

1168. Identification of a certain mechanism's recoiling mass by limited memory least square method

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Abstract. This paper is concerned with the identification problem of a certain mechanism's dynamic parameters in its operation. This mechanism can be simplified as three rigid body structure joined by a prismatic pair, a revolute pair and a fixed constraint. The unrestraint dynamic equations of the mechanism can be obtained by using multi-rigid-body theory. In order to acquire unknown dynamic parameters, such as displacement, velocity and acceleration, a field experiment was designed. Then by choosing limited memory least square method and using the experiment results, one mass of rigid body could be identified. Finally, the calculative mass was compared to the "real" mass which was consulted in the specification book of this mechanism. The whole process shows that the rigid dynamic model of this mechanism and the method of identification are both effective.

Keywords: multi-body dynamics, mass, identification, limited memory least square method.

1. Introduction

The structure of machinery is becoming more and more complex. As a result that the mass matrix and the stiffness matrix of the whole system is also complex presenting time-varying. So, it's difficult for researchers to describe the dynamic model of some machinery accurately.

At present, research field is mainly divided into two direction, forward problem and inverse problem. Based on mathematical model, forward problem is concerned with the structure's dynamic response under different excitation. Q. Chen analyzed a certain mechanism dynamic response by using the wavelet transform method [1]. By means of multi-body transfer matrix method, X. Rui obtained the model function's accurate expression [2]. Inverse problem, namely, dynamic parameter identification problem, uses some model class to find an equivalent model upon input and output. X. Liu, D. Jing and W. Zhuang provide many identifying methods of time-varying parameters in their papers [3-5]. Because of least square method's simple principle and fast convergence, it is widely used in identifying system parameters. As one of least square method's improved algorithms, limited memory least square method can effectively overcome "data saturation" phenomenon when it is applied to time-varying identification. X. Chen and C. Chen proved its effect in their work [6]. But, most of inverse problem dynamic equations lack physical significance and it's hard to examine the result of identification intuitively.

In this paper, taking a certain mechanism as the research object, we can identify the time-varying mass effectively when combining mechanism dynamic model with limited memory least square method. Because the mass has physical significance, the result of identification can be compared to the static measurement in order to examine the method's effect on time-varying parameters.

2. Problem formulation

2.1. Dynamic model

It is supposed that the mechanical system is moving in the vertical plane of symmetry during being operated and consists of three rigid parts [7], namely, recoiling part B_3 , tipping part B_2 and

carriage part B_1 . Recoiling part moves linear relative to tipping part expressed as x while tipping part rotates relative to carriage part expressed as $\theta + \beta$. The whole mechanism's motion is expressed as the horizontal motion ξ , the vertical movement η and the rotation α . The external force is made up of the gravity acting on each rigid-body, the breech force P_{KH} working on B_3 and the constraining force coming from C and D points on the ground. The internal force consists of the R_f between B_3 and B_2 , the M_C between B_2 and B_1 . So, the dynamic model can be simplified into three-rigid-body structure and is shown in Fig. 1.

