A comparative study of time-marching schemes for fluid-structure interactions

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(Accepted 8 September 2014)

Abstract. Four types of partitioned time-marching schemes, namely the iterative staggered serial (ISS) scheme, the conventional serial staggered serial (CSS) scheme, the generalized serial staggered scheme (GSS), and the serial staggered scheme with fluid loads predictions (FPSS), are presented for accuracy comparisons of nonlinear fluid-structure interactions (FSI). A 2-DOF aeroelastic model for an airfoil is used as an example to illustrate the effects of different control parameters of the schemes. Some modifications are made to the schemes to improve the FSI simulation accuracy. The numerical results show that the GSS and FPSS are accurate and robust if the default parameters are adopted. Moreover, the accuracy of FPSS can be further improved by simply tuning the control parameters.

Keywords: fluid-structure interaction, partitioned schemes, time-marching, aeroelastic.

1. Introduction

Fluid-structure interaction (FSI) is a typical multi-field problem, where the simulations of fluid field, structural field and mesh movement are involved. The FSI simulation is always a challenging problem due to the huge computational effort and the data exchange accuracy between different domains. The numerical simulations of FSI problems can be generally divided into three groups, namely the monolithic schemes, the strongly-coupled schemes and the loosely-coupled schemes [1-3]. The monolithic methods are more accurate but much less efficient than the coupled methods.

To improve the computational efficiency of FSI problems, some partitioned schemes are devised. In the partitioned schemes, the fluid and structural part are discretized and solved separately, and data exchanges are necessary at the FSI interfaces at each time step. For the strongly-coupled schemes, several iterations may be required to enforce the equilibrium of interaction force and boundary movement at the FSI interface [3-4]. The computational efficiency can be further improved using the loosely-coupled schemes, where the iterations in each time step are removed by properly design the time-marching schemes [2]. The time-marching schemes must be carefully designed since they are crucial to accuracy, robustness and stability of the loosely-coupled methods. In this paper, we summarize and compare the available time-marching schemes, and make some modifications to improve the accuracy. The purpose of this research is to find proper highly-efficient time-marching schemes for a newly developed FSI simulation platform for rotarycraft, which combines the CFD solver based on gradient smoothing method [5-8] and traditional finite element structural solver.

There are three more sections in this paper. In Section 2, the algorithms of strongly- and loosely-coupled time-marching schemes are introduced and discussed. In Section 3, the 2-DOF nonlinear aeroelastic model is used as the numerical example to show the properties of different time-marching schemes, and the effects of tunable parameters are also discussed. Some conclusions are drawn in the last section.

2. Time-marching schemes for FSI problems

In this section, four kinds of partitioned schemes are introduced. These algorithms are designed for time-marching schemes of general FSI problems, and the CFD and structural solver can be any available ones.

2.1. Iterative serial staggered (ISS) schemes

In the ISS schemes, the equilibrium of force and velocity (and/or displacement) on the fluid-structure interface is enforced by some iterations in each time step. The basic steps for modified iterative serial staggered (ISS) scheme equipped with load and displacement relaxations can be written as following:

(1) Start from the solutions of previous step, $\mathbf{x}^n$, $\mathbf{v}^n$ and $\mathbf{a}^n$ are vectors of the structural solutions for displacement, velocity and acceleration, and $\mathbf{F}_f^n$ is the unsteady fluid loads.

(2) Iteration steps from t^n to t^{n+1}. Set initial condition for iteration as:

$$\mathbf{X}^{(0)} = \mathbf{X}^n, \quad \mathbf{X} = [\mathbf{x}^T \quad \mathbf{v}^T \quad \mathbf{a}^T]^T, \ \mathbf{F}_f^{(0)} = \mathbf{F}_f^n. \tag{1}$$

(3) If not converged, repeat (4)-(7):

(4) Update iterative structural results:

$$\mathbf{X}^{(k+1)} = \mathbb{S}\left(\mathbf{K}, \mathbf{C}, \mathbf{M}, \mathbf{X}^n, \mathbf{F}_f^{(k)}\right). \tag{2}$$

