Product reliability evaluation based on manufacturing process information fusion

Chuanliang Zhang[1], Wei Dai[2], Yu Zhao[3]
School of Reliability and Systems Engineering, Beihang University, Beijing 100191, China
[1]Corresponding author
E-mail: [1]*zcls07@163.com*, [2]*dw@buaa.edu.cn*, [3]*lpds07@126.com*

(Accepted 28 September 2014)

Abstract. To make full use of multi-source information to evaluate product reliability, as in the case of quick response manufacturing demand, the Bayes information fusion method based on the information entropy is proposed. Firstly, the information entropy in the fusion of multi-source information reliability with the Bayes method is analyzed, and the premise of using the Bayes method for information fusion is pointed out. Based on the forming process of product reliability is analyzed, the reasonable source is determined. By comparing the similar degree of current processing finished products and similar products, different prior distributions are determined. And then the Bayes method is used for information fusion to evaluate product reliability. Finally, the case study on reliability evaluation of radar phase-shift unit is demonstrated to prove the feasibility and effectiveness of the proposed method.

Keywords: reliability evaluation, information entropy, information fusion, Bayes method, similar degree.

1. Introduction

Reliability assessment is an important content of reliability work, its purpose is to measure whether the reliability of the product reaches the expected goal and to point out the weak links of products. Assessing the reliability with utilizating the fault data collected by the actual use of the product is the most accurate way [1]. However, as in the case of quick response manufacturing demand, the sample that used for life testing was small. Thus the current research that based on a large sample used for life testing was difficult to be applied.

However, can be used to evaluate product reliability, there is multi-source information accumlated in the stage of manufacture and produce. To make full use of multi-source information to evaluate product reliability, the Bayes method was usually used. For the purpose of information fusion is to obtain the more clear understanding of the evaluation goal through the information added, therefore its essence is to reduct the uncertainty of the goal [2]. The information entropy was most commonly used to describe the information uncertainty. Thus the Introduction of the information entropy was given first, and then the Bayes assessment method based on it was given in this part. Then based on the forming process of product reliability is analyzed, the reasonable source is determined. By comparing the similar degree of current processing finished products and similar products, different prior distributions are determined. And then the Bayes method is used for information fusion to evaluate product reliability. Finally, the case is demonstrated to prove the feasibility and effectiveness of the proposed method.

2. Bayes method based on the information entropy

2.1. Instruction of information entropy

The information entropy is mainly used to describe the uncertainty of the system state. It was defined as:

$$E(x) = -\int_{-\infty}^{+\infty} f(x)\ln f(x)\,dx, \tag{1}$$

where the $f(x)$ is a probability density function of the random variable distribution.

To obtaine the probability density function of random variables with using the information entropy, the corresponding constraints were usually given. In many probability density functions, which are satisfied to the given constraints, to take the maximum entropy for the best probability density function is the principle of maximum entropy.

2.2. Bayes method

The paper selected product life as its reliability index. Assuming the finished product life X is a random variable and subject to the distribution density function $f(x|\theta)$, where θ is the unknown parameters. If a small sample experiment was carried out after completion of processing, and the collected sample data was $X = (x_1, x_2, \dots, x_n)$. The likelihood function of these sample datas can be obtained as $p(x|\theta) = \prod_{i=1}^{n} p(x_i|\theta)$.

If the prior distribution of the parameter θ, which was obtained according to the different sources, was $\pi_i(\theta)$, i =1, 2,…, n, and the incorporating prior density function was $\pi(\theta)$, the posterior density function $\pi(\theta|x)$ of the parameter θ can be obtained according to the Bayes formula as [3]:

$$\pi(\theta|x) = \frac{\pi(\theta)p(x|\theta)}{\int_{\Theta} \pi(\theta)p(x|\theta)d\theta}, \tag{2}$$

where the Θ represents the range of values of the parameter θ. Combined with the sample data $X = (x_1, x_2, \cdots, x_n)$, the point estimate of the θ can be obtained according to the Bayes formula as:

$$\hat{\theta} = \frac{\frac{1}{\sigma^2}}{\frac{1}{\sigma^2} + \frac{1}{s^2}} \bar{Y} + \frac{\frac{1}{s^2}}{\frac{1}{\sigma^2} + \frac{1}{s^2}} \mu_\theta, \tag{3}$$

where the σ^2 and $\bar{Y}$ are the mean and the variance of the sample data, whilie the s^2 and μ_θ are the mean and the variance of the posterior density function.

Through the above analysis, the key to use Bayes method is how to determine a reasonable fusion prior distribution depending on the different prior distribution. The fution method based on information entropy was given below.

