

2455. Bifurcation characteristics of torsional-horizontal coupled vibration of rolling mill system

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Abstract. In this paper, the static bifurcation and dynamic bifurcation of rolling mill system with torsional-horizontal coupling vibration are studied. Firstly, the dynamic equation of torsional-horizontal coupled vibration of rolling mill main drive system is established. Rolling mill is driven by the gear pair, so the torsional vibration of the gear pair, the horizontal vibration and the friction factor are considered in the dynamic equation. Then the equivalent low-dimensional bifurcation equation, which can reveal the system nonlinear characteristics, is obtained using Lyapunov-Schmidt method, and the system static bifurcation characteristics are studied using singularity theory. Lastly, the bifurcation condition and stability of the system with dynamic Hopf bifurcation are studied using Hopf bifurcation theorem. Numerical simulation of the actual parameter values confirms the analytical results.

Keywords: rolling mill, lubrication friction, Lyapunov-Schmidt method, singularity theory, bifurcation.

1. Introduction

With the development of modern steel rolling industry, rolling mill is required to have high precision and high dynamic performance. However, unstable vibration has been recognized as a major restriction in rolling productivity [1], so many scholars have been devoted to the study of rolling mill vibration. A large number of researchers have studied rolling mill system from the perspective of the torsional vibration. Liu H. [2] established a dynamic torsional vibration equation of a rotary nonlinear dynamic system and discussed the dynamic stability by the Hopf bifurcation theory. Scholars have made great achievements in the study of the mechanical systems dynamics. Shen Y. J. [3, 4] used singularity theory to analyze the dynamic response of a semi-active control system. Yang S. P. [5, 6] analyzed a vehicle suspension system's singularity completely and studied the effects of system parameters on the bifurcation parameters and unfold parameters in detail. By theoretical analysis, it is shown that the design of parameters has a close relation with the system's stability. According to the situation of bifurcation parameters, the reasonable parameters can achieve system stability [7, 8]. Li X. [9] studied the bursting phenomenon in a piecewise mechanical system with different time scales and provided important theoretical basis on the mechanical manufacturing and engineering practice. Shi P. [10], Liu G. [11] and Liu S. [12] established a nonlinear torsional vibration model of rolling mill system, respectively.

At the same time, some scholars have studied the dynamic behavior of rolling mill from the perspective of vertical and horizontal vibration. Yang X. [13] presented a vertical vibration model based on lubrication and friction theory. Hou D. [14, 15] established a vertical-horizontal coupling vibration dynamic model of rolling mill. It was found that the reasonable parameters can effectively restrain the bifurcation of system. Xu Y. [16] established a dynamic coupling model of the four-roll mill in horizontal and vertical. The stability of cold rolling mill system is discussed under different working conditions. Considering the friction between the roller and the workpiece, Zhang R. [17] derived a horizontal motion non-linear equation of the single-roll driving mill and analyzed the effect of the stiffness fluctuation on the horizontal nonlinear resonance of rolling mill.

The purpose of this paper is to study horizontal-torsional coupled vibration characteristics of rolling mill system. As far as we know, rolling mill vibration is influenced by several coupling factors, which should be taken into account as much as possible. It was found that the rolling speed, the nonlinear friction and the thickness of the strip have strong influences on the stability of rolling mill [18-20]. The research also found that there are a lot of horizontal-torsional coupling vibration phenomena in rolling mill [21]. In the actual rolling process, the horizontal vibration and torsional vibration are closely related to the change of friction force. Under certain conditions, these vibrations are coupled with each other. Therefore, it is necessary to study these two kinds of coupling vibration characteristics of rolling mill. In this paper, the influence of variable friction factors is considered and a horizontal-torsional coupled vibration model of rolling mill is established. Based on this model, dynamic behavior of system is researched and simulated. Besides, it reveals the characteristics and rules in instability of vibration system, and promotes the development of the research on rolling mill coupled vibration system.

The paper is organized as follows. Section 2 establishes a torsional-horizontal coupling vibration dynamic model of rolling mill system. The static bifurcation of the system is studied, and the topological structure of the static bifurcation is obtained in Sections 3. Section 4 studies the system dynamic Hopf bifurcation, and verifies dynamic behavior by numerical simulation. Section 5 concludes this paper.

2. The coupling vibration model with friction of rolling mill

In the study of rolling mill driven by the motor, in order to simplify the model and conform to actual physical meaning, rolling mill drive system is usually abstracted into two parts, which are the motor terminal and the roller terminal according to lumped mass method. In rolling mill drive system, the motor and the roller are connected through the driving system, which consist of gearwheel box, universal couplings, arc-gear synchronous belt and so on. For reflecting main influencing factors of the actual process, the connecting part can be regarded as a gear pair system, which is composed of the active gear and the passive gear. Because the motor and the driving gear, the roller and the passive gear are connected through a rigid shaft, the motor and the driving gear, the roller and the passive gear can be regarded as a concentrated mass, respectively. The model of rolling mill main drive system with the gear pair is as follows.