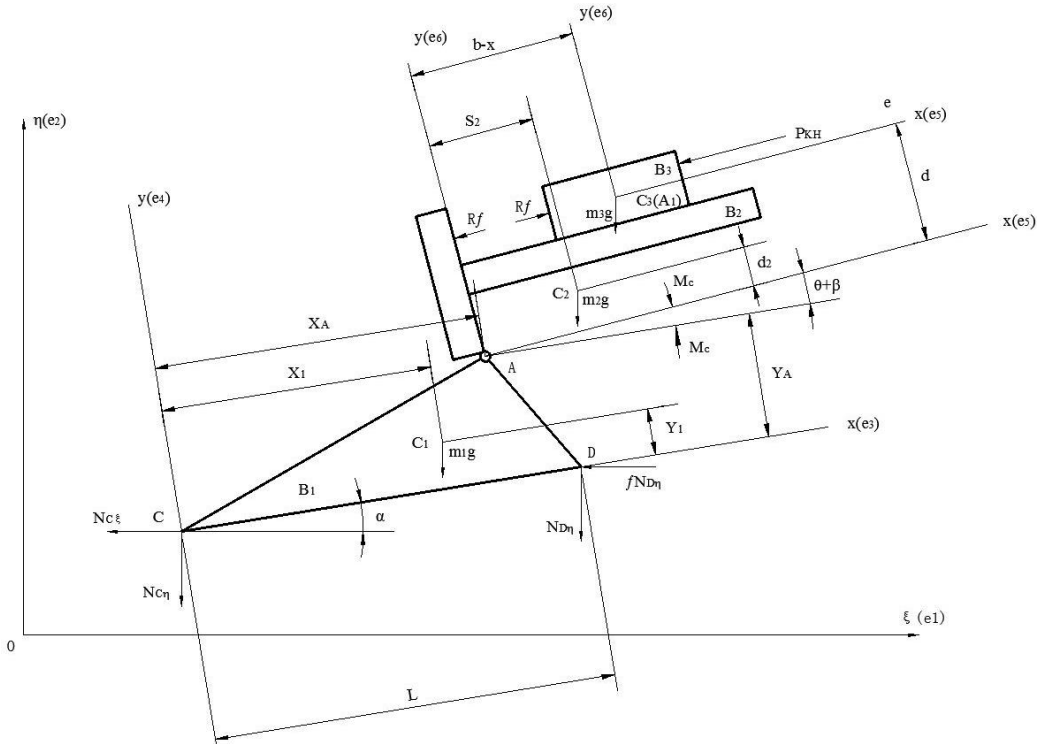


Fig. 1. The dynamic model of the mechanism

In Fig. 1, $o\xi\eta$ is chosen as the inertial reference frame; the points C_1, C_2, C_3 are center of mass on the rigid body B_1, B_2, B_3 ; the points C, A, A_1 are the base points; the coordinate system CXY, AXY, A_1XY is built on these base points; the unit base vector is expressed as $e_1, e_2, e_3, e_4, e_5, e_6$ and e_7 , where e_7 is established by right hand rule; m_1, m_2, m_3 represents each rigid body's mass; m_1, m_2, m_3 is the rotational inertia relative to C, A, A_1 ; the force $N_{C\xi}$ and $N_{C\eta}$ is the constraining force coming from the ground [8-10].

2.2. Dynamic equations

The absolutely velocity and acceleration of each rigid body is illustrated in Eq. (1):

$$\left\{ \begin{array}{l} v_{p1} = \dot{\xi}e_1 + \dot{\eta}e_2, \\ a_{p1} = \ddot{\xi}e_1 + \ddot{\eta}e_2, \\ v_{p2} = \dot{\xi}e_1 + \dot{\eta}e_2 - Y_A\dot{\alpha}e_3 + X_A\dot{\alpha}e_4, \\ a_{p2} = \ddot{\xi}e_1 + \ddot{\eta}e_2 - (Y_A\ddot{\alpha} + X_A\dot{\alpha}^2)e_3 + (X_A\ddot{\alpha} - Y_A\dot{\alpha}^2)e_4, \\ v_{p3} = \dot{\xi}e_1 + \dot{\eta}e_2 - Y_A\dot{\alpha}e_3 + X_A\dot{\alpha}e_4 - d(\dot{\alpha} + \dot{\beta})e_5 - \dot{x}e_5 + (b - x)(\dot{\alpha} + \dot{\beta})e_6, \\ a_{p3} = \left\{ \begin{array}{l} \ddot{\xi}e_1 + \ddot{\eta}e_2 - (Y_A\ddot{\alpha} + X_A\dot{\alpha}^2)e_3 + (X_A\ddot{\alpha} - Y_A\dot{\alpha}^2)e_4 \\ - [\dot{x} + d(\ddot{\alpha} + \ddot{\beta}) + (b - x)(\dot{\alpha} + \dot{\beta})^2]e_5 \\ + [(b - x)(\ddot{\alpha} + \ddot{\beta}) - d(\dot{\alpha} + \dot{\beta})^2 - 2\dot{x}(\dot{\alpha} + \dot{\beta})]e_6 \end{array} \right\}, \\ v_{c1} = \dot{\xi}e_1 + \dot{\eta}e_2 - Y_1\dot{\alpha}e_3 + X_1\dot{\alpha}e_4, \\ a_{c1} = \ddot{\xi}e_1 + \ddot{\eta}e_2 - (Y_1\ddot{\alpha} + X_1\dot{\alpha}^2)e_3 + (X_1\ddot{\alpha} - Y_1\dot{\alpha}^2)e_4, \\ v_{c2} = \dot{\xi}e_1 + \dot{\eta}e_2 - Y_A\dot{\alpha}e_3 + X_A\dot{\alpha}e_4 - d_2(\dot{\alpha} + \dot{\beta})e_5 + s_2(\dot{\alpha} + \dot{\beta})e_6, \\ a_{c2} = \left\{ \begin{array}{l} \ddot{\xi}e_1 + \ddot{\eta}e_2 - (Y_A\ddot{\alpha} + X_A\dot{\alpha}^2)e_3 + (X_A\ddot{\alpha} - Y_A\dot{\alpha}^2)e_4 \\ - [d_2(\ddot{\alpha} + \ddot{\beta}) + s_2(\dot{\alpha} + \dot{\beta})^2]e_5 + [s_2(\ddot{\alpha} + \ddot{\beta}) - d_2(\dot{\alpha} + \dot{\beta})^2]e_6 \end{array} \right\}, \\ v_{c3} = v_{p3}, \\ a_{c3} = a_{p3}. \end{array} \right. \quad (1)$$