(5) Structural results relaxation:

$$\mathbf{X}^{(k+1)} = (1-\alpha)\mathbf{X}^{(k+1)} + \alpha\mathbf{X}^k. \tag{3}$$

(6) Update fluid loads:

$$\mathbf{F}_f^{(k+1)} = \mathbb{F}\left(\mathbf{X}^{(k+1)}\right). \tag{4}$$

(7) Fluid loads relaxation:

$$\mathbf{F}_f^{(k+1)} = (1-\beta)\mathbf{F}_f^{(k+1)} + \beta\mathbf{F}_f^k. \tag{5}$$

(8) Prepare data for next step:

$$\mathbf{X}^n = \mathbf{X}^{(k+1)}, \ \mathbf{F}_f^n = \mathbf{F}_f^{(k+1)}. \tag{6}$$

The ISS schemes are accurate and can be slightly tuned to improve the computational efficiency. However, the iterative execution of both fluid and structural solvers can be extremely time-consuming, especially for large scale engineering problems. Thus, the loosely-coupled schemes, where no iteration is involved, are designed to reduce the computational difficulties of FSI problems.

2.2. Conventional serial staggered (CSS) schemes

The CSS scheme is the simplest and most straightforward loosely-coupled method for FSI computation [2, 9]. In this paper, the CSS schemes are also slightly modified to add the relaxation steps for fluids and structural results. The relaxation patterns will be introduced in GSS and FPSS schemes.

2.3. Generalized serial staggered (GSS) schemes

It is stated in [2, 10] that the temporal accuracy of the staggered partitioned schemes can be greatly improved by adding a prediction step of the structural part. This kind of schemes is named as the GSS schemes. The structural prediction is realized by extrapolation based on the kinetic equations using the high order derivatives of displacement. By adding the structural predictor, GSS schemes provide a new configuration to calculate the fluid loads. The basic steps of GSS schemes can be summarized as following:

(1) Start from the structural solutions of previous one or two steps, $\mathbf{X}^{n-1}$ and $\mathbf{X}^{n}$, and the fluid loads from the previous step $\mathbf{F}_f^n$.

(2) Structural predictor for displacements:

$$\mathbf{x}_p^{n+1} = \mathbf{x}^n + \alpha_1 \Delta t \mathbf{v}^n + \alpha_2 \Delta t (\mathbf{v}^n - \mathbf{v}^{n-1}). \tag{7}$$

(3) Update the fluid loads using the predicted configuration:

$$\mathbf{F}_f^{n+1} = \mathbb{F}\left(x_p^{n+1}\right). \tag{8}$$

(4) Relaxation of fluid loads:

$$\mathbf{F}_f = (1-\beta)\mathbf{F}_f^{n+1} + \beta \mathbf{F}_f^n. \tag{9}$$

(5) Correct structural solutions using the structural solver:

$$\mathbf{X}^{n+1} = \mathbb{S}\left(\mathbf{K}, \mathbf{C}, \mathbf{M}, \mathbf{X}^n, \mathbf{F}_f^{n+1}\right). \tag{10}$$

(6) Update data for next step:

$$\mathbf{X}^{n-1} = \mathbf{X}^n, \quad \mathbf{X}^n = \mathbf{X}^{n+1}, \quad \mathbf{F}_f^n = \mathbf{F}_f^{n+1}. \tag{11}$$

There are two types of structural predictors: the first one predicts the structural configuration at t^{n+1} (denoted as GSS here) and the second one at $t^{n+1/2}$ (denoted as GSS0.5 here). For each type, the first- or second-order accuracy can be achieved by choosing α_1 and α_2 [2, 10].

2.4. Serial staggered scheme with fluid force prediction (FPSS)

In the previous subsection, the structural predictors are used to improve stability and accuracy of the time-marching schemes for FSI problems. More recently, Dettmer and Peric [11] proposed a new staggered scheme for fluid-structure interaction based on fluid load predictor. In this paper, we modify this scheme by adding structural relaxation as in the CSS and GSS schemes to improve the flexibility and accuracy of FPSS. The algorithm of the serial staggered scheme with fluid force prediction can be summarized as following:

(1) Start from the fluid solutions of two previous steps, $\mathbf{F}_f^{n-1}$ and $\mathbf{F}_f^n$, and the structural solution $\mathbf{X}^n$ from the previous step.