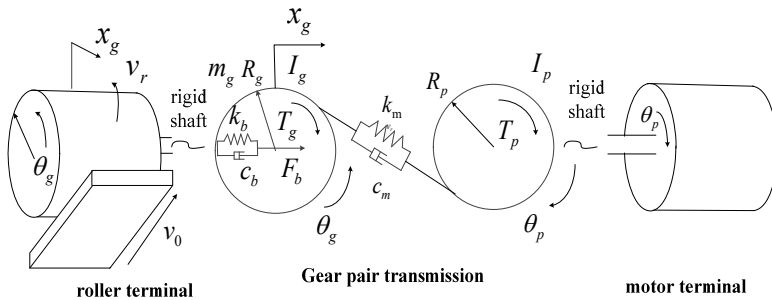


Fig. 1. The model of rolling mill main drive system with the gear pair

I_p, I_g are the rotational inertia of the input and the load after centralized mass, respectively. T_p, T_g are the input torque and the load torque, respectively. Since the input torque is volatile, its general form is $T_p = T_a + T_b \cos(\omega t)$. θ_p, θ_g are the vibratory angular displacements of the input and the load, respectively. c_m, k_m are the meshing damping coefficient, the meshing stiffness, respectively. R_p, R_g are the base radius of the input and the load, respectively. c_b, k_b are the bearing damping and the bearing stiffness on the driven shaft, respectively. F_b is the force between the bearing and the driven gear on the drive shaft m_g, x_g are the mass of driven gear, the horizontal displacement of the load center, respectively v_r, v_0 are the roller linear speed, the stable

workpiece speed, respectively. F_f is the friction between the roller and the workpiece.

In the actual working process of rolling mill, the roller is not often in contact with the workpiece directly, and it is necessary to do some lubrication. So, there is a lubricant film between the roller and the workpiece, and the friction force is a kind of lubrication friction force. Studies have shown that there is a correspondence between the friction coefficient and the thickness of the oil film. It can be affected by a variety of factors, such as the rolling speed, cold reduction ratio, the rolling out entry tension and so on. Base on the theory of fluid mechanics and Hill's formula, simplified the dynamics process and emphasized the influence of the lubricant viscosity coefficient η and the speed of the roll line v_r , the model of oil film thickness under certain conditions can be simplified as:

$$\varepsilon = b_0\eta v_r + c_0. \tag{1}$$

Among them, b_0, c_0 are the corresponding coefficients.

In the rolling process, the corresponding data of film thickness and friction factor can be obtained through appropriate measuring and calculating, and the expression of the friction factor can be acquired by fitting the data. Expressing the friction coefficient in the form of index, the friction coefficient can be described as:

$$\mu = a_0 + ae^{-b\eta v_r + c}, \tag{2}$$

where, a_0, a, b are the liquid friction coefficient, the dry friction coefficient, the corresponding coefficient of the friction, respectively. When the roller is in horizontal vibration, the linear speed of the roller v_r can be expressed as:

$$v_r = v_0 + \dot{x}_g, \tag{3}$$

where v_0 is the stable workpiece speed; \dot{x}_g is the horizontal vibration speed. Substituting Eq. (3) into Eq. (2) and performing Taylor expansion, expanding retention in first several items have:

$$\mu = a_0 + \mu_0 \left(1 - d\dot{x}_g + \frac{1}{2}d^2\dot{x}_g^2 - \frac{1}{6}d^3\dot{x}_g^3 \right), \tag{4}$$

where $d = b\eta$, and $\mu_0 = ae^{-dv_0 + c}$ is the coefficient of friction when the rolling speed is v_0 . At this point the friction between the roller and the workpiece is:

$$F_f = \mu F_l, \tag{5}$$

where F_l represents the rolling force.

According to the Newton's law, the dynamic equation of the system is:

$$\begin{cases} I_p \ddot{\theta}_p + R_p c_m \dot{z} + R_p k_m z = T_p, \\ I_g \ddot{\theta}_g - R_g c_m \dot{z} - R_g k_m z = -T_g, \\ m_g \ddot{x}_g + c_b \dot{x}_g + k_b x_g + c_m \dot{z} + k_m z = F_b - F_f. \end{cases} \tag{6}$$

The transmission error of the linear displacement is defined as:

$$z = R_p \theta_p - R_g \theta_g + x_g. \tag{7}$$

Substituting Eq. (6) with the linear displacement transfer error z , and considering the friction force F_f in the formula, further simplified results can be expressed as:

$$\begin{cases} m_e \ddot{z} - m_e \ddot{x}_g + c_m \dot{z} + k_m z = F_1 + F_2 \cos(\omega \bar{t}), \\ m_g \ddot{x}_g + c_b \dot{x}_g + k_b x_g + c_m \dot{z} + k_m z = F_b - F_l \left[a_0 + \mu_0 \left(1 - d \dot{x}_g + \frac{1}{2} d^2 \dot{x}_g^2 - \frac{1}{6} d^3 \dot{x}_g^3 \right) \right], \end{cases} \quad (8)$$

where $m_e = I_p I_g / (I_p R_g^2 + I_g R_p^2)$, $F_1 = T_a / R_p = T_g / R_g$, $F_2 = m_e R_p T_b / I_p$.

Eq. (8) is the dynamics equation of rolling mill main drive system, which takes horizontal-torsional coupled vibration, lubrication friction and other factors into account. The dynamic behavior of rolling mill is very complex. The research shows that there is obvious horizontal chatter in the rolling process, and source of horizontal vibration is a tribological behavior between the coexistence of fluid lubrication and dry friction motion in rolling interface. Thus, taking the roller horizontal vibration and torsion vibration coupling effect into account, the model is built. Then the friction factor between roller and workpiece is analyzed in detail. Regarding the friction as the main factor, the coupling vibration of rolling mill is analyzed and the influence of parameter variation on vibration buckling is given. It can provide relative guidance for the design of actual system. In the following, the friction coefficient d would be taken as the main research parameter to analyze dynamic behavior of rolling mill.