Assuming there was no information, the prior distribution of the parameter θ was $\pi_0(\theta)$. The uncertainty of the θ can be obtained according to the Eq. (1) as:

$$E_0(X) = -\int_{\Theta} \pi_0(\theta|x) \ln \pi_0(\theta|x) d\theta. \tag{4}$$

Make $m(x) = \int_{\Theta} p(x|\theta)\pi(\theta)d\theta$ as the marginal distribution density of the X under $\pi(\theta)$. Empirical Bayes theory [4] considers that the size of the marginal distribution density reflects the average size of the possibility of a sample under the prior distribution. Thus in the case of the absence of information, the mean uncertainty of θ was [5]:

$$I_0 = \sum_{j=1}^{n} E_0(x_j) m_0(x_j). \tag{5}$$

While the priori distribution $\pi_i(\theta)$ was known, the posterior distribution $\pi_i(\theta|x)$ of parameter θ was determined according to Eq. (2). Thus the marginal distribution density of X was $m_i(x)$, and the mean uncertainty $E_i(X)$ and the average uncertainty I_i of θ can be obtained. Because of the priori distribution $\pi_i(\theta)$, the reduction of the uncertainty of parameter θ was:

$$\Delta I_i = I_0 - I_i, \quad i = 1, 2, \dots, n. \tag{6}$$

The greater ΔI_i represents the greater elimination of the uncertainty of θ. Thus the fused prior distribution can be obtained as:

$$\pi(\theta) = \sum_{i=1}^{n} \frac{\Delta I_i}{\sum_{i=1}^{n} \Delta I_i} \pi_i(\theta). \tag{7}$$

3. Determination of the source of information

All the inherent reliability of product will be formed through processing stage after the completion of its design. Due to design is the start stage in product development, thus the reliability of the design phase for the product has been studied for a large number of research results [6]. However, even the same design specifications of products by different processes in manufacturing, the inherent reliability will also have great differences. Practice shows that the 65 percent problems exposed in the using course of the product are caused by the cause in processing cycle. Therefore, the sources of information of reliability assessment were determined through analyzing the factors affecting the reliability of the product during processing in this paper.

At the processing stage, the product reliability is formed during the processing of the quality characteristics, thus whether the product reliability can be guaranteed in the process or not was determined by whether the quality characteristic of the process meets the requirements of specifications or not [1]. Different quality characteristics have varied influence on the product reliability. And the critical quality characteristics, which have great influence on the product reliability, can be determined through the judgment of experts. Therefore, the degree of reliability can be reflected through assessing the level the critical quality characteristics meeted the technological requirements.

And each quality characteristic was final formed often requiring multichannel working procedure processing, and each process has many factors that influencing on it. Thus when the information on the quality characteristics of the product is not readily available, the relationship between critical process factors and critical quality characteristics can be found out. And the degree of product reliability can be reflected through the assessment of critical process factors.

In addition, life testing of small samples usually will be carried out after the processing is complete, and the information relating to product reliability also can be achieved by a small amount of data. And the information of reliability of the similar products can also be obtained through user feedback. Therefore, in reliability assessment, the sources of information were determined as critical quality characteristics, critical process factors and product reliability. Among them, product reliability information includes current product reliability and similar product reliability information.

4. Determination of different prior distributions

The life of current product can be estimated through comparing the degree of similarity between the current product and similar product.

The analysis above shows that product reliability is largely determined in the processing process, thus the level of product reliability can be reflected by the critical quality characteristics and the critical technical factors that affected it. Therefore, the degree of similarity between the products can be reflected through comparing the level the critical quality characteristics and the critical technical factors, which affected the reliability, meeting the technological requirements of the current products and similar products.

4.1. The prior distribution based on critical quality characteristics

Assuming the weight of the critical quality characteristics were w_i $(i = 1, 2, \ldots, n)$, the optimized model can be obtained as:

$$MaxH_1(W) = \frac{2(n-1)}{n} - \sum_{i=1}^{n} \left| w_i - \frac{1}{n} \right|, \ \ s.t. \sum_{\substack{i=1 \\ w_i \in H}}^{n} w_i = 1, \ \ w_i \in [0,1], \ \ i = 1, 2, \ldots, n. \tag{8}$$

Among which, H was the value range of each weight.

The value of each w_i can be obtained through Eq. (8). Assuming the values of the degree that critical quality characteristics of the current products and similar products meeting the technological requirements were M_{K_i} and $M_{K_i}^*$. And the degree of similarity between the products and similar products can be defined as:

$$\xi_K = \frac{\sum_{i=1}^{n} \omega_i \cdot M_{K_i}}{\sum_{i=1}^{n} \omega_i \cdot M_{K_i}^*}. \tag{9}$$

Assuming the life data of similar product, which was feed backed from user, was $X_0 = (x_1, x_2, \ldots, x_n)$, and the life data of current product, which through the source of the critical quality characteristics can be expressed as $X_K = (\gamma_K x_1, \gamma_K x_2, \ldots, \gamma_K x_n)$. Through the data X_K, the prior distribution $\pi_K(\theta)$ of θ based on critical quality characteristics can be determined.

