Evolution of acoustic waves in heaped granular material

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Abstract. The propagation and structure of acoustic finite amplitude wave in heaped granular material is studied in this paper. The equations describing the evolution of arbitrary wave disturbances of finite amplitude velocity (density, or temperature) perturbations have been developed, with correction for heat transfer and momentum exchange between the phases under significantly unsteady conditions. The extreme cases of long- and short-wave interactions between phases are obtained. The resulting equations incorporate integral terms containing the amplitude of the perturbation. These formulae are valid for describing the evolution of waves at any ratio between the time of the internal process and the characteristic period of perturbation. Because of the high interaction between the skeleton and internal gas, the finite amplitude wave is largely dissipative. It will be shown that during the wave propagation, decrease of the wave amplitude with increasing distance is described by an exponential curve.

Keywords: heaped granular material, perturbation, propagation, heat, momentum, transfer, evolution.

Nomenclature

M	Porosity of the heaped granular medium, which is equal to the part of the unit volume occupied by the gas phase [m³/m³]
p	Pressure in the gas phase [N/m²]
V	Variable volume [m³]
δ	The particle radius [m]
r	Radial coordinate [m]
x	Instantaneous coordinate in the direction of propagated perturbation [m]
t	Time [sec]
T	Period of perturbation [sec]
l	Wavelength of perturbation [m]
ρ_p	Density of the particle's or granular material [kg/m³]
ρ	The gas density [kg/m³]
u	The gas velocity [m/sec]
ϑ	The gas temperature [K]
ϑ_3	Theta function, as mentioned in [11]
θ	Relative temperature of a particle [K]
q_P	Heat flux between a single particle and surrounding gas [W/m²]
Q	The overall heat flux from the gas to the particles cloud [W/m²]
R	Specific gas constant [J/molk]
c_v	Specific heat of the gas at constant volume [J/kgk]
c_P	Specific heat of the particle's material [J/kgk]
κ	$\kappa = c_P/c_v$
C	Speed of sound [m/sec] $C = \sqrt{R\vartheta_0}$
τ_P	Characteristic relaxation time for particle's temperature [sec], $1/\tau_P = 8\pi\delta mac_P/c_v$
a	Thermal diffusivity of the particle's material [m²/sec]
λ_P	Thermal conductivity of the particle's material [W/mk]
ν	Kinematic viscosity [m²/sec]
0	Subscript, refers to conditions in the front of wave

x Subscript, refers to differentiation with respect to x coordinate
t Subscript, refers to differentiation with respect to the time coordinate
r Subscript, refers to differentiation to radial coordinate

1. Introduction

Many real life problems could be modelled by some simplified models. For example rusted surface of metal could represented as heaped granular material. These materials are also used widely in industrial technologies, for example, in powder metallurgy, powder technology, chemical industry and many in other industrial applications. The relationships governing flows in these types of situations have been described by Marble [1], Davidson [2,3], Popov [4], Rytov and Vladimirsky [5], and Isakovich [6], who investigated the propagation of weak (primarily acoustic) pulses in gases containing liquid droplets or solid particles. Nikolaevskiy et al. [7] studied propagation of acoustic waves in porous media. These media could be modeled by an assembly of the spherical particles, with population m in a given unit volume, considered constant, regularly packed with the area between them filled with the air. It is feasible to consider similar approximation when we are studying infinite porous media. Any disturbances, which can happen in the area next to this heaped granular material, will propagate through the media. The signal will go through the rigid phase and also through the gas phase of the porous media.

All interactions between the phases were treated by applying the Newton-Richman law for description of the heat transfer and the Stokes law for description of the mechanical interaction. However, the application of these laws is valid only in the case of quasisteady processes, whereas for unsteady conditions, which are rather frequently encountered in practice, it is incorrect to represent heat transfer and friction between phases in this form. Nigmatulin [8] demonstrated that the heat transfer coefficient under transient conditions is up to 50 times larger than in the steady state. The present work concentrates on the derivation and analysis of a single equation for the description of the evolution of a finite amplitude perturbation, allowing for relaxation processes induced by unsteady heat and momentum transfer between the phases.

2. Interphase exchange processes

For the propagation of the non-steep pressure waves with finite amplitude through the heaped granular material we assume that the wave propagates through the porous part of the media. This porous media is represented as a fill made up of spherical particles of uniform radius, Fig. 1, with finite amplitude perturbations propagate through. So the process of wave propagation will be described by the system of equations for the gas phase with consideration of interphase exchange processes due to rigid boundaries formed by steady particles.

To consider the interaction between phases we will use results obtained for wave propagation in the media having spherical particles [9]. Interphase heat exchange is accounted using relation for the single particle's heat flux, which is taken in the form:

$$q_p = \lambda_p \left(\frac{\partial \theta}{\partial r}\right)_{r=\delta} = -\frac{2\lambda_p}{\delta} \sum_{n=1}^{\infty} \frac{\partial}{\partial t} \int_0^t \vartheta(y) \exp\left[-n^2\pi^2 \frac{a}{\delta^2}(t-y)\right] dy. \tag{1}$$

In the case when there is a relative motion between the heaped granular material skeleton and the surrounding gas we can use for the momentum exchange next expression given in reference [10] as follows:

$$F = 2\pi\rho\delta^3 \left(\frac{1}{3}u_t + \frac{3\nu}{\delta^2}u + \frac{3}{\delta}\sqrt{\frac{\nu}{\pi}} \int_0^t u_y \frac{dy}{\sqrt{t-y}}\right). \tag{2}$$

The first term in Eq. (2) allows for the induced mass and is significant in analyzing the motion being of gas bubbles in a liquid. This term can be treated negligible in case of solid particles or interaction solid boundaries with moving gas, as insignificant. The second term in Eq. (2) is a frictional drag, which is much smaller than the Basset force (third term of Eq. (2) in the case of unsteady interaction of the flow with the boundaries. In our special case the momentum exchange is described only by the third term.