3. Static bifurcation analysis of high dimensional nonlinear rolling mill system

Selecting z_0 as the nominal scale, and letting:

$$\begin{aligned} \omega_1 &= \sqrt{\frac{k_m}{m_e}}, \quad t = \omega_1 \bar{t}, \quad \Omega = \frac{\omega}{\omega_1}, \quad y_1 = \frac{z}{z_0}, \quad y_2 = \frac{x_g}{z_0}, \quad \omega_2 = \sqrt{\frac{k_b}{m_g}}, \quad \zeta_1 = \frac{c_m}{(2m_e \omega_1)}, \\ k_1 &= 1, \quad P_1 = \frac{F_1}{(z_0 m_e \omega_1^2)}, \quad P_2 = \frac{F_2}{(z_0 m_e \omega_1^2)}, \quad \zeta_2 = \frac{c_b}{(2m_g \omega_1)}, \quad \xi_3 = \frac{c_m}{(2m_g \omega_1)}, \\ k_2 &= \frac{\omega^2}{\omega_1^2}, \quad k_3 = \frac{k_m}{(m_g \omega_1^2)}, \quad P_3 = \frac{F_b}{(m_g z_0 \omega_1^2)}, \quad P_4 = \frac{F_l (a_0 + \mu_0)}{(m_g z_0 \omega_1^2)}, \\ \alpha_1 &= d \omega_1, \quad \alpha_2 = \frac{1}{2} d^2 \omega_1^2, \quad \alpha_3 = \frac{1}{6} d^3 \omega_1^3. \end{aligned}$$

Thus, the dimensionless system model can be expressed as:

$$\begin{cases} \ddot{y}_1 - \ddot{y}_2 + 2\zeta_1 \dot{y}_1 + k_1 y_1 = P_1 + P_2 \cos(\Omega t), \\ \ddot{y}_2 + 2\zeta_2 \dot{y}_2 + k_2 y_2 + 2\zeta_3 \dot{y}_1 + k_3 y_1 = P_3 - P_4 (1 - \alpha_1 \dot{y}_2 + \alpha_2 \dot{y}_2^2 - \alpha_3 \dot{y}_2^3). \end{cases} \quad (9)$$

Letting $x_1 = y_1$, $x_2 = \dot{y}_1$, $x_3 = y_2$, $x_4 = \dot{y}_2$, and the state equation of the system can be expressed as:

$$\dot{x} = f(t, x) = \begin{pmatrix} x_2 \\ -(k_1 + k_3)x_1 - 2(\zeta_1 + \zeta_3)x_2 - k_2 x_3 - (2\zeta_2 - \alpha_1 P_4)x_4 \\ -\alpha_2 P_4 x_4^2 + \alpha_3 P_4 x_4^3 + P_1 + P_2 \cos(\Omega t) + P_3 - P_4 \\ x_4 \\ -k_3 x_1 - 2\zeta_3 x_2 - k_2 x_3 - (2\zeta_2 - \alpha_1 P_4)x_4 - \alpha_2 P_4 x_4^2 + \alpha_3 P_4 x_4^3 + P_3 - P_4 \end{pmatrix}, \quad (10)$$

where $x = (x_1, x_2, x_3, x_4)$.

3.1. Lyapunov-Schmidt reduction

Ignoring the influence of the external excitation, Eq. (10) becomes autonomous system.

Let:

$$P_1 = P_2 = 0, \quad a_1 = -(k_1 + k_3), \quad b_1 = -2(\zeta_1 + \zeta_3), \quad c_1 = -k_2, \quad d_1 = -(2\zeta_2 - \alpha_1 P_4),$$

$$l_1 = -\alpha_2 P_4, \quad l_2 = \alpha_3 P_4, \quad l_3 = P_3 - P_4, \quad a_2 = -k_3, \quad b_2 = -2\zeta_3.$$

The equation of motion can be expressed as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = a_1 x_1 + b_1 x_2 + c_1 x_3 + d_1 x_4 + l_1 x_4^2 + l_2 x_4^3 + l_3, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = a_2 x_1 + b_2 x_2 + c_1 x_3 + d_1 x_4 + l_1 x_4^2 + l_2 x_4^3 + l_3. \end{cases} \quad (11)$$

Then Eq. (11) can be expressed as:

$$\dot{X} = f(X), \quad (12)$$

where $X = (x_1, x_2, x_3, x_4)^T$ are the state variables of the dynamic system and $f(X) = (f_1, f_2, f_3, f_4)^T$ are the corresponding nonlinear vector functions.