According to Eq. 1, the base vector is expressed in Eq. (2):

$$e_p^v = \begin{bmatrix} e_1e_2 & 0 & 0 & 0 \\ e_1e_2 & X_Ae_4 - Y_Ae_3 & 0 & 0 \\ e_1e_2 & -Y_Ae_3 + X_Ae_4 - de_5 + (b - x)e_6 & -de_5 + (b - x)e_6 & -e_5 \end{bmatrix}. \quad (2)$$

The absolutely angular velocity and angular acceleration of each rigid body is obtained in Eq. (3):

$$\left\{ \begin{array}{l} \omega_1 = \dot{\alpha}e_7, \\ \dot{\omega}_1 = \ddot{\alpha}e_7, \\ \omega_2 = (\dot{\alpha} + \dot{\beta})e_7, \\ \dot{\omega}_2 = (\ddot{\alpha} + \ddot{\beta})e_7, \\ \omega_3 = (\dot{\alpha} + \dot{\beta})e_7, \\ \dot{\omega}_3 = (\ddot{\alpha} + \ddot{\beta})e_7. \end{array} \right. \quad (3)$$

According to Eq. (3), the angular base vector is expressed in Eq. (4):

$$e^{\omega} = \begin{bmatrix} 0 & 0 & e_7 & 0 & 0 \\ 0 & 0 & e_7 & e_7 & 0 \\ 0 & 0 & e_7 & e_7 & 0 \end{bmatrix}. \quad (4)$$

The principal vector and principal moment of the external force is shown in Eq. (5) and Eq. (7):

$$R' = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}, \quad (5)$$

where:

$$\begin{cases} R_1 = \left\{ \begin{aligned} &-(N_{C\xi}\cos\phi_1 + N_{C\eta}\sin\phi_1 + fN_{D\eta}\cos\phi_1 + N_{D\eta}\sin\phi_1 + m_1g\sin\phi_1)e_3 \\ &+(N_{C\xi}\sin\phi_1 - N_{C\eta}\cos\phi_1 + fN_{D\eta}\sin\phi_1 - N_{D\eta}\cos\phi_1 - m_1g\cos\phi_1)e_4 \end{aligned} \right\}, \\ R_2 = -R_f e_5 - m_2g\sin\phi_2 e_5 - m_2g\cos\phi_2 e_6, \\ R_3 = -(P_{KH} - R_f)e_5 - m_3g\sin\phi_2 e_5 - m_3g\cos\phi_2 e_6. \end{cases} \quad (6)$$

$$L'_p = \begin{bmatrix} L_{p1} \\ L_{p2} \\ L_{p3} \end{bmatrix}, \quad (7)$$

where:

$$\begin{cases} L_{p1} = (M_C - N_{D\eta}L_C\cos\phi_1 + fN_{D\eta}L_C\sin\phi_1 - m_1gX_1\cos\phi_1 + m_1gY_1\sin\phi_1)e_7, \\ L_{p2} = (-M_C + R_f d_1 - m_2gs_2\cos\phi_2 + m_2gs_2\sin\phi_2)e_7, \\ L_{p3} = (P_{KH}e)e_7. \end{cases} \quad (8)$$

The principal vector and principal moment of the inertia force is shown in Eq. (9) and Eq. (11):

$$R^* = -Ma_C = - \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} a_{C1} \\ a_{C2} \\ a_{C3} \end{bmatrix} = \begin{bmatrix} -m_1 a_{C1} \\ -m_2 a_{C2} \\ -m_3 a_{C3} \end{bmatrix} = \begin{bmatrix} R_1^* \\ R_2^* \\ R_3^* \end{bmatrix}, \quad (9)$$

where:

$$\begin{cases} R_1^* = -m_1 \left\{ \begin{aligned} &\ddot{\xi}e_1 + \ddot{\eta}e_2 - (Y_1\ddot{\alpha} + X_1\dot{\alpha}^2)e_3 + (X_1\ddot{\alpha} - Y_1\dot{\alpha}^2)e_4, \\ &\ddot{\xi}e_1 + \ddot{\eta}e_2 - (Y_A\ddot{\alpha} + X_A\dot{\alpha}^2)e_3 + (X_A\ddot{\alpha} - Y_A\dot{\alpha}^2)e_4 \end{aligned} \right\}, \\ R_2^* = -m_2 \left\{ \begin{aligned} &-[d_2(\ddot{\alpha} + \ddot{\beta}) + s_2(\dot{\alpha} + \dot{\beta})^2]e_5 + [s_2(\ddot{\alpha} + \ddot{\beta}) - d_2(\dot{\alpha} + \dot{\beta})^2]e_6 \end{aligned} \right\}, \\ R_3^* = -m_3 \left\{ \begin{aligned} &\ddot{\xi}e_1 + \ddot{\eta}e_2 - (Y_A\ddot{\alpha} + X_A\dot{\alpha}^2)e_3 + (X_A\ddot{\alpha} - Y_A\dot{\alpha}^2)e_4 \\ &-[\ddot{x} + d(\ddot{\alpha} + \ddot{\beta}) + (b-x)(\dot{\alpha} + \dot{\beta})^2]e_5 \\ &+ [(b-x)(\dot{\alpha} + \dot{\beta}) - d(\dot{\alpha} + \dot{\beta})^2 - 2\dot{x}(\dot{\alpha} + \dot{\beta})]e_6 \end{aligned} \right\}. \end{cases} \quad (10)$$

$$\begin{aligned} L_p^* &= -\rho_C \times Ma_p - \dot{G}_p = - \begin{bmatrix} \rho_{C1} & 0 & 0 \\ 0 & \rho_{C2} & 0 \\ 0 & 0 & \rho_{C3} \end{bmatrix} \times \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} a_{p1} \\ a_{p2} \\ a_{p3} \end{bmatrix} - \begin{bmatrix} \dot{G}_{p1} \\ \dot{G}_{p2} \\ \dot{G}_{p3} \end{bmatrix} \\ &= \begin{bmatrix} \rho_{C1} \times (-m_1 a_{p1}) - \dot{G}_{p1} \\ \rho_{C2} \times (-m_2 a_{p2}) - \dot{G}_{p2} \\ \rho_{C3} \times (-m_3 a_{p3}) - \dot{G}_{p3} \end{bmatrix} = - \begin{bmatrix} -\rho_{C1} \times (-m_1 a_{p1}) + \dot{G}_{p1} \\ -\rho_{C2} \times (-m_2 a_{p2}) + \dot{G}_{p2} \\ -\rho_{C3} \times (-m_3 a_{p3}) + \dot{G}_{p3} \end{bmatrix} = \begin{bmatrix} L_{p1}^* \\ L_{p2}^* \\ L_{p3}^* \end{bmatrix}, \end{aligned} \quad (11)$$

where:

$$\begin{cases} L_{p1}^* = \{m_1(X_1\sin\phi_1 + Y_1\cos\phi_1)\ddot{\xi} - m_1(X_1\cos\phi_1 - Y_1\sin\phi_1)\ddot{\eta} - J_1\ddot{\alpha}\}e_7, \\ L_{p2}^* = \left\{ \begin{aligned} &m_2(s_2\sin\phi_2 + d_2\cos\phi_2)\ddot{\xi} - m_2(s_2\cos\phi_2 - d_2\sin\phi_2)\ddot{\eta} \\ &-[J_2 + m_2(s_2X_A + d_2Y_A)\cos(\phi_2 - \phi_1) + m_2(s_2Y_A - d_2X_A)\sin(\phi_2 - \phi_1)]\ddot{\alpha} \\ &-J_2\ddot{\beta} - m_2[(s_2X_A + d_2Y_A)\sin(\phi_2 - \phi_1) - (s_2Y_A - d_2X_A)\cos(\phi_2 - \phi_1)]\dot{\alpha}^2 \end{aligned} \right\}e_7, \\ L_{p3}^* = -\{J_{3C}\ddot{\alpha} + J_{3C}\ddot{\beta}\}e_7. \end{cases} \quad (12)$$

The generalized force and the generalized moment of force in the whole dynamic model can

be expressed in Eq. (13) and Eq. (14):

$$(e_p^v)^T \cdot R' = \begin{bmatrix} e_1 & e_1 & e_1 \\ e_2 & e_2 & e_2 \\ 0 & X_A e_4 - Y_A e_3 & -Y_A e_3 + X_A e_4 - d e_5 + (b-x)e_6 \\ 0 & 0 & -d e_5 + (b-x)e_6 \\ 0 & 0 & -e_5 \end{bmatrix} \cdot \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \quad (13)$$

$$= \begin{bmatrix} e_1 \cdot (R_1 + R_2 + R_3) \\ e_2 \cdot (R_1 + R_2 + R_3) \\ (X_A e_4 - Y_A e_3) \cdot R_2 + [-Y_A e_3 + X_A e_4 - d e_5 + (b-x)e_6] \cdot R_3 \\ [-d e_5 + (b-x)e_6] \cdot R_3 \\ -e_5 \cdot R_3 \end{bmatrix}$$

$$(e^\omega)^T \cdot L'_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_7 & e_7 & e_7 \\ 0 & e_7 & e_7 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} L_{p1} \\ L_{p2} \\ L_{p3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ e_7 \cdot (L_{p1} + L_{p2} + L_{p3}) \\ e_7 \cdot (L_{p2} + L_{p3}) \\ 0 \end{bmatrix} \quad (14)$$

