Finite amplitude pressure wave propagation in chemically active heterogeneous mediums

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Abstract. This paper presents a mathematical analysis of pressure wave propagation in gas media containing particles and proposed mathematical model of the process. It is assumed that particles consist of the material which could be ignited in the surrounding gas if the temperature rises to necessary level. Due to pressure wave propagating in the media the temperature in the gas phase of the mixture is rising. Depending on the wave amplitude and wave profile three different scenarios are possible. The first scenario: the pressure wave is propagating in the heterogeneous media and due to low amplitude of the wave is dissipating in the mixture containing particles as a result of interphase exchange processes and the temperature of the gas does not reach values necessary for the beginning of the chemical reaction. The second scenario: the pressure wave amplitude propagating in the mixture is sufficient for temperature of the gas surrounding particles to reach level necessary for the beginning of chemical reaction but due to wave profile that temperature level is held for a short period of time and the chemical reaction is not started. The third scenario: the pressure wave propagating in the heterogeneous media containing chemically active mixture has necessary amplitude and profile to create temperature level and hold it for the time necessary for the beginning of chemical reaction. In the last scenario it is possible to have different developments of the pressure wave propagation also. Depending on the concentration of particles in the mixture energy release as a result of heat generation due to chemical reaction can be less than necessary to support pressure wave propagation as the wave energy is spent on the dissipation in the process of propagation. It is possible that generated energy will support steady propagation of the pressure wave in the mixture if the energy dissipated due to interphase exchange processes will be fully recovered by heat generation from chemical reaction. If the heat generation from chemical reaction is prevailing over the energy dissipation the process is accelerating and as result we will have process which will be leading to increase of the pressure wave amplitude depending on the particles concentration. The proposed model estimates the wave evolution under different conditions of interphase interactions and possibility to have chemical reaction. Conservation equations describing the propagation and structure of finite amplitude perturbations in such a medium, with correction for heat transfer, momentum exchange and heat generation from chemical reaction between the phases, have been employed to obtain the wave profile during the pressure wave propagation. The model is capable of describing the evolution of waves at any ratio between characteristic times of the internal processes and the characteristic period of the pressure wave. The solution can be used to determine the profile and energy dissipation or generation during pressure wave propagation through the gas medium with suspended particles.

Keywords: pressure wave, heterogeneous media, wave energy, chemical reaction, mathematical modelling.

Nomenclature

δ	Particle's radius
n	Number of particles in the unit volume
ρ, P, u, ϑ	Variable density, pressure, velocity, and temperature of the gas phase
R	Specific gas constant
θ	Variable temperature of particle
Q, F, q	Heat flux, drag force, and heat generated due to chemical reaction between phases

c_v	Specific heat at constant volume
a	The thermal diffusivity of the particle's material
x, t, r	Variables of space, time and radial distance, respectively
$\dot{m}$	Mass rate of chemical reaction
h	Calorific value of the particle's material
c_o	Concentration of the gas phase
$\overline{r}$	Stoichiometric coefficient
D	Diffusion coefficient
ϑ_s	Combustion temperature at the initial pressure
L	Latent heat
c_p	Heat capacity at the constant pressure
η	Unit function

1. Introduction

The propagation of a pressure wave in a heterogeneous medium consisting of the particles suspended in a gas is commonly found in the environment. This type of medium is referred to as gas suspension. Examples of gas suspensions include dusty environment in different industrial environments or in natural conditions of mines. It is assumed that volume concentration of particles in gas suspension is small so we can neglect particles' interaction. When a pressure disturbance is present in the air, the presence of the suspended particles affects the propagation and energy of the disturbance. It is believed that understanding these interactions between the carrier (gas) phase and suspended phase provides explanations in various physical phenomena, such as air-particles momentum and heat exchange [1, 2].

In this paper, we propose a mathematical model describing the evolution of wave profile in gas suspension at the different conditions. To account for the interphase interactions, heat and momentum exchange between gas and suspended phases are considered as well as possible heat generated due to possible chemical reaction. Mathematical model is presented and discussed to evaluate different scenarios of pressure wave propagating through gas suspension with possibility to have chemical reaction activated due to processes accompanying wave propagation.

2. Mathematical model of wave propagation through gas suspension

We are considering the medium which consists of equal radius spherical particles surrounded by gas with constant number of particles per unit of volume. This medium is assumed to be homogeneous provided sufficient number of particles is suspended in the gas over the distance of a wavelength. The initially we assume that gas suspension is in equilibrium, that means that the particles are considered to have the same density and temperature. Due to low particles' concentration, the volume occupied by these particles is much smaller compared to the volume of gas and exchange processes occur only between gas and the particles. We can consider proposed medium to be treated as a continuum, and we can neglect the interaction between suspended particles due to low probability of the particles colliding with each other. Considering above assumptions we can take system of equations which will describe the gas phase with particles' input which will be presented by momentum and heat exchange terms as well as heat generated by the chemical reaction between phases of gas suspension. This system is similar to [3, 4] with additional heat source due to chemical reaction and it is able to describe pressure wave propagation in the gas suspension:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \tag{1}$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + R \frac{\partial (\rho \vartheta)}{\partial x} = F, \tag{2}$$

$$\rho c_v \frac{\partial \vartheta}{\partial t} + \rho c_v u \frac{\partial \vartheta}{\partial x} + \rho R \vartheta \frac{\partial u}{\partial x} = Q + Fu + q, \tag{3}$$

$$P = \rho R \vartheta, \tag{4}$$

$$\frac{\partial \theta}{\partial t} - a \frac{\partial^2 \theta}{\partial r^2} - \frac{2a}{r} \frac{\partial \theta}{\partial r} = 0. \tag{5}$$