At the equilibrium point \bar{X} , Eq. (12) satisfies:

$$f(\bar{X}) = 0. \quad (13)$$

Letting $Df(\bar{X})$ represent the Jacobian matrix of $f(X)$ at the equilibrium point, the Lyapunov stability is dependent on the eigenvalues of the Jacobian matrix. The Jacobian matrix $Df(\bar{X})$ can be expressed as:

$$Df(\bar{X}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ a_1 & b_1 & c_1 & d_1 + 2l_1 x_4 + 3l_2 x_4^2 \\ 0 & 0 & 0 & 1 \\ a_2 & b_2 & c_1 & d_1 + 2l_1 x_4 + 3l_2 x_4^2 \end{pmatrix}. \quad (14)$$

The value of the determinant is:

$$\begin{aligned} \det(L) &= \lambda^4 - (b_1 + d_1 + 2l_1 x_4 + 3l_2 x_4^2) \lambda^3 \\ &+ (b_1 d_1 + 2b_1 l_1 x_4 + 3b_1 l_2 x_4^2 - b_2 d_1 - 2b_2 l_1 x_4 - 3b_2 l_2 x_4^2 - a_1 - c_1) \lambda^2 \\ &+ (b_1 c_1 - b_2 c_1 + a_2 d_1 + 2a_2 l_1 x_4 + 3a_2 l_2 x_4^2) \lambda \\ &+ a_1 d_1 + 2a_1 l_1 x_4 + 3a_1 l_2 x_4^2 + a_1 c_1 - a_2 c_1 = 0. \end{aligned} \quad (15)$$

There is a zero eigenvalue when $a_1 d_1 + 2a_1 l_1 x_4 + 3a_1 l_2 x_4^2 + a_1 c_1 - a_2 c_1 = 0$, and L is a singular matrix.

It can be known that $\dim N(L) = 1$. Letting v_0 represent a basis vector of null space $N(L)$, through calculating, $v_0 = (-c_1/a_1, 0, 1, 0)$. The adjoint operator of L is the conjugate transpose matrix L^* . Since L is a real matrix, the conjugate transpose matrix L^* is also a real matrix, and satisfies $L^* = L^T$, we can obtain that $\dim N(L^*) = 1$ since L is a Fredholm operator with zero index. One basis vector $v_1 = (a_1 b_2/a_2 - b_1, 1, a_1 e/a_2 - a_2 e/a_2, -a_1/a_2)$ of $N(L^*)$ is obtained by calculating, where $e = d_1 + 2l_1 x_4 + 3l_2 x_4^2$. So, the four-dimensional Euclid space R^4 can be decomposed as follows:

$$R^4 = N(L) \oplus M = R(L) \oplus N(L^*). \quad (16)$$

Defining the projection operator and the complementary projection operator as:

$$P: R^4 \rightarrow R(L), \quad (17)$$

$$I - P: R^4 \rightarrow N(L^*) = \text{span}\{v_1\}. \quad (18)$$

Then the equation can be expressed as:

$$f(x, l_2) = 0. \tag{19}$$

Eq. (19) is equivalent to:

$$\begin{cases} pf(v + w, l_2) = 0, \\ (I - P)f(v + w, l_2) = 0. \end{cases} \tag{20}$$

where $v \in N(L) = span(v_0)$, $w \in M$. According to the definition of inner product in Euclid space, $(I - P)f(v + w, l_2) = 0$ is equivalent to:

$$\langle f(v + w, l_2), v_1 \rangle = 0. \tag{21}$$

Then, the following equation is obtained:

$$\frac{a_1^2 b_1}{a_2} + \frac{2a_1 c_1 e}{a_2} - \frac{2a_1 e}{a_2} - \frac{a_1}{a_2} - a_1 b_1 + a_1 b_2 - a_2 b_1 - c_1 e + b_1 + b_2 + 1 + 6l_3 + F = 0. \tag{22}$$

Since $a_1 d_1 + 2a_1 l_1 x_4 + 3a_1 l_2 x_4^2 + a_1 c_1 - a_2 c_1 = 0$, rewriting this equation and substituting it into Eq. (22), we have:

$$Ax_4^4 + Bx_4^2 + Cx_4 + D = 0, \tag{23}$$

where

$$\begin{aligned} A &= \frac{8a_1^2 l_2^2}{a_2(a_1 - a_2)} - \frac{18a_1 l_2^2}{a_1 - a_2}, & B &= \frac{4a_1^2(2l_1^2 + 3d_1 l_2)}{a_2(a_1 - a_2)} + \frac{4a_1(2l_1^2 - 3d_1 l_2)}{a_1 - a_2} - \frac{6a_1 l_2}{a_2}, \\ C &= \frac{8a_1^2 d_1 l_1}{a_2(a_1 - a_2)} - \frac{4a_1 l_1}{a_2} + 8c_1 l_1, \\ D &= F + b_1 + b_2 + 6l_3 - a_1 b_1 + a_1 b_2 - a_2 b_1 - \frac{a_1}{a_2} + \frac{a_1^2 b_1}{a_2} \\ &\quad - \frac{2a_1 d_1^2}{a_1} - a_2 - \frac{2a_1 d_1}{a_2} + \frac{2a_1^2 d_1^2}{a_2(a_1 - a_2)} + 1. \end{aligned}$$

Eq. (23) is a quartic algebraic equation and any univariate equation with a higher order can eliminate the second highest item by a proper transformation.

3.2. Singularity analysis

Define a new state variable $m = x_4$, then the bifurcation equation can be written as:

$$G(m, n, \alpha, \beta) = m^4 + \beta m^2 + \alpha m + n, \tag{24}$$

where α, β are the open fold parameters of the bifurcation equation, m, n are the physical parameters of the system, respectively.