The generalized inertia force and the generalized inertia moment of force is obtained in a similar way.

Then, by substituting Eq. (1)-(14) into Eq. (15)-(16), the generalized force and the generalized inertia force is established in Eq. (17)-(27):

$$Q = e^T \cdot \begin{bmatrix} R' \\ L'_p \end{bmatrix} = (e_p^v)^T \cdot R' + (e^\omega)^T \cdot L'_p, \quad (15)$$

$$Q^* = e^T \cdot \begin{bmatrix} -M a_c \\ -\rho_c \times M a_c - \dot{G}_p \end{bmatrix} = (e_p^v)^T \cdot (-M a_c) - (e^\omega)^T \cdot (-\rho_c \times M a_c - \dot{G}_p) \\ = (e_p^v)^T \cdot R^* + (e^\omega)^T \cdot L_p^*, \quad (16)$$

$$\begin{cases} Q_1 = -P_{KH} \cos \phi_2 - N_{C\xi} - f N_{D\eta}, \\ Q_2 = -P_{KH} \sin \phi_2 - (m_1 + m_2 + m_3)g - N_{C\xi} - N_{D\eta}, \\ Q_3 = \begin{cases} P_{KH}[e + d - X_A \sin(\phi_2 - \phi_1) + Y_A \cos(\phi_2 - \phi_1)] - m_1 g(X_1 \cos \phi_1 - Y_1 \sin \phi_1) \\ -(m_2 + m_3)g(X_A \cos \phi_1 - Y_A \sin \phi_1) - m_2 g(s_2 \cos \phi_2 - d_2 \sin \phi_2) \\ -m_3 g[(b-x) \cos \phi_2 - d \sin \phi_1] - N_{D\eta} L \cos \phi_1 + f N_{D\eta} L \sin \phi \end{cases}, \\ Q_4 = P_{KH}(e + d) - m_2 g(s_2 \cos \phi_2 - d_2 \sin \phi_2) - m_3 g[(b-x) \cos \phi_2 - d \sin \phi_1] - M_C, \\ Q_5 = P_{KH} - R_f + m_3 g \sin \phi_2, \end{cases} \quad (17)$$

$$Q_i^* = -(m_1 + m_2 + m_3)\ddot{\xi} + (m_1 X_1 + m_2 X_A + m_3 X_A) \sin \phi_1 + (m_1 Y_1 + m_2 Y_A + m_3 Y_A) \cos \phi_1 + [m_2 s_2 + m_3(b-x)] \sin \phi_2 + (m_2 d_2 + m_3 d_2) \cos \phi_2 \ddot{\alpha} \\ + \{[m_2 s_2 + m_3(b-x)] \sin \phi_2 + (m_2 d_2 + m_3 d_2) \cos \phi_2\} \ddot{\beta} + m_3 \cos \phi_2 \ddot{x} + f_1 \\ = a_{22} \ddot{\xi} + a_{13} \ddot{\alpha} + a_{14} \ddot{\beta} + a_{15} \ddot{x} + f_1, \quad (18)$$

where:

$$f_1 = \{(m_1 X_1 + m_2 X_A + m_3 X_A) \cos \phi_1 - (m_1 Y_1 + m_2 Y_A + m_3 Y_A) \sin \phi_1 \\ + [m_2 s_2 + m_3(b-x)] \cos \phi_2 - (m_2 d_2 + m_3 d_2) \sin \phi_2\} \dot{\alpha}^2 \\ + \{[m_2 s_2 + m_3(b-x)] \cos \phi_2 - (m_2 d_2 + m_3 d_2) \sin \phi_2\} (\dot{\beta}^2 + 2\dot{\alpha}\dot{\beta}) \\ - 2m_3 \dot{x} (\dot{\alpha} + \dot{\beta}) \sin \phi_2, \quad (19)$$