4.2. The prior distribution based on critical process factors

Assuming the critical process factors which have a significant impact on product reliability were Y_j (j =1, 2,..., m), and the weights that influencing the product reliability were λ_j (j =1, 2,..., m) and meet $\sum_{i=1}^{m} \lambda_i = 1$. In order to get the value of each λ_j, firstly supposed the weights of the critical process factors (Y_j) influencing on the critical quality characteristics (K_i) were μ_{ij} (i =1, 2,...,n; j =1, 2,..., m), and $\sum_{i=1}^{m} \mu_{ij} = 1, j$ =1, 2,..., n. According to the experience of experts, the range of the values of each μ_{ij} can be given, and using Eq. (8) each values of μ_{ij} can be obtained. And according to Eq. (10), the values of each can be calculated:

$$\lambda_j = \sum_{i=1}^{n} w_i \mu_{ij}. \tag{10}$$

Assuming the values of the degree that critical process factors of the current products and similar products meeting the technological requirements were M_{Y_j} and $M_{Y_j}^*$, and the degree of similarity ξ_Y between the products and similar products can be obtained. Assuming another life data of similar product, which was feedbacked from user, was $X_0^* = (x_1^*, x_2^*, \ldots, x_n^*)$, and the life data of current product, which through the source of the critical process parameters can be expressed as $X_Y = (\gamma_Y x_1^*, \gamma_Y x_2^*, \ldots, \gamma_Y x_n^*)$. Through the data XY, the prior distribution $\pi_Y(\theta)$ of θ based on critical process parameters can be determined.

5. Application cases

All ferrite phase shifter unit is the core device of phased-array radar. S factory has recently introduced a new process, and supposes to obtain the life of the ferrite phase shifter unit under this kind of technology. But the manufacturing costs of the ferrite phase shifter unit are high, and the development cycle is becoming shorter with the performance and functional requirements of the product becoming more and more. It is difficult to sample life test after the processing is completed. Thus the proposed method was used to assess the life of these ferrite phase shifter units.

Step one: determine the different sources of information. To test small samples of the ferrite phase shifter unit after the processing is complete, and obtain the life data as: (1.3802, 1.5963, 1.8667, 1.5624, 1.5335, 1.5442, 1.4813, 1.1091, 1.3025, 1.1319, 1.3900, 1.7236).

To take advantage of proposed method to evaluate the life of current products, each 100 of life data of two similar products (A and B) were collected.

Through failure mechanism analyzing and FMEA analysis of the ferrite phase shifter unit, the critical quality attributes and critical process factors that affect the life expectancy were obtained and shown in Table 1.

According to the experience of experts, the scales of critical quality characteristics of the current product and product A were obtained and shown in Table 2; and the scales of critical process factors of the current product and product B were obtained and shown in Table 3.

Table 1. Critical quality attributes (CQA) and critical process factors (CPF)

No.	Steps	CPF	CQA
1	External	Grinding wheel size (Y_1), feed (Y_2), roller speed (Y_3)	Linearity (K_1)
2	End-face	Grinding wheel size (Y_4), table flatness (Y_5), table feed rate (Y_6)	Parallelism (K_2)
3	Adhesive	Curing temperature (Y_7), curing time (Y_8), curing pressure (Y_9)	Bonding strength (K_3)
4	Cylindrical	Grinding wheel size (Y_{10}), feed (Y_{11}), roller speed (Y_{12})	Roughness (K_4)
5	Coated	Cavity temperature (Y_{13}), coating (Y_{14}), sputtering (Y_{15})	Thickness (K_5)

Step two: determination of different priors. Firstly the density distribution function of the experimental data was determined as:

$$p(x|\mu,\sigma)=\frac{1}{\sqrt{2\pi}\times 0.985}e^{-\frac{(x-4.135)^2}{1.940}}. \tag{11}$$

In order to obtain prior distributions based on critical quality characteristics and prior distribution based on critical process factors, the weight w_i (i =1, 2, 3, 4) of the critical quality characteristics influencing on the product life and the weight λ_j (j =1, 2, 3, 4) of the critical process factors influencing on the product life were need to be determined.