Selecting $g_0(m, n) = m^4 + n$ as the Golubitsky-Schaeffer normal form, according to recognition condition, the necessary and sufficient condition for $G(m, n, \alpha, \beta)$ to be a versal unfolding of $g_0(m, n) = m^4 + n$ is:

$$detA(0,0,0,0) \neq 0, \tag{25}$$

where A is, a matrix constructed by some partial derivatives of g and G , it can be expressed as:

$$\begin{pmatrix} g_n & g_{nm} & g_{nmm} \\ G_\alpha & G_{\alpha m} & G_{\alpha mm} \\ G_\beta & G_{\beta m} & G_{\beta mm} \end{pmatrix}. \tag{26}$$

Thus:

$$\det A(0,0,0,0) = \begin{vmatrix} 1 & 0 & 0 \\ m & 1 & 0 \\ m^2 & 2m & 2 \end{vmatrix}_{m=0} = 2 \neq 0,$$

so $g_0(m, n) = m^4 + n$ is a 2-parameter unfolding and α, β are the unfolding parameter.

The topology of Eq. (24) and the effects of the open fold parameters on the bifurcation behavior will be studied by singularity theory.

The following point sets are obtained based on the definition of the transition set:

I: Bifurcation point set:

$$B = \{(\alpha, \beta) | G = G_m = G_n = 0\} = \Phi.$$

II: Lag point sets:

$$H_1 = \{(\alpha, \beta) | G = G_m = G_{mm} = 0\} = \{(\alpha, \beta) | \alpha = 0\},$$

$$H_2 = \{(\alpha, \beta) | G = G_m = G_{mm} = 0\} = \left\{ (\alpha, \beta) | \alpha + \frac{4}{3}\beta \sqrt{-\frac{\beta}{6}} = 0, \beta \leq 0 \right\}.$$

III: Double limits point set:

$$D = \{(\alpha, \beta) | G = G_m = 0\} = \left\{ (\alpha, \beta) | -\frac{2\beta^2}{9} + \alpha \sqrt{-\frac{\beta}{3}} = 0, \beta \leq 0 \right\}.$$

IV: Transition set:

$$\Sigma = H_1 \cup H_2 \cup D.$$

The transition set Σ is shown in Fig. 2.

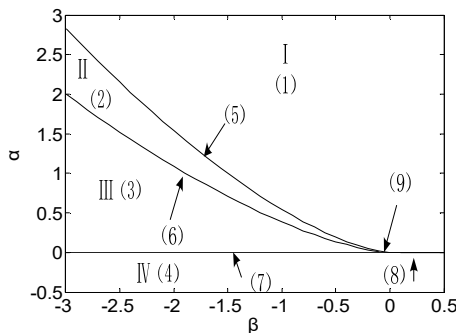


Fig. 2. Transition set

It can be seen that the unfolding parametric space $\alpha - \beta$ is divided into four sub regions by Σ .

The bifurcation diagrams of each subregion and its boundary are shown in Fig. 3, a total of nine parts with Fig. 3(a)-(i) including the boundary. Among them, Fig. 3(a)-(d) are four different bifurcation regions divided by the transition set and Fig. 3(e)-(i) are bifurcation curves in the transition set. The topology of each sub region and its boundary are shown in Fig. 3. It can be seen from Fig. 3 that each sub region and its boundary have different bifurcation forms, and the system vibration mode will change when the unfolding parameters pass by the transition set.