$$\begin{aligned}
 Q_2^* &= -(m_1 + m_2 + m_3)\ddot{\eta} \\
 &\quad - \{(m_1X_1 + m_2X_A + m_3X_A)\cos\phi_1 - (m_1Y_1 + m_2Y_A + m_3Y_A)\sin\phi_1 \\
 &\quad + [m_2s_2 + m_3(b-x)]\cos\phi_2 - (m_2d_2 + m_3d)\sin\phi_2\}\ddot{\alpha} \\
 &\quad - \{[m_2s_2 + m_3(b-x)]\cos\phi_2 - (m_2d_2 + m_3d)\sin\phi_2\}\ddot{\beta} + m_3\sin\phi_2\ddot{x} + f_2 \\
 &= a_{22}\ddot{\eta} + a_{23}\ddot{\alpha} + a_{24}\ddot{\beta} + a_{25}\ddot{x} + f,
 \end{aligned} \tag{20}$$

where:

$$\begin{aligned}
 f_2 &= \{(m_1X_1 + m_2X_A + m_3X_A)\sin\phi_1 - (m_1Y_1 + m_2Y_A + m_3Y_A)\cos\phi_1 \\
 &\quad + [m_2s_2 + m_3(b-x)]\sin\phi_2 - (m_2d_2 + m_3d)\cos\phi_2\}\dot{\alpha}^2 \\
 &\quad + \{[m_2s_2 + m_3(b-x)]\sin\phi_2 - (m_2d_2 + m_3d)\cos\phi_2\}(\dot{\beta}^2 + 2\dot{\alpha}\dot{\beta}) \\
 &\quad - 2m_3\dot{x}(\dot{\alpha} + \dot{\beta})\cos\phi_2,
 \end{aligned} \tag{21}$$

$$\begin{aligned}
 Q_3^* &= \{(m_1X_1 + m_2X_A + m_3X_A)\sin\phi_1 + (m_1Y_1 + m_2Y_A + m_3Y_A)\cos\phi_1 \\
 &\quad + [m_2s_2 + m_3(b-x)]\sin\phi_2 + (m_2d_2 + m_3d)\cos\phi_2\}\ddot{\xi} \\
 &\quad - \{(m_1X_1 + m_2X_A + m_3X_A)\cos\phi_1 - (m_1Y_1 + m_2Y_A + m_3Y_A)\sin\phi_1 \\
 &\quad + [m_2s_2 + m_3(b-x)]\cos\phi_2 - (m_2d_2 + m_3d)\sin\phi_2\}\ddot{\eta} \\
 &\quad - \{J_1 + J_2 + J_{3C} + (m_2 + m_3)(X_A^2 + Y_A^2) + m_3[(b-x)^2 + d^2] \\
 &\quad + 2[m_2s_2 + m_3(b-x)][X_A\cos(\phi_2 - \phi_1) + Y_A\sin(\phi_2 - \phi_1)] \\
 &\quad - 2(m_2d_2 + m_3d)[X_A\sin(\phi_2 - \phi_1) - Y_A\cos(\phi_2 - \phi_1)]\}\ddot{\alpha} \\
 &\quad - \{J_2 + J_{3C} + m_3[(b-x)^2 + d^2] + [m_2s_2 + m_3(b-x)] \\
 &\quad \times [X_A\cos(\phi_2 - \phi_1) + Y_A\sin(\phi_2 - \phi_1)] - (m_2d_2 + m_3d) \\
 &\quad \times [X_A\sin(\phi_2 - \phi_1) - Y_A\cos(\phi_2 - \phi_1)]\}\ddot{\beta} \\
 &\quad + \{m_3[X_A\sin(\phi_2 - \phi_1) - Y_A\cos(\phi_2 - \phi_1)] - m_3d\}\ddot{x} + f_3 \\
 &= a_{31}\ddot{\xi} + a_{32}\ddot{\eta} + a_{33}\ddot{\alpha} + a_{34}\ddot{\beta} + a_{35}\ddot{x} + f,
 \end{aligned} \tag{22}$$

where:

$$\begin{aligned}
 f_3 &= \{[m_2s_2 + m_3(b-x)][X_A\sin(\phi_2 - \phi_1) - Y_A\cos(\phi_2 - \phi_1)] \\
 &\quad + (m_2d_2 + m_3d)[X_A\cos(\phi_2 - \phi_1) + Y_A\sin(\phi_2 - \phi_1)]\}(\dot{\beta}^2 + 2\dot{\alpha}\dot{\beta}) \\
 &\quad + 2m_3[X_A\cos(\phi_2 - \phi_1) + Y_A\sin(\phi_2 - \phi_1) + (b-x)]\dot{x}(\dot{\alpha} + \dot{\beta}),
 \end{aligned} \tag{23}$$

