrically irregular shell are discovered.

Keywords: mathematical modeling, hydroelasticity, viscous liquid, coaxial cylindrical shells, geometrically irregular shell, vibration.

1. Introduction

The pipelines are one of the basic elements in modern devices and various engineering constructions applied in aviation. Such devices are represented by pipelines of the hydraulic drive, cooling, greasing, fuel supply, dosaging system, power cylinder and ram-type pairs [1]. The studies of the mechanical systems, consisting of coaxial geometrically regular cylindrical shells with pressure pulsations or under vibrations were conducted earlier [2-8].

The present paper considers the pipeline system of ring like profile with elastic geometrically irregular outer shell of final length and absolutely rigid inner cylinder, interacting with viscous incompressible liquid between them under vibration for defining and regulating oscillations of "shell-fluid" system.

It should be noted that this problem is relevant because real mechanical systems are constantly exposed to different vibration impacts from both external and internal sources. Vibration leads to emerging substantial oscillations of pipeline systems. At resonance frequencies the walls velocities of the pipeline elastic shells will be maximum, and it can lead to ruptures in a fluid and emergence of vibrational cavitation. Gas bubbles bursting on walls, lead to pipeline walls damage and therefore to cavitation wear [9].

2. Statement of the problem

Let us consider the mechanical model presented on Fig. 1.

The outer shell is an elastic cylindrical geometrically irregular shell. The inner shell can be considered as absolutely rigid cylinder. Viscous incompressible liquid is placed between the indicated cylindrical shells. The radius of outer ribbed shell medium surface equals R, and its thickness at the places, without rigid ribs equals h_0. The length of cylindrical shells l are equal, and elastic shifts of an outer ribbed shell are much less than the width δ of the cylindrical slot. The width $\delta = R_1 - R_2 \ll R_2$ of the ring crossed section cylindrical slot, formed by two shells, is less than outer radius R_2 of the inner shell and inner radius R_1 of an outer shell. The medium

surface radius R is much larger than the width of an outer shell $h_0 = 2(R - R_1)$. The temperature effects in the system discussed above are not considered.

In the present mechanical model we will determine amplitude-frequency characteristic of outer geometrically irregular shell sagging.
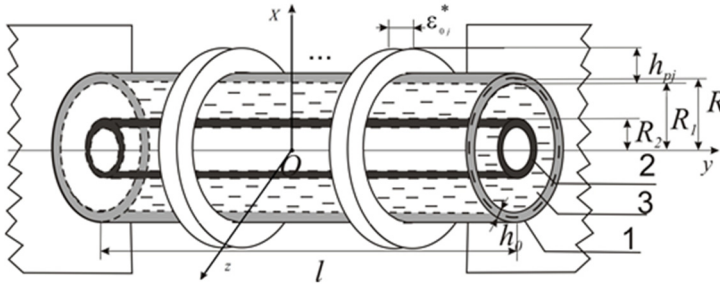

Fig. 1. The mechanical model

3. The theory

The movements of viscous incompressible liquid in coordinate system r, θ, y rigidly connected with the center of coordinates, are presented by Navier-Stokes equations and by continuity equation for an axes symmetric case in a scalar state [10, 11]:

$$
\begin{aligned}
W_r &= -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} + \frac{\partial^2 V_r}{\partial y^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta} - \frac{V_r}{r^2}\right), \\
W_\theta &= -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\left(\frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{\partial^2 V_\theta}{\partial y^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta}{r^2}\right), \\
W_y &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 V_y}{\partial r^2} + \frac{1}{r}\frac{\partial V_y}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V_y}{\partial \theta^2} + \frac{\partial^2 V_y}{\partial y^2}\right), \\
\frac{\partial V_r}{\partial r} &+ \frac{V_r}{r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_y}{\partial y} = 0,
\end{aligned}
\tag{1}
$$

where:

$$
\begin{aligned}
W_r &= W_{1z1}\cos\theta + W_{1x1}\sin\theta + \frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_y\frac{\partial V_r}{\partial y} - \frac{V_\theta^2}{r}, \\
W_\theta &= -W_{1z1}\sin\theta + W_{1x1}\cos\theta + \frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_y\frac{\partial V_\theta}{\partial y} + \frac{V_r V_\theta}{r}, \\
W_y &= \frac{\partial V_y}{\partial t} + V_r\frac{\partial V_y}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_y}{\partial \theta} + V_y\frac{\partial V_y}{\partial y},
\end{aligned}
$$

where p – pressure of viscous incompressible liquid; V_y, V_r, V_θ – are the components of liquid velocity vector $(\bar{n}_r, \bar{j})$ in which the beginning is located in the center of the inner shell O; t – time; ν – kinematic coefficient of viscosity; r – distance from an axis Oy; ρ – liquid density, $W_{1x1} = E_x\omega^2\sin(\omega t)$, $W_{1z1} = E_z\omega^2\sin(\omega t)$ – vibroacceleration of a base which is attached to a mechanical system into inertial space.

The boundary conditions for cylindrical axes on impenetrable surface of outer and inner shells in a cylindrical clearance for Eq. (1) are:

$$V_r = \frac{\partial u_3}{\partial t}, \quad V_\theta = \frac{\partial u_2}{\partial t}, \quad V_y = \frac{-\partial u_1}{\partial t} \text{ at } r = R_2 + \delta + \grave{u_3},$$
$$V_r = 0, \quad V_\theta = 0, \quad V_y = 0 \text{ at } r = R_2,$$

(2)

where $u_1 = u_1(y, \theta, t)$ – elastic longitudinal shell shift which is positive in the direction $\bar{n}_s$, opposite in the direction $\bar{j}$; $u_2 = u_2(y, \theta, t)$ – circumferential elastic movement in the direction $\bar{n}_\theta$, $u_3 = u_3(y, \theta, t)$ –shell sagging which is positive in the direction $\bar{n}$, coinciding with $\bar{n}_r$ and opposite in a direction to a curvature center; $\bar{u} = u_1\bar{n}_s + u_2\bar{n}_\theta + u_3\bar{n}$ – vector displacement of the elastic shell.

The outer surface of a pipe outer shell represents geometrically irregular shell, having n of rigidity ribs which altitude changes stepwise. The ribs present outer frames. The strengthening of geometrically irregular shell at butt ends has free attaching [11].

The dynamic equations of a ribbed shell based on Kirchhoff-Love hypotheses, proceeding from the received Hamilton integral variational principle, in cylindrical axes are:

$$\frac{\partial}{\partial y}\left(\frac{Eh_0}{1-\mu_0^2}\left\{\left[-\frac{\partial u_1}{\partial y} + \mu_0\frac{1}{R}\left(\frac{\partial u_2}{\partial \theta} + u_3\right)\right] \cdot k_1(y)\right.\right.$$
$$+ \left.\left(\mu_0\frac{1}{R^2}\left(\frac{\partial u_2}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right) - \frac{\partial^2 u_3}{\partial y^2}\right) \cdot h_0 k_2(y)\right\} + \frac{1}{R}\frac{\partial}{\partial \theta}\left(\frac{Eh_0}{2(1+\mu_0)}\left\{\left(\frac{\partial u_2}{\partial y} - \frac{1}{R}\frac{\partial u_1}{\partial \theta}\right) \cdot k_1(y)\right.\right.$$
$$+ \left.\left.\frac{2}{R}\left(\frac{\partial u_2}{\partial y} - \frac{\partial^2 u_3}{\partial y \partial \theta}\right) \cdot h_0 k_2(y)\right\}\right) = -h_0\rho_0\frac{\partial^2 u_1}{\partial t^2}k_1(y) - q_s,$$
$$\frac{\partial}{\partial y}\left(\frac{Eh_0}{2(1+\mu_0)}\left\{\left(\frac{\partial u_2}{\partial y} - \frac{1}{R}\frac{\partial u_1}{\partial \theta}\right) \cdot k_1(y) + \frac{2}{R}\left(\frac{\partial u_2}{\partial y} - \frac{\partial^2 u_3}{\partial y \partial \theta}\right) \cdot h_0 k_2(y)\right\}\right)$$
$$+ \frac{1}{R}\frac{\partial}{\partial \theta}\left(\frac{Eh_0}{1-\mu_0^2}\left\{\left(\frac{1}{R}\left(\frac{\partial u_2}{\partial \theta} + u_3\right) - \mu_0\frac{\partial u_1}{\partial y}\right)k_1(y)\right.\right.$$
$$+ \left.\left.\left(\frac{1}{R^2}\left(\frac{\partial u_2}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right) - \mu_0\frac{\partial^2 u_3}{\partial y^2}\right)h_0 k_2(y)\right\}\right)$$
$$+ \frac{1}{R^2}\frac{\partial}{\partial \theta}\left(\frac{Eh_0^3}{12(1+\mu_0^2)}\left(\frac{1}{R^2}\left(\frac{\partial u_2}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right) - \mu_0\frac{\partial^2 u_3}{\partial y^2}\right)k_3(y)\right.$$
$$+ \left.\frac{Eh_0}{1-\mu_0^2}\left(-\mu_0\frac{\partial u_1}{\partial y} + \frac{1}{R}\left(\frac{\partial u_2}{\partial \theta} + u_3\right)\right) \cdot h_0 k_2(y)\right)$$

(3)

$$+ \frac{2}{R}\frac{\partial}{\partial y}\left(\frac{Eh_0^3}{12(1+\mu_0)}\frac{1}{R}\left(\frac{\partial u_2}{\partial y} - \frac{\partial^2 u_3}{\partial y \partial \theta}\right)k_3(y) + \frac{Eh_0}{2(1+\mu_0)}\left(\frac{\partial u_2}{\partial y} - \frac{1}{R}\frac{\partial u_1}{\partial \theta}\right) \cdot h_0 k_2(y)\right)$$
$$= h_0\rho_0\left(W_{1x1}\sin\theta - W_{1z1}\cos\theta\frac{\partial^2 u_2}{\partial t^2}\right)k_1(y) - q_\theta,$$
$$\frac{\partial^2}{\partial y^2}\left(\frac{Eh_0^3}{12(1-\mu_0^2)}\left(\mu_0\frac{1}{R^2}\left(\frac{\partial u_2}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right) - \frac{\partial^2 u_3}{\partial y^2}\right)k_3(y)\right.$$
$$+ \left.\frac{Eh_0}{1-\mu_0^2}\left(\frac{\partial u_1}{\partial y} + \mu_0\frac{1}{R}\left(\frac{\partial u_1}{\partial \theta} + u_3\right)\right)h_0 k_2(y)\right)$$
$$+ \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2}\left(\frac{Eh_0^3}{12(1-\mu_0^2)}\left(\frac{1}{R^2}\left(\frac{\partial u_2}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right) - \mu_0\frac{\partial^2 u_3}{\partial y^2}\right)k_3(y)\right.$$
$$+ \left.\frac{Eh_0}{1-\mu_0^2}\left(-\mu_0\frac{\partial u_1}{\partial y} + \frac{1}{R}\left(\frac{\partial u_2}{\partial \theta} + u_3\right)\right) \cdot h_0 k_2(y)\right)$$
$$+ \frac{2}{R}\frac{\partial}{\partial \theta \partial y}\left(\frac{Eh_0^3}{12(1+\mu_0)}\frac{1}{R}\left(\frac{\partial u_2}{\partial y} - \frac{\partial^2 u_3}{\partial y \partial \theta}\right)k_3(y)\right)$$