(2) Fluid loads predictor based on linear extrapolation:

$$\mathbf{F}_f^p = \mathbf{F}_f^n + \left(\mathbf{F}_f^n - \mathbf{F}_f^{n-1}\right). \tag{12}$$

(3) Update the structural solution using the predicted loads:

$$\mathbf{X}^{n+1} = \mathbb{S}\left(\mathbf{K}, \mathbf{C}, \mathbf{M}, \mathbf{X}^n, \mathbf{F}_f^p\right). \tag{13}$$

(4) Relaxation of structural results:

$$\mathbf{X}^{n+1} = (1-\alpha)\mathbf{X}^{n+1} + \alpha\mathbf{X}^{n}. \tag{14}$$

(5) Fluid force correction using the fluid solver:

$$\mathbf{F}_f^{n+1} = \mathbb{F}(\mathbf{X}^{n+1}). \tag{15}$$

(6) Update data for next step:

$$\mathbf{F}_\mathrm{f}^{n-1} = \mathbf{F}_f^{n},\ \ \mathbf{F}_f^{n} = \mathbf{F}_f^{n+1},\ \ \mathbf{X}^{n} = \mathbf{X}^{n+1}. \tag{16}$$

3. Numerical examples

3.1. 2-DOF aerodynamic model

In this paper, a 2-DOF aeroelastic problem shown as Fig. 1 is used to compare the presented time-marching schemes. All the definitions of structural parameters and flow conditions can be found in [12]. The equations of motion for this aerodynamic model can be written as:

$$\begin{bmatrix} m_T & m_w x_\alpha b \\ m_w x_\alpha b & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix}, \tag{17}$$

where h denotes the plunging DOF and α the pitching DOF. The nonlinear torsional stiffness is considered. In this example, the Theodorsen's model is used to calculate the unsteady aerodynamic loads due to the plunging and pitching movements of the airfoil, and the conventional Newmark method is used as the structural solver. Here, we only focus on the coupling process, i.e., the accuracy comparisons of the time-marching schemes and the influences of the tunable parameters. The amplitude and phase accuracy are evaluated as:

$$\varepsilon_A = \frac{A_N - A_R}{A_R}, \quad \varepsilon_\Phi = \frac{t_N - t_R}{T_R}, \tag{18}$$

where A_N, A_R, t_N and t_R denote the amplitude and peak time of the numerical and reference solutions of the limit cycle oscillations (LCO), and T_R denotes the period of the reference solutions of LCO.

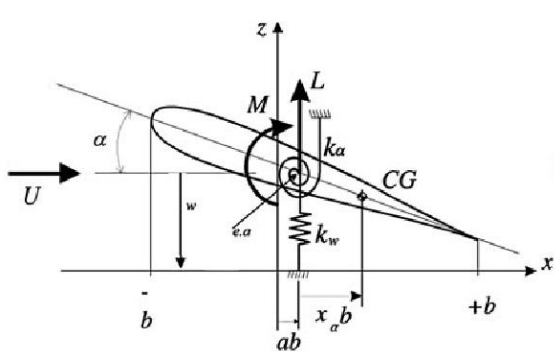

Fig. 1. 2-DOF aeroelastic model [12]

3.2. Influence of relaxation parameters

As introduced previously, the relaxation parameters can be tuned for the structural solutions or the fluid loads. These parameters should be properly tuned to obtain the correct and accurate results. The numerical results of the 2-DOF aeroelastic system obtained using various schemes with different relaxation parameters are summarized in Table 1.