$$\begin{aligned}
 Q_4^* &= \{[m_2s_2 + m_3(b-x)]\sin\phi_2 + (m_2d_2 + m_3d)\cos\phi_2\}\ddot{\xi} \\
 &\quad - \{[m_2s_2 + m_3(b-x)]\cos\phi_2 - (m_2d_2 + m_3d)\sin\phi_2\}\ddot{\eta} \\
 &\quad - \{J_2 + J_{3C} + m_3[(b-x)^2 + d^2] + [m_2s_2 + m_3(b-x)] \\
 &\quad \times [X_A\cos(\phi_2 - \phi_1) + Y_A\sin(\phi_2 - \phi_1)] - (m_2d_2 + m_3d) \\
 &\quad \times [X_A\sin(\phi_2 - \phi_1) - Y_A\cos(\phi_2 - \phi_1)]\}\ddot{\alpha} - \{J_2 + J_{3C} + m_3[(b-x)^2 + d^2]\}\ddot{\beta} \\
 &\quad + m_3d\ddot{x} + f_4 = a_{41}\ddot{\xi} + a_{42}\ddot{\eta} + a_{43}\ddot{\alpha} + a_{44}\ddot{\beta} + a_{45}\ddot{x} + f,
 \end{aligned} \tag{24}$$

where:

$$\begin{aligned}
 f_4 &= -\{[m_2s_2 + m_3(b-x)][X_A\sin(\phi_2 - \phi_1) - Y_A\cos(\phi_2 - \phi_1)] \\
 &\quad + (m_2d_2 + m_3d)[X_A\cos(\phi_2 - \phi_1) + Y_A\sin(\phi_2 - \phi_1)]\}\dot{\alpha}^2 + 2m_3(b-x)\dot{x}(\dot{\alpha} + \dot{\beta}),
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 Q_5^* &= m_3\cos\phi_2\ddot{\xi} + m_3\sin\phi_2\ddot{\eta} + m_3[X_A\sin(\phi_2 - \phi_1) - Y_A\cos(\phi_2 - \phi_1)]\ddot{\alpha} - m_3d\ddot{\beta} \\
 &\quad - m_3\ddot{x} + f = a_{51}\ddot{\xi} + a_{52}\ddot{\eta} + a_{53}\ddot{\alpha} + a_{54}\ddot{\beta} + a_{55}\ddot{x} + f_5,
 \end{aligned} \tag{26}$$

where:

$$f_5 = -m_3[X_A\cos(\phi_2 - \phi_1) + Y_A\sin(\phi_2 - \phi_1)]\dot{\alpha}^2 - m_3(b-x)(\dot{\alpha} + \dot{\beta}). \tag{27}$$

Finally, the whole mechanism dynamic equations are established as Eq. (28):

$$A\ddot{q} = F, \tag{28}$$

where:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{15} \\ a_{21} & a_{22} & \cdots & a_{25} \\ \cdots & \cdots & \cdots & \cdots \\ a_{51} & a_{52} & \cdots & a_{55} \end{bmatrix}, \tag{29}$$

$$q = [\xi \quad \eta \quad \alpha \quad \beta \quad x]^T, \tag{30}$$

$$F = [-(Q_1 + f_1) \quad -(Q_2 + f_2) \quad -(Q_3 + f_3) \quad -(Q_4 + f_4) \quad -(Q_5 + f_5)]^T. \tag{31}$$

3. Field experiment

In order to identify the time-varying system parameters, like time-varying mass, a field experiment is designed and conducted.

3.1. Experimental equipment

Experiment Equipment mainly consists of two parts, the data collection system and the high-speed photography system.

The data collection system consists of Brüel&Kjær acceleration sensor, Qiwei angular acceleration sensor, Lianneng pressure sensor, Kistler charge amplifier, Dewetron data collector, Dewetron software and so on.

The high-speed photography system consists of an IDT Y3-S2 high-speed camera, a high-light LED light, a nikkor 50 mm/1.2D camera lens, a nikkor 400 mm/2.8D camera lens, ProAnalyst dynamic target capture software and some other ancillary equipment.

3.2. Experiment results

The angular velocity of the recoiling part is shown in Fig. 2 and the active force of the mechanism system is shown in Fig. 3.