$$+ \frac{Eh_0}{2(1+\mu_0)}\left(\frac{\partial u_2}{\partial y} - \frac{1}{R}\frac{\partial u_1}{\partial \theta}\right) \cdot h_0 k_2(y)\right) - \frac{1}{R}\left(\frac{Eh_0}{1-\mu_0^2}\left\{\left(\frac{1}{R}\left(\frac{\partial u_2}{\partial \theta} + u_3\right) - \mu_0 \frac{\partial u_1}{\partial y}\right)k_1(y)\right.\right.$$

$$+ \left(\frac{1}{R^2}\left(\frac{\partial u_2}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right) - \mu_0 \frac{\partial^2 u_3}{\partial y^2}\right)h_0 k_2(y)\right\}\right)$$

$$= -h_0 \rho_0 \left(W_{1x1}\sin\theta + W_{1z1}\cos\theta + \frac{\partial^2 u_3}{\partial t^2}\right)k_1(y) - q_n,$$

where:

$$k_{1j} = \left(1 - \frac{h_0}{h_{pj}}\right)\frac{h_{pj}}{h_0}, \quad k_{2j} = \left(1 - \frac{h_0}{h_{pj}}\right)\frac{h_{pj}^2}{2h_0^2}, \quad k_{3j} = \left(1 - \frac{h_0}{h_{pj}}\right)\left(4 - 2\frac{h_0}{h_{pj}} + \frac{h_0^2}{h_{pj}^2}\right)\frac{h_{pj}^3}{h_0^3}.$$

$\Delta\Gamma_{yj} = \Gamma(y - y_j) - \Gamma(y - y_j - \varepsilon_{0j}^*)$, $\Gamma(y)$ – is a unit function Heaviside on the longitudinal coordinate y; μ_0 – Poisson's ratio of shell material; y_j – is a point, where the rib appears on the longitudinal coordinate; ρ_0 – shell material density; E – shell material Young modulus:

$$q_s = -\left[P_{ry}\cos(\bar{n},\bar{n}_r) + P_{\theta y}\cos(\bar{n},\bar{n}_\theta) + P_{yy}\cos(\bar{n},\bar{j})\right]\big|_{r=r^{(1)}},$$

$$q_\theta = \left[P_{r\theta}\cos(\bar{n},\bar{n}_r) + P_{\theta\theta}\cos(\bar{n},\bar{n}_\theta) + P_{\theta y}\cos(\bar{n},\bar{j})\right]\big|_{r=r^{(1)}},$$

$$q_n = \left[P_{rr}\cos(\bar{n},\bar{n}_r) + P_{r\theta}\cos(\bar{n},\bar{n}_\theta) + P_{ry}\cos^{(i)}(\bar{n},\bar{j})\right]\big|_{r=r^{(1)}},$$

$$r^{(1)} = R_2 + \delta + u_3, \quad P_{rr} = -p + 2\rho v\frac{\partial V_r}{\partial r}, \quad P_{r\theta} = \rho v\left(\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} + \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right),$$

$$P_{\theta\theta} = -p + 2\rho v\left(\frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}\right), \quad P_{ry} = \rho v\left(\frac{\partial V_y}{\partial r} + \frac{\partial V_r}{\partial y}\right), \quad P_{yy} = -p + 2\rho v\frac{\partial V_y}{\partial y}, \qquad (4)$$

$$P_{\theta y} = \rho v\left(\frac{\partial V_\theta}{\partial y} + \frac{1}{r}\frac{\partial V_y}{\partial \theta}\right), \quad \cos(\bar{n},\bar{n}_r) = \frac{r}{|\bar{N}|}, \quad \cos(\bar{n},\bar{n}_\theta) = -\frac{1}{|\bar{N}|}\frac{\partial u_3}{\partial \theta},$$

$$\cos(\bar{n},\bar{j}) = -\frac{r}{|\bar{N}|}\frac{\partial u_3}{\partial y}, \quad |\bar{N}| = \left\{r^2\left[1 + \frac{\partial u_3}{\partial y}\right]^2 + \left(\frac{\partial u_3}{\partial \theta}\right)^2\right\}^{\frac{1}{2}}, \quad (a_0)^2 = \frac{(h_0)^2}{12R^2}.$$