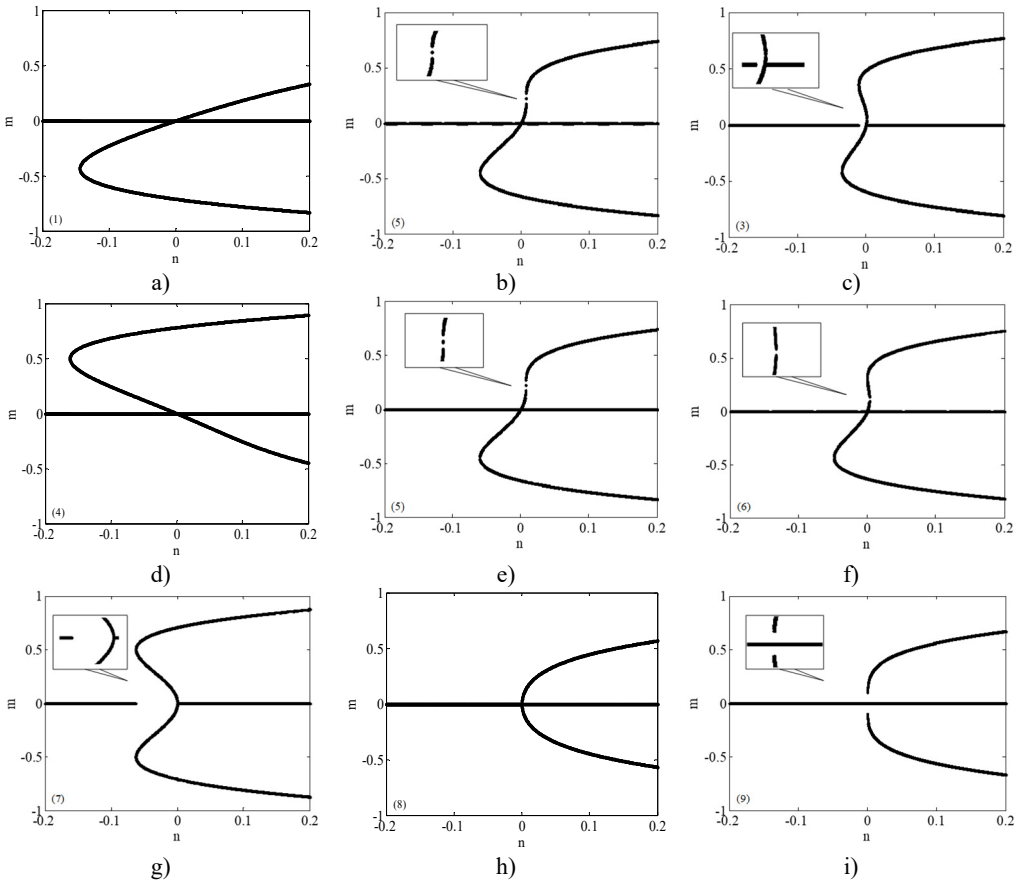


Fig. 3. Bifurcation diagram

4. Hopf bifurcation analysis of high dimensional nonlinear rolling mill system

4.1. The equilibrium point analysis

In last section, the main drive system of rolling mill can be expressed as:

$$\dot{x} = f(x, d). \tag{27}$$

The equilibrium point of the Eq. (27) can be transferred to the coordinate origin by simple linear transformation. Coordinate translation does not change the dynamic characteristics, so it is of significance to study the dynamic characteristics of the system equilibrium point at the origin point. When the equilibrium point is transformed to the coordinate origin, the Eq. (13) can be turned into the following equation:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = a_1x_1 + b_1x_2 + c_1x_3 + d_1x_4 + l_1x_4^2 + l_2x_4^3, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = a_2x_1 + b_2x_2 + c_1x_3 + d_1x_4 + l_1x_4^2 + l_2x_4^3. \end{cases} \quad (28)$$

After linearizing, the Jacobian matrix at the origin can be expressed as

$$J(x_0, b) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ a_1 & b_1 & c_1 & d_1 \\ 0 & 0 & 0 & 1 \\ a_2 & b_2 & c_1 & d_1 \end{pmatrix}. \quad (29)$$

The characteristic equation of the $J(x_0, \alpha_2)$ is:

$$\lambda^4 + p_1\lambda^3 + p_2\lambda^2 + p_3\lambda + p_4 = 0, \quad (30)$$

where:

$$p_1 = -d_1 - b_1p_2 = -c_1 + b_1d_1 - b_2d_1 - a_1p_3 = b_1c_1 - b_2c_1 + a_1d_1 - a_2d_1p_4 = a_1c_1 - a_2c_1.$$

It is easy to verify that the characteristic polynomial of Eq. (30) is equivalent to:

$$(\lambda^2 + r_1\lambda + s_1)(\lambda^2 + r_2\lambda + s_2) = 0, \quad (31)$$

where:

$$r_1 = \frac{p_1 + \sqrt{8u_0 + p_1^2 - 4p_2}}{2}, \quad r_2 = \frac{p_1 - \sqrt{8u_0 + p_1^2 - 4p_2}}{2}, \quad (32)$$

$$s_1 = u_0 + \frac{P_1u_0 - P_3}{\sqrt{8u_0 + p_1^2 - 4p_2}}, \quad s_2 = u_0 - \frac{P_1u_0 - P_3}{\sqrt{8u_0 + p_1^2 - 4p_2}}, \quad (33)$$

$$u_0 = \frac{p_2}{6} + \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}, \quad (34)$$

$$p = \frac{p_1p_3 - 4p_4}{4} - \frac{p_2^2}{12}, \quad q = \frac{p_1p_2p_3 + 8p_2p_4 - 3p_1^2p_4 - 3p_3^2}{24} + \frac{p_2^2}{54}. \quad (35)$$

Therefore, the Eq. (31) can be written as the following two equations:

$$\lambda^2 + r_1\lambda + s_1 = 0, \quad (36)$$

$$\lambda^2 + r_2\lambda + s_2 = 0. \quad (37)$$

Further, letting:

$$\Delta_1 = r_1^2 - 4s_1, \quad \Delta_2 = r_2^2 - 4s_2. \quad (38)$$

Fixing other system parameters as constants, when the friction coefficient d changes, the characteristic root of Eq. (30) has two important distributions:

Situation 1: when $\Delta_1 < 0, r_1 > 0, r_2 = 0, s_2 > 0$, Eq. (36) has a pair of conjugate complex roots with negative real parts, and Eq. (37) has a pair of pure imaginary roots.

Situation 2: when $\Delta_2 < 0, r_2 > 0, r_1 = 0, s_1 > 0$, Eq. (36) has a pair of pure imaginary roots,

and Eq. (37) has a pair of conjugate complex roots with negative real parts.

4.2. Hopf bifurcation analysis

The friction condition has great influence on the dynamic behavior of rolling mill system. So, friction coefficient d is selected as a major parameter to analyze dynamic characteristics. The Situation 1 and Situation 2 all show that characteristic Eq. (30) has a pair of pure imaginary roots and a pair of conjugate complex roots with negative real parts. When the roots satisfy the two situations mentioned above, Hopf bifurcation is very likely to occur.