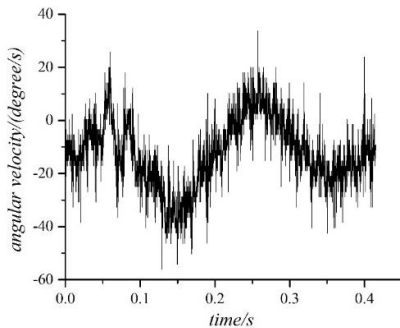


Fig. 2. The angular velocity of the recoiling part

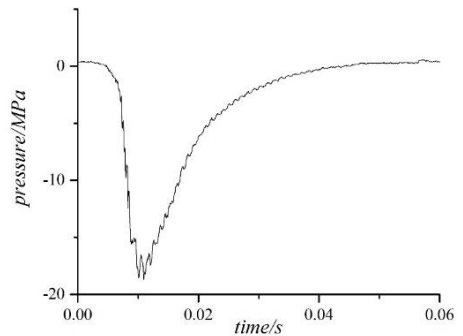


Fig. 3. The active force of the mechanism system

4. Numerical calculation

4.1. Identification method

The model can be expressed in Eq. (32) [11-13]:

$$A(z^{-1})y(k) = B(z^{-1})y(k - d) + \xi(k), \tag{32}$$

where $\xi(k)$ is white noise, and the structure parameters m, n, d are known. And:

$$\begin{cases} A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + \dots + a_mz^{-m}, \\ B(z^{-1}) = b_0 + a_1z^{-1} + a_2z^{-2} + \dots + a_nz^{-n}. \end{cases} \quad (33)$$

The target of identification is to establish $m + n + 1$ parameters according to the measurable input and output. The model can be described in least square method and is expressed in Eq. (34):

$$\begin{aligned} y(k) = & -a_1y(k-1) - \dots - a_{n_a}y(k-n_a) + b_0u(k-d) \\ & + \dots + b_{n_b}u(k-d-n_b) + \xi(k) = \varphi^T(k)\theta + \xi(k), \end{aligned} \quad (34)$$

where $\varphi(k)$ is the data vector, θ is the parameter need to be identified. And:

$$\begin{cases} \varphi(k) = [-y(k-1), \dots, -y(k-n_a), u(k-d), \dots, u(k-d-n_b)]^T \in R^{(n_a+n_b+1) \times 1}, \\ \theta = [a_1, \dots, a_{n_a}, b_0, \dots, b_{n_b}]^T \in R^{(n_a+n_b+1) \times 1}. \end{cases} \quad (35)$$

So, the identification equation of the rigid body mechanism model can be expressed in Eq. (36):

$$m_3\ddot{q}' = F'. \quad (36)$$

4.2. Identification results

The time-varying identified mass of the recoiling part is illustrated in Fig. 4.

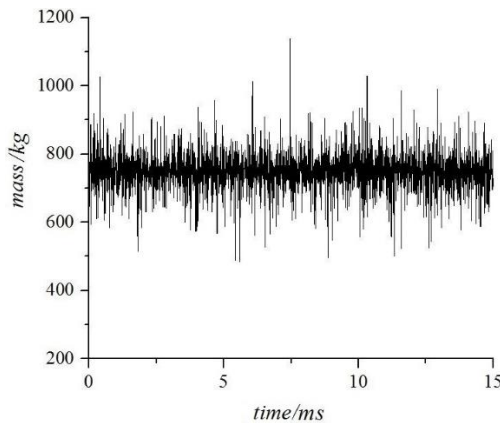


Fig. 4. The time-varying identified mass of the recoiling part

The angular velocity data contains the stress wave noise, as a result that the identified mass of the recoiling part consists of calculative deviation. Although, this time-varying identified mass can reflect the calculative consequence ignoring physical significance.

A relative percentage error (RPE) is defined to compare the result of the identification which is expressed in Eq. (37):

$$RPE = \frac{\sum |m_{true} - m_{identify}|}{\sum |m_{true}|} \times 100\%. \quad (37)$$

In Fig. 4, the identified mass of the recoiling part is about 740 kg. After comparing with the “real” mass which was consulted in mechanism specification book, the RPE is finally established at about 5.4 %.

5. Conclusions

In this paper, the multi-rigid-body theory and the limited memory least square method is applied to identify the time-varying mass of the recoiling part.

The mass of the recoiling part has physical significance so that the result of identification can be compared to the static measurement which is consulted in mechanism specification book. Limited memory least square method can overcome "data saturation" phenomenon effectively and it can be applied to time-varying identification in this situation. The result reflects that limited memory least square method and the process of establishing the mechanism dynamic equations are accurate in identifying the time-varying mass. At the same time, a new kind of identification method is provided to make the parameters more intuitive and can be applied in identifying dynamic loads which is difficult to measure.

At the present stage, the dynamic model is described as rigid body model. In the future, this model would be instead of flexible multi-body model to make the equations more accurately. And more parameters would be identified such as the mass and the stiffness of the structure, the active force from the propellant powder, the inertia force of the mechanism.

Acknowledgments

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