1169. Comparison of the calculated method to the driving voltage applied across the lay in single and double layers of piezoelectric material of active sound absorption

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Abstract. Piezoelectric material can be used as a main component of devices, such as transducers, energy exchangers and arresters. Due to its excellent mechanics and electric coupling performances, piezoelectric material can also be utilized in control system of sound and vibration. However, there have not been any publications outlining the basic equations of reflection or transmission coefficients of driving voltage applied across the layers (single or double) of piezoelectric material. In this paper, two methods – the theoretical method and the electro-acoustic analogy method – are used in order to compare the driving voltage applied across the single and the double layer of active sound surfaces of piezoelectric material. Computational results indicate that the proposed theoretical models are correct and applicable in practical implementations.

Keywords: piezoelectric material, active sound absorption, driving voltage.

1. Introduction

Several investigators have recently studied a new class of materials – the piezoelectric composite. These materials usually combine a piezoelectric ceramic and a polymeric solid, and exhibit rather large piezoelectricity. On the other hand, their densities are much lower than those of usual piezoelectric materials and therefore offer better acoustic coupling in low-density media, such as water or flesh. Consequently, piezoelectric materials can be used to form nonpolar and transducer without the need for casting and grinding.

A material that is both piezoelectric and flexible can also be utilized to cover the exterior of a structure, producing an acoustically active surface. In principle, such a surface (with proper arrays of electrodes) could be electrically driven in such a way to become nonreflecting of non-transmitting to incident sound.

At present, there are two cases that describe the initial evaluation of the piezoelectric material in acoustically active surfaces: (1) as a single layer transducer, which is designed to prevent either the reflection or the transmission of a normally incident plane sound wave, and (2) as a double transducer, which is designed to prevent simultaneously both the reflection and the transmission of the incident plane sound wave.

A number of studies have been carried out regarding the properties of piezoelectric material and its use in hydrophones and sonar arrays [1-5]. And yet, there have not been any publications outlining the basic equations of reflection or transmission coefficients of driving voltage applied across the lay of piezoelectric material. In addition to this, there has not been any comparison between the measurements of these coefficients and the values calculated from the complex elastic, dielectric and piezoelectric constants. It is the purpose of the paper to (1) compare the calculated method to the driving voltage applied across a single piezoelectric material layer, and (2) compare the calculated method to the driving voltage applied across double piezoelectric material layers.

2. Single layer piezoelectric material

The arrangement of the single layer piezoelectric material is shown in Fig. 1.

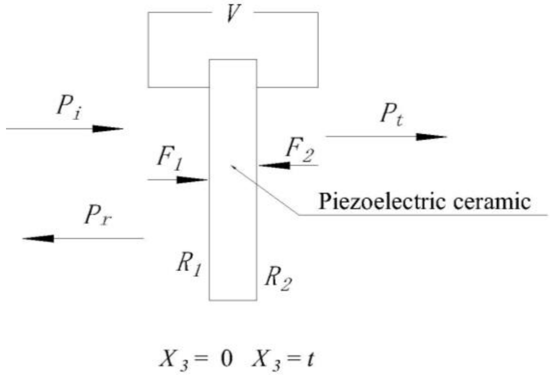

Fig. 1. The arrangement of the single layer piezoelectric material

2.1. Theoretical method

Several authors have presented work on the following topics: the derivation of the relation between normal forces on opposite faces of a thin piezoelectric material, the velocities of those faces, the voltage across and current through the piezoelectric material. Consider a piezoelectric material, which consists of a non-conducting piezoelectric layer of thickness t much smaller than the dimensions of its faces, each of area A. Denoting rectangular coordinates by (x_1, x_2, x_3), we define the x_3 axis normal to the faces of the piezoelectric material so that its thickness extends from $x_3 = 0$ (face 1) to $x_3 = t$ (face 2).

For harmonic electric and/or acoustic excitation at angular frequency ω, we represent (1) all time-dependent quantities by the usual product of constant complex amplitude and (2) a time-dependent exponential. For example, at time t the force f_1 on face 1 of the piezoelectric material is $f_1(t) = F_1 e^{j\omega t}$, where F_1 is the complex amplitude of that force. Thus we define F_i and U_i ($i = 1, 2$) as complex amplitudes of the normal forces and velocities of the piezoelectric material faces. We also define V and I as the complex amplitudes of the voltage across and current into the piezoelectric material. The sign conventions of these quantities are indicated in Fig. 1.

The relation between the complex amplitudes described above can be found using the fact that for $0 < x_3 < t$. The elastic stress and strain, and the electric displacement and field intensity must satisfy the following: the usual mechanical laws of motion, Maxwell's equations, and the constitutive piezoelectric equations for the piezoelectric material. The interaction between these fields depends on the orientation of the symmetry directions of the material relative to the piezoelectric material faces. Let us assume that the material is isotropic in the $x_1 - x_2$ direction and is polarized in the x_3 direction. If field disturbances are limited to the x_3 direction, they may be related using only the elements c_t^D, e_{33}, and ε_{33}^S of the elastic stiffness, piezoelectric stress constant, and dielectric impermissibility matrices. As a result of these assumptions, one obtains [6]:

$$F_1 = -j\frac{A\rho v_x}{\tan(kd)}U_1 - j\frac{A\rho v_x}{\sin(kd)}U_2 - j\frac{e_{33}}{\omega}I, \quad (1)$$

$$F_2 = -j\frac{A\rho v_x}{\sin(kd)}U_1 - j\frac{A\rho v_x}{\tan(kd)}U_2 - j\frac{e_{33}}{\omega}I, \quad (2)$$

$$V = -j\frac{e_{33}}{\omega}U_1 - j\frac{e_{33}}{\omega}U_2 - j\frac{\varepsilon_{33}^S d}{\omega A_0}I. \quad (3)$$

In the above, $v_x = \sqrt{\frac{c_t^D}{\rho}}$ is the velocity of longitudinal elastic (acoustic) waves traveling in the thickness direction of the piezoelectric material. ρ is the density of the piezoelectric material, and $k = \frac{\omega}{v_x}$ is the acoustic number.