According to the experience of experts, the range of values of each w_i was given as: $0.18 \leq w_1 \leq 0.5$, $0.25 \leq w_2 \leq 0.55$, $0.16 \leq w_3 \leq 0.4$, $0.15 \leq w_4 \leq 0.25$, $0.12 \leq w_5 \leq 0.2$. And according to Eq. (9), the value of each was obtained as follows: w_1 =0.1939, w_2 = 0.25, w_3 = 0.1900, w_4 = 0.1913, w_5 = 0.1748. In order to get the value of each λ_j, supposed the weights of the critical process factors influencing on the critical quality characteristics were μ_{ij} (i =1, 2,..., 5; j =1, 2,..., 15). Table 2 shows that the critical process factors of each process only have an impact on the critical quality characteristics under the same process, thus the range of weighted values was given.

Table 2. The scales of critical qualities characteristics

	K_1	K_2	K_3	K_4	K_5
M	0.84	0.85	0.83	0.86	0.82
A	0.92	0.94	0.97	0.89	0.93

Table 3. The scales of critical process factors

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}
M	0.85	0.86	0.85	0.82	0.83	0.82	0.84	0.81	0.80	0.76	0.82	0.81	0.84	0.81	0.82
B	0.92	0.89	0.91	0.86	0.86	0.85	0.82	0.85	0.84	0.80	0.87	0.83	0.89	0.84	0.87

According to Eqs. (8) and (10), the value of each can was obtained, and according to Eq. (9), the result can be obtained as $\gamma_K = 0.935$ and $\gamma_Y = 0.893$. Combined Table 1, the current product lifetime data based on the information of the similar products can be obtained. The prior distributions based on critical quality characteristics and the prior distribution based on critical process factors were obtained as:

$$\pi_1(\theta) = \frac{1}{\sqrt{2\pi} \times 1.25} e^{-\frac{(x-3.715)^2}{3.125}}, \quad \pi_2(\theta) = \frac{1}{\sqrt{2\pi} \times 0.96} e^{-\frac{(x-4.876)^2}{1.8432}}. \tag{12}$$

Step three: determination of the fused prior distribution. Supposed that $\theta \in [1, 8]$, and the prior distribution with no information of θ is the uniform distribution on the interval of $[1, 8]$, calculated and obtained $I_0 = 0.914$, $I_K = 0.484$, $I_Y = 0.420$. According to Eq. (7), the fused prior distribution was obtained as:

$$\pi(\theta) = 0.465 \times \pi_1(\theta) + 0.535 \times \pi_2(\theta) = \frac{1}{\sqrt{2\pi} \times 0.776} e^{-\frac{(x-4.336)^2}{1.203}}. \tag{13}$$

Step four: statistical inference. By the calculation of the second step, the variance and the mean value of the sample data were 0.97 and 4.135; by the results obtained in step three, the variance and the mean value of the fusion prior distribution were 0.602 and 4.336. According to the Eq. (3) and the result could be obtained $\hat{\theta} = 4.328$. Therefore the average life of these balls was 432.8 hours.

6. Conclusion

To respond quickly to manufacturing demand and evaluate the reliability of processed products, the Bayes method is usually used to merge multiple source information. Based on the forming process of product reliability is analyzed, the reasonable source is determined. By comparing the similar degree of current processing finished products and similar products, different prior distributions are determined. And then the Bayes method is used for information fusion to evaluate product reliability. Finally, the case study on reliability evaluation of radar phase-shift unit is demonstrated to prove the feasibility and effectiveness of the proposed method.

References

[1] **Savchuk V. P., Martz H. F.** Bayes reliability estimation using multiple sources of prior information: binomial sampling. IEEE Trans on Reliability, Vol. 43, Issue 1, 1994, p. 138-144.

[2] **Philippe Smets** The transferable belief model was builded for expert judgments and reliability problems. Reliability Engineering and System Safety, Vol. 38, 1992, p. 59-66.

[3] **Zhang Y. M., Ji Q.** Active and dynamic information fusion for multisensory system with dynamic Bayesian networks. IEEE Transaction on Systems, Man and Cybernetics-Part B: Cybernetics, Vol. 36, Issue 2, 2006, p. 467-472.

[4] **Park K. S., Kim S. H., Yoom W. C.** Establishing srict dominated between alternatives with special type of incomplete information. European of Journal Operational Research, Vol. 96, Issue 2, 1997, p. 398-406.

[5] **Jardine Anderson P. M., Mann D. S.** Application of the Weibull proportional hazards model to aircraft and marine engine failure data. Quality and Reliability Engineering International, Vol. 3, Issue 2, 1987, p. 77-82.

[6] **Chen Y., Jin J.** Quality-reliability chain modeling for system-reliability analysis of complex manufacturing processes. Reliability, IEEE Transactions on, Vol. 54, Issue 3, 2005, p. 475-488.