The boundary conditions of outer geometrically irregular shell free attaching look like:

$$\frac{\partial u_1}{\partial y} = 0, \quad u_2 = 0, \quad u_3 = 0, \quad \frac{\partial^2 u_3}{\partial y^2} = 0 \quad \text{at} \quad y = \pm\frac{l}{2}. \qquad (5)$$

The bound hydroelasticity problem Eq. (1-5) is solved in the undimensional variables by perturbation method [12-15] with the use of two small parameters describing the relative width of liquid layer and relative outer elastic geometrically irregular shell sagging by presupposing the harmonic law of system vibration. Such approach allows linearizing the problem of fluid dynamics. Liquid movements velocity and pressure in liquid layer, received in the course of hydrodynamics equations solution, allow to find the stresses acting on the shell. They are substituted in elastic geometrically irregular cylindrical shell dynamic equations. As a result, the system of integrodifferential equations is received.

Bubnov-Galerkin method in the longitudinal coordinate is applied to the solution of integrodifferential equations system [11, 15], where elastic shells movements are presented as trigonometric series in ζ and harmonic function in θ:

$$u_1 = u_m U_1 = u_m \sum_{k=1}^{\infty} \sin\left(\frac{2k-1}{2}\pi\zeta\right)\{(a_{10C}\cos\theta + a_{10S}\sin\theta)\sin(\tau + \varphi_{u1}) + a_{10O}\},$$

$$u_2 = v_m U_2 = v_m \sum_{k=1}^{\infty} \cos\left(\frac{2k-1}{2}\pi\zeta\right)\{a_{20S}\cos\theta + a_{20C}\sin\theta + a_{20O}\}\sin(\tau + \varphi_{u2}), \qquad (6)$$

$$u_3 = w_m U_3 = w_m \sum_{k=1}^{\infty} \cos\left(\frac{2k-1}{2}\pi\zeta\right)\{(a_{30C}\cos\theta + a_{30S}\sin\theta)\sin(\tau + \varphi_{u3}) + a_{30O}\}.$$

Thus, as a result of the solution of hydroelasticity problem, we shall get the expressions for outer elastic ribbed shell elastic shifts.

It was shown earlier that for the study of the resonance amplitude-frequency characteristic in solving the equations of the dynamics of the shell it is sufficient to take only the first term of the series Eq. (6) [16].

By solving the considered system of simple equations, we get new coefficients in the expressions of outer geometrically irregular shell sagging, which in the dimensional state will become:

$$u_{30} = \cos\left(\frac{\pi\zeta}{2}\right)\left(a_{30O} + \sqrt{\frac{An}{Dn}} \times \left(\begin{array}{c} E_z\omega^2\cos\theta\sin(\omega t + \varphi_{z0} + \eta(\omega)) \\ +E_x\omega^2\sin\theta\sin(\omega t + \varphi_{x0} + \eta(\omega)) \end{array}\right)\right). \qquad (7)$$

We get the corresponding matching amplitude-frequency characteristic of outer shell sagging:

$$A(\omega) = \sqrt{\frac{An}{Dn}}, \qquad (8)$$

where An, Dn– expression system-dependent parameters are not discussed here due to where cumbersomeness; $\eta(\omega)$ – phase frequency characteristic.

4. Conclusions

Thus, the mathematical model of the mechanical system consisting of two coaxial cylindrical shells of final length, with the free attaching at butt ends is constructed. The outer shell is geometrically irregular, and the inner one is an absolutely rigid cylinder. The mechanical system is exposed to vibration. Such model of the system considers the liquid movement inertias and elasticity of final length outer shell, having rigid ribs. The constructed model allowed to define the expressions for outer geometrically irregular shell amplitude frequency characteristics. The proposed model allows to choose optimal parameters, proceeding from the known frequency range of vibration and necessary parameters of wear hardness, at the stage of mechanisms projects.

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