If we set $F_1 = F_2 = 0$, the electrical impedance $Z_{el} = \frac{V}{I}$ of the air-loaded piezoelectric material may be obtained from Eqs. (1)-(3). The result is:

$$Z_{el} = Z_0\left\{1 - \left(\frac{e_{33}}{c_t^D \varepsilon_{33}^S}\right)\left[\frac{\tan\left(\frac{kd}{2}\right)}{\frac{kd}{2}}\right]\right\}. \tag{4}$$

The material parameters c_t^D, e_{33}, and ε_{33}^S can thus be obtained by fitting Eq. (4) to measurements of electrical impedance versus frequency.

Suppose the above piezoelectric material separates two semi-infinite media of specific acoustic impedance R_1 and R_2 bounded by face 1 and 2, respectively. If a plane acoustic sound wave is normally incident on face 1, the net force F_1 results from the superposition of the incident and reflected sound waves, while F_2 is due to the transmitted sound wave. Letting P_i, P_r and P_t be the complex pressure amplitudes of the incident, reflected and transmitted sound waves, respectively, we have:

$$F_1 = (P_i + P_r)A_0, \tag{5}$$
$$F_2 = P_t A_0, \tag{6}$$
$$\frac{P_i - P\_r}{R_1} = U_1, \tag{7}$$
$$\frac{-P_t}{Z_1} = U_2. \tag{8}$$

Insertion of Eq. (5)-(8) into Eq. (1)-(3) yields:

$$\left(-\sin\theta - j\frac{R_x}{R_1}\cos\theta\right)P_i - \left(\sin\theta - j\frac{R_x}{R_1}\cos\theta\right)P_r + j\frac{R_x}{R_2}P_t - j\frac{e_{33}}{\omega}\sin\theta J = 0, \tag{9}$$
$$-j\frac{R_x}{R_1}P_i - j\frac{R_x}{R_1}P_r - \left(\sin\theta + j\frac{R_x}{R_2}\cos\theta\right)P_t - j\frac{e_{33}}{\omega}\sin\theta J = 0, \tag{10}$$
$$j\frac{e_{33}}{\omega R_1}P_i - j\frac{e_{33}}{\omega R_1}P_r - j\frac{e_{33}}{\omega R_2}P_t + V + \frac{j\varepsilon_{33}^S d}{\omega}J = 0. \tag{11}$$

In the above, $\theta = kd$, and $R_x = \rho v_x$ is the specific acoustic impedance of the piezoelectric material.

In Eq. (9)-(11) $J = \frac{I}{A_0}$ is the complex amplitude of the current density into face 1 of the piezoelectric material.

Eq. (9)-(11) are simultaneous equations relating the complex amplitudes P_i, P_r, P_t, V and J of five sigmoidally varying quantities. If two of these amplitudes are specified, the equations may be solved for the remaining three. If P_i is set to desired incident sound wave amplitude and $P_r = 0$, thus we can have:

$$\left(A_0 + \frac{Z_{E1} + Z_{E2}}{Z_1}\right)P_i = \frac{Z_{E2}}{Z_1}P_t - \frac{N}{j\omega C_0}I, \tag{12}$$

$$\left(\frac{Z_{E1}+Z_{E2}}{Z_1}-A_0\right)P_t+\frac{N}{j\omega C_0}I=-\frac{Z_{E2}}{Z_1}P_i, \tag{13}$$

$$\frac{N}{j\omega C_0 Z_1}P_t-V+\frac{1}{j\omega C_0}I=-\frac{N}{j\omega C_0 Z_1}P_i, \tag{14}$$

where $Z_{E1}=j\rho C_t^{\mathrm{D}}\tan\frac{kd}{2}$, $Z_{E1}=\frac{-j\rho c_t^D}{\sin kd}$, $C_0=\frac{\varepsilon_{33}^S}{d}$, $N=\frac{e_{33}}{t}$, d, ρ, C_t^D, ε_{33}^S, e_{33} are thickness, density, elastic coefficient, dielectric constant, and coupling constant of the piezoelectric material, respectively.

2.2. Electro-acoustic analogy method

The structure of the piezoelectric material used in this paper is shown in Fig. 2.

In the Fig. 2, d is the thickness of the piezoelectric material. U_1, U_2 are the vibration velocity in the direction of thickness. F_1, F_2 are force press on the piezoelectric material. V is voltage. According to the structure of the piezoelectric material, the equivalent circuit figuration is attained and shown in Fig. 3.

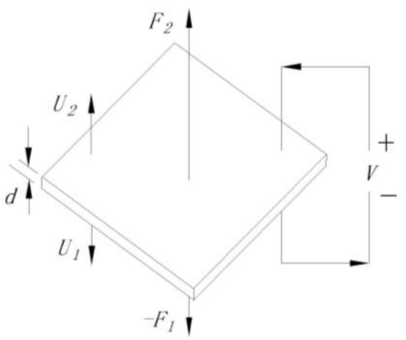

Fig. 2. Structure of the piezoelectric ceramic