This system consists first four of the continuity Eq. (1), the momentum Eq. (2), the energy Eq. (3), and the Eq. (4) of state for ideal gas, and Eq. (5) is one-dimensional heat exchange equation of a spherical particle [3]. Values for Q and F are taken in the form which accounting any type of interphase interaction from steady state to fully unsteady process including intermediate one. Such as the interphase momentum exchange for the spherical particle could be accounted by the Basset-Boussinesq-Oseen formulation [4] and interphase heat exchange could be accounted by Duhamel's integral [5] presented below for single particle respectively:

$$f = 2\pi n \delta^3 \rho \left[\frac{1}{3} \frac{du}{dt} + \frac{3\nu}{\delta^2} u + \frac{3}{\delta} \sqrt{\frac{\nu}{\pi}} \int_0^t \frac{d}{dy} \frac{dy}{\sqrt{t-y}} \right], \tag{6}$$

$$\theta(r,t) = -\sum_{k=1}^{\infty} \frac{2\delta}{k\pi r} (-1)^{k+1} \sin\left(\frac{k\pi r}{\delta}\right) \frac{\partial}{\partial t} \int_0^t \vartheta(y) \exp\left[-\frac{ak^2\pi^2(t-y)}{\delta^2} \right] dy. \tag{7}$$

The heat generated by chemical reaction could be taken in the form:

$$q = \frac{\dot{m} h c_0}{\bar{r}}. \tag{8}$$

Mass rate of chemical reaction we can take for surface reaction on spherical particle [6] in the form:

$$\dot{m} = 4\pi \delta \rho D \ln \left[1 + \frac{c_p(\vartheta - \vartheta_s) + \frac{c_o h}{\bar{r}}}{L} \right]. \tag{9}$$

Any other combustion mechanism could be considered as well.

Substituting all above values Eq. (6)-(9) into system of equations we can describe process of pressure wave propagation through gas suspension with the possibility to have chemical reaction between particles and surrounding gas.

If we consider propagation of small but finite amplitude pressure wave moving in the positive direction of a semi limited space we can obtain a single equation describing the evolution of the pressure wave [7]. This equation will be applicable for the certain values amplitudes of the wave. In the case of wave amplitude grown to high values it will be necessary to make adjustments. In the stated limitations we can obtain single equation for dimensionless value of velocity, pressure, or temperature of a gas surrounding particles in gas suspension which will describe the wave propagation in the presence of interphase interactions. We will write this equation for dimensionless velocity of the wave propagation through the medium in the next form:

$$\frac{\partial u}{\partial x}-\frac{\kappa^3-\kappa^2+4\kappa-2}{\kappa(\kappa+1)}u\frac{\partial u}{\partial \tau}+\frac{T}{\tau_T}\frac{\rho_p}{\rho_o}\frac{\kappa-1}{\kappa+1}\frac{\partial}{\partial \tau}\int_0^\tau u\left\{\vartheta_3\left[0,e^{-\frac{\pi^2 a}{\delta^2}(\tau-y)}\right]-1\right\}dy$$
$$-\frac{\eta(\tau-\tau_i)}{\kappa+1}\frac{T}{\tau_r}\frac{hc_o}{c_v\bar{r}\vartheta_0}\ln\left[1+\frac{M}{\tau_v}\frac{T}{\kappa+1}u+(1+u+\kappa Ru)\frac{M}{(\kappa+1)\sqrt{\pi}}\frac{T}{\sqrt{\tau_v}}\int_0^\tau u\frac{dy}{\sqrt{\tau-y}}\right.$$
$$\left.+\frac{c_p(u+1-u_s)(\kappa-1)\vartheta_0+\frac{hc_o}{\bar{r}}}{L}\right](u+1)=o. \tag{10}$$

This Eq. (10) is describing propagation of pressure wave and evolution of its profile. It is also considering possibility of chemical reaction in case if the wave propagating through gas phase is rising gas temperature to the necessary level during certain time needed for ignition.

3. Conclusion

This mathematical model is able to describe propagation of the pressure wave with wave profile changes during propagation taking in account interphase exchange processes and development of chemical reaction between phases due to temperature rise in the wave. In case of chemical reaction has not been initiated the wave amplitude will be reducing due to energy dissipation during propagation as a result of heat and momentum exchange. If the chemical reaction is initiated amplitude of the pressure wave will be rising due to the energy released by chemical reaction. The rate of the wave amplitude increase will be defined by the rate of energy released by the chemical reaction. In this case three different scenarios are available: if the energy released by the chemical reaction is not sufficient it will support propagation of pressure wave until reaction will be continued; in the case if the energy from chemical reaction will be sufficient to replace energy spent by the wave on dissipation we may receive wave which will be self-supporting and will become steady during propagation through the media; in the case when energy released by chemical reaction is prevailing over the dissipation of energy amplitude of the wave will be increasing and that wave may grow to the significant value and shock wave can be developed.

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