These Eqs. (1) and (2) are describing the exchange terms considered for a single particle. In our case we need to consider porosity and from there derive terms for the granular material situation.
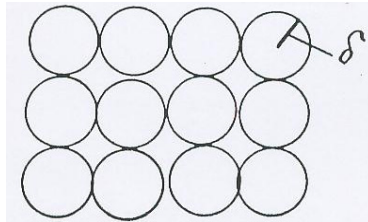

Fig. 1. Model of porous media

When we consider the porous media in Fig. 1, it is possible to model this media as an infinite volume regularly packed by spherical particles of the uniform radius δ with interparticle space occupied by gas.

If finite-amplitude perturbation propagate through an infinite heaped granular material, modeled by the absolutely rigid solid skeleton (porosity $M = $ const) then we can assume that only part of the surface of those particles will participate in the interphase heat and momentum exchange.

The equations of conservation for the mass, momentum and energy, and the equation of state need to be formulated only for the gaseous phase with correction for the thermal and momentum interaction between phases. Neglecting viscosity and thermal conductivity in the conservation equations, the following set of equations, analogous to that given by Nikolayevskiy et al. [7] can be obtained for describing the evolutions of waves in granular material with allowance for thermal and force interaction:

$$M\rho_t + M(\rho u)_x = 0,$$

$$M\rho u_t + M\rho u u_x + MR(\rho v)_x = 6(1-M)\pi\rho\delta v u + \frac{9}{2}(1-M)\frac{\rho}{\delta}\sqrt{\frac{v}{\pi}}\int_0^t \frac{u_y dy}{\sqrt{t-y}},$$

$$M\rho c_v v_t + M\rho c_v u v_x + M\rho R v u_x =$$

$$-6(1-M)\frac{a}{\delta^2}\rho_p c_p \frac{\partial}{\partial t}\int_0^t v\left\{\vartheta_3\left[0,\exp\left(-\frac{\pi^2 a}{\delta^2}(t-y)\right)\right]-1\right\}dy \tag{3}$$

$$+\frac{9}{2}(1-M)\frac{\rho}{\delta}\sqrt{\frac{v}{\pi}}u\int_0^t \frac{u_y dy}{\sqrt{t-y}}.$$

We have assumed here that we are able to use the equation of state for ideal gas and also we can neglect the first term of the relation for the particle's drag force, because it is responsible for the induced mass.

The term describing heat transfer between phases in this energy equation can have another form if the medium is modeled by let us say, cylinders or plates, instead of modelling by an ensemble of spherical particles.

Eq. (3) could be reduced to a single equation for waves with small, but finite amplitude,

traveling in the positive direction using technique described in [9]. The following equation for the evolution of waves in porous material in the presence of two relaxation processes due to unsteady heat and momentum transfer between the phases can be obtained:

$$
u_x - \frac{\kappa - 1}{2\kappa} u_\tau - \frac{\kappa + 1}{2} u u_\tau - 3 \frac{1 - M}{M} \frac{\pi \delta \nu}{\kappa} u (1 + u) - \frac{9}{2} \frac{1 - M}{M \delta} \sqrt{T \frac{\nu}{\pi}} \int_0^\tau \frac{u_y}{\sqrt{\tau - y}}
$$
$$
-9 \frac{1 - M}{M} \frac{\kappa}{\delta} \sqrt{T \frac{\nu}{\pi}} u \int_0^\tau \frac{u_y dy}{\sqrt{\tau - y}}
$$
$$
+6 \frac{1 - M}{M} \frac{T}{\tau_p} \frac{\rho_p}{\rho_0} \frac{\partial}{\partial \tau} \int_0^\tau [1 + (\kappa - 1)u] \left\{ \vartheta_3 \left[0, \exp\left(-\frac{\pi^2 a}{\delta^2} (\tau - y) \right) \right] - 1 \right\} dy = 0.
$$

(4)

It is thus seen that the propagation of waves in bulk granular material can be described by a single, nonlinear evolution equation. It is evident that the principal contribution to the decay of waves is made by the unsteady interaction between phases. These effects make a much greater contribution to decay of waves compare to viscosity and heat conduction.

3. Conclusion

The obtained Eq. (4) describing the propagation of the perturbations through and porous part of the heaped granular material is known as the Burger's equation. This equation is well known and it has as analytical and numerical solutions. It describes the propagation of the waves in two-phase medium with the dissipation related to the heat and momentum exchange between phases. The evolution Eq. (4) allows for an easier way to consider the propagation of the perturbations, as compared to the initial systems of the Eq. (3). The derived dimensionless Eq. (4) allows considering the dissipation of the pressure pulses in porous media with different phase properties. This equation enables calculation of the wave profile evolution and defines the effects of the interphase exchange processes on the propagation of the perturbations. The derived equation is well known and it facilitates the analysis and evaluation of the effects of interphase exchange processes on the dissipation of the pressure pulses in any two phase system.

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