In order to find the Hopf bifurcation point and prove the existence of limit cycles, in general, we can calculate all the eigenvalues of Jacobian matrix $J(x_0, d)$, and then judge whether characteristic roots pass through the imaginary axis with the change of parameters. But the amount of calculations is very large and it is difficult to write the analytic expression of characteristic roots. Thus, the Hurwitz determinant is used to determine the existence of the Hopf bifurcation.

The characteristic equation of matrix $J(x_0, d)$ is:

$$\lambda^4 + p_1\lambda^3 + p_2\lambda^2 + p_3\lambda + p_4 = 0, \tag{39}$$

where:

$$\Delta'_1 = p_1, \quad \Delta'_2 = \begin{vmatrix} p_1 & 1 \\ p_3 & p_2 \end{vmatrix}, \quad \Delta'_3 = \begin{vmatrix} p_1 & 1 & 0 \\ p_3 & p_2 & p_1 \\ 0 & p_4 & p_3 \end{vmatrix}, \tag{40}$$

where Δ'_i are the Hurwitz determinants. Select $d = d^*$ as a Hopf bifurcation point, the conditions of Hopf bifurcation are:

$$\begin{cases} p_i(d^*) > 0, & i = (1, 2, 3, 4), \\ \Delta'_i(d^*) > 0, & i = (1, 2), \\ \Delta'_3(d^*) = 0, \\ \left. \frac{d(\Delta'_3(d))}{d(d)} \right|_{d=d^*} \neq 0. \end{cases} \tag{41}$$

Take $k_1 = 1, k_2 = 1, k_3 = 0.6, \zeta_1 = 0.38, \zeta_2 = 0.38, \zeta_3 = 0.09, P_3 = 1, P_4 = 1$. Hopf bifurcation occurs at $d^* = 1.2324$.

Supposing that the pure imaginary roots are $\pm\omega_0 i$, U and W are left and right normalized characteristic vectors of $\omega_0 i$, respectively:

$$UA = \omega_0 iU, \quad AW = \omega_0 iW, \quad UW = 1. \tag{42}$$

Letting:

$$\rho = Re(-Uf_{xxx}WWW^* + 2Uf_{xx}WA^{-1}(0)f_{xx}WW^* + Uf_{xx}W^*[A(0) - 2i\omega_0 I]^{-1}f_{xx}WW), \tag{43}$$

where W^* is conjugate complex of $W, A(0) = A(x_0, d)|_{d=d^*}$:

$$f_{xxx}WWW^* = \left. \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} \left(\frac{\partial f(x, d)}{\partial x} \times W \right) \times W \right) \times W^* \right|_{d=d^*, x=x_0}. \tag{44}$$

The stability of bifurcating periodic solutions is decided by the sign of ρ . With $d^* = 1.2324$,

$\rho = -0.0900 < 0$ can be obtained by calculation. Bifurcating periodic solutions aren't asymptotical stable when $\rho < 0$. The results show that it is a subcritical bifurcation point.

4.3. Numerical simulation

Numerical Simulation of the dynamical behavior is made near the bifurcation point. For subcritical bifurcation, when $d = 1.2100 < d^*$. It can be known that the equilibrium point is asymptotically stable from Fig. 4. The unstable periodic motion may occur in the region of $d = 1.2280$.

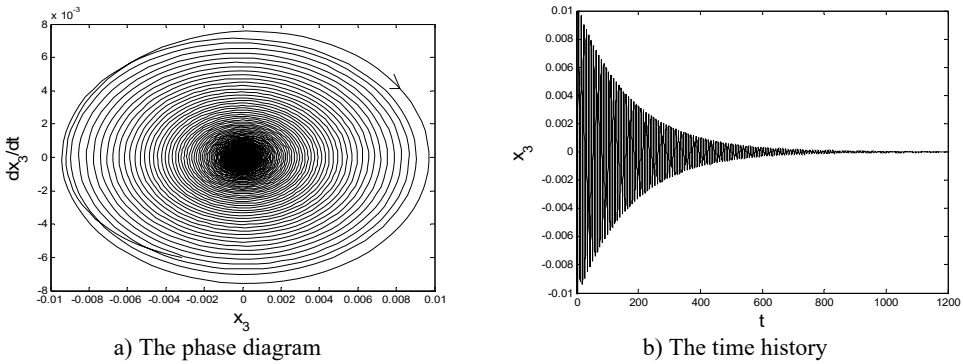


Fig. 4. Asymptotic stability of the equilibrium point

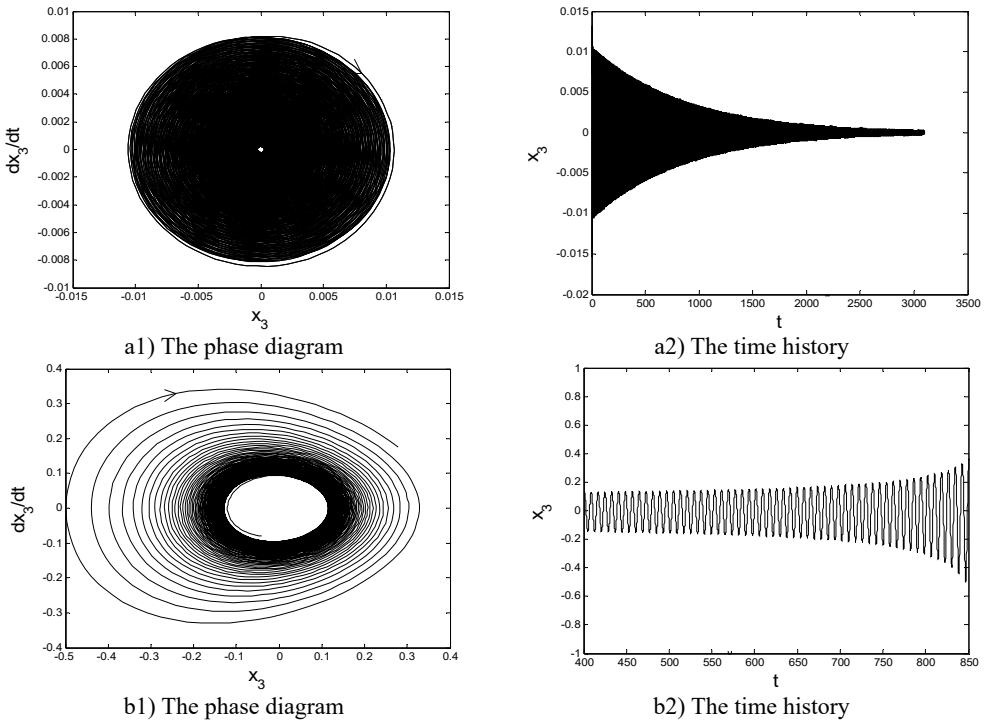


Fig. 5. Time history responses and phase diagram with $d = 1.2280$ and initial values are a) $[0.01 \ 0.01 \ 0.01 \ 0.01]$, b) $[0.12 \ 0.12 \ 0.12 \ 0.12]$

Initial values have a certain influence on the trajectory. When initial point is $[0.01 \ 0.01 \ 0.01 \ 0.01]$, which is close to the equilibrium point, Fig. 5(a1), (a2) show that time

history responses are stable and phase diagram is stable focus. When initial point is $[0.12 \ 0.12 \ 0.12 \ 0.12]$, which is far away from the equilibrium point, Fig. 5(b1), (b2) show that time history responses are unstable and phase diagram gradually expands away from a certain limit cycle. Region of attraction on initial point can be obtained by numerical simulation. In the current parameter system, when initial point exceeds $[0.09 \ 0.09 \ 0.09 \ 0.09]$, the time history responses will be unstable.

When $d = 1.2432 > d^*$, the equilibrium point of the system is unstable as shown in Fig. 6, in which case the roller loses its stability and self-oscillation can be generated.

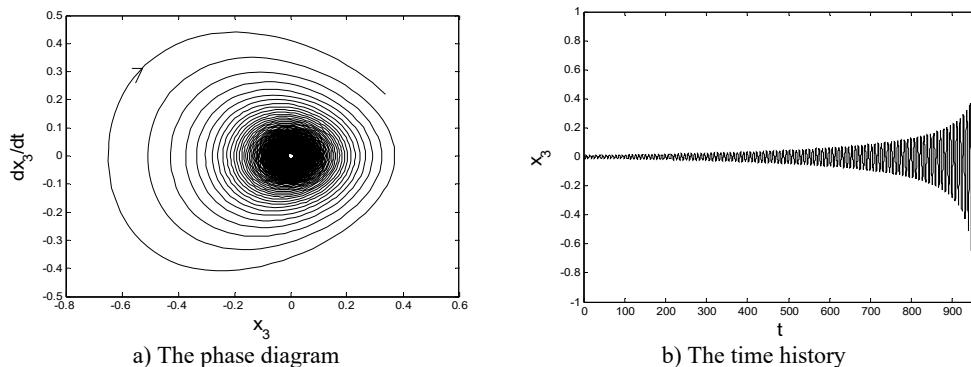


Fig. 6. Asymptotic stability of the equilibrium point

In summary, the system shows a complex dynamic behavior. It can be known that the friction coefficient d and the initial condition of the system are directly related to the stability of the system. When the value of friction coefficient d is much smaller than the bifurcation point, the system is asymptotically stable; when the friction coefficient d near the bifurcation point $d^* = 1.2324$, the stability of the system is mainly affected by the initial values. When the initial values are close to the equilibrium point, the system is stable; and the stability becomes worse when the initial value is far away from the equilibrium point. When the value of friction coefficient d is larger than the value of bifurcation parameter, the system is divergent, and the phenomenon of the self-excitation oscillation may occur. In the vicinity of the bifurcation point, the system may have a violent shock and even lead to complex dynamic phenomena. These phenomena should be avoided in the actual production process, so as to ensure the smooth running of the whole rolling mill system.

5. Conclusions

In this paper, a torsional-horizontal coupling vibration dynamic equation of rolling mill system is established to analyze the bifurcation characteristics. For the four-dimensional dynamic equation, firstly, the dimension is reduced by Lyapunov-Schmidt reduction method, the topological structure of the system static bifurcation is obtained by singularity analysis, and the parameter condition, which makes the system stable, is given. Then the friction coefficient d is selected as the bifurcation parameter, the dynamic Hopf bifurcation of the system is studied, and the dynamic behavior of the system is verified by numerical simulation. The results show that the system may be unstable when the friction coefficient d changes around the bifurcation point. These phenomena should be avoided in the actual production process to ensure the safe operation of the whole system. It provides a theoretical guide for the study of the horizontal-rotational coupling mechanical vibration.

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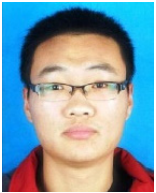
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