Table 1. Relative errors with different relaxation parameters

Schemes	Parameters			Δt = 5.0×10^{-3} s			Δt = 1.0×10^{-2} s		
	α_1	α_2	β	ε_h (%)	ε_α (%)	ε_Φ (%)	ε_h (%)	ε_α (%)	ε_Φ (%)
ISS	0		0	–0.2227	–0.1048	5.456	–0.9870	–0.8223	24.22
ISS	0		0.5	0.1418	0.0830	8.342			
ISS	0.5		0.5	–0.1767	–0.0765	8.343			
CSS	0		0	63.79	34.99	–40.73			
CSS	0		–1	2.013	2.162	6.900	9.414	10.39	35.77
CSS	0.013		–1	0.1159	0.8527	N/A			
GSS	1.0	0.5	0	–0.4281	–0.2785	5.456	–2.177	–2.047	12.67
GSS	1.0	0	0	0.3859	–0.1253	5.456	2.005	–0.1105	24.22
GSS	1.0	0.5	0.5	34.44	19.37	–42.81			
GSS0.5	0.5	0.125	0	34.81	19.37	–42.81			
GSS0.5	0.5	0.125	–0.5	0.5145	0.6045	5.465	2.175	2.264	21.33
GSS0.5	0.5	0	–0.5	0.6984	0.6294	5.546	3.282	3.118	24.22
FPSS	0		0	2.004	2.149	6.900	9.421	10.38	36.81
FPSS	–0.02		0	0.06704	1.242	–0.288			
FPSS	–0.05		0				2.181	4.862	1.126
FPSS	–0.065		0				0.054	3.427	–10.42

According to the numerical results, we can have the following comments:

(1) For ISS schemes, the LCO results are not sensitive to the relaxation parameters, but the number of iterations in each step can be greatly influenced by changing these two parameters.

(2) For CSS schemes, load relaxation is necessary to obtain the correct response results of the aeroelastic system. We have $\mathbf{F}_f = \mathbf{F}_f^{n+1} + \left(\mathbf{F}_f^{n+1} - \mathbf{F}_f^{n}\right)$ when $\beta = –1$, which means a load prediction based on linear extrapolation is applied to approximate the loads at the end of the current step. When the structural relaxation parameters are slightly changed, better amplitude accuracy can be achieved, but the frequency is not correct, which is absolutely unacceptable for dynamic analysis.

(3) For GSS schemes, no load relaxation is needed, i.e., $\beta = 0$ can produce the correct results. However, for the GSS0.5 schemes, $\beta = -0.5$ is needed to achieve the correct response results. Compared the CSS, GSS and GSS0.5 schemes, we can see that when the structural loads are extrapolated (CSS) or calculated using a predicted structural configuration (GSS and GSS0.5) at the end of the current time step, correct response results can be obtained, but with different accuracy. Moreover, the first- and second-order structural predictors achieve similar response accuracy for both GSS and GSS0.5 schemes.

(4) For FPSS schemes, the applied loads are also extrapolated at the end of the current time step same as the CSS scheme with $\beta = –1$, and thus the two schemes have similar accuracy when the default parameters are adopted. However, different from the CSS scheme, the structural relaxation provides a new configuration to correct the fluid loads for next step. Therefore, the accuracy of FPSS can be improved by slightly tuning the structural relaxation parameter.

(5) When the time step becomes two times larger, the errors of all the schemes also become several times larger. However, the FPSS can also be tuned to achieve better accuracy for amplitudes or phase.

4. Conclusions and discussions

In this paper, four types of partitioned time-marching schemes for general fluid-structure interaction problems are presented. Although the iterative serial staggered (ISS) has the best accuracy and robustness of all the four present kinds of schemes, it is not our first choice for practical engineering due to the huge computational burden. The conventional serial staggered (CSS) schemes, the general serial staggered schemes without and with half time offset (GSS and GSS0.5) and the serial staggered schemes with fluid loads prediction (FPSS) are loosely-coupled methods, where no iterations are involved. Therefore, these three kinds are preferred for large scale problems. According to the numerical results of the 2-DOF aeroelastic model, we can see that the GSS and GSS0.5 are accurate and quite robust. However, the FPSS schemes can be tuned via changing the structural relaxation parameter to achieve better accuracy, especially the phase accuracy.

It should be noted that the phase errors are significant for all the schemes, even for the iterative schemes. This could possibly be alleviated by improving the accuracy of the structural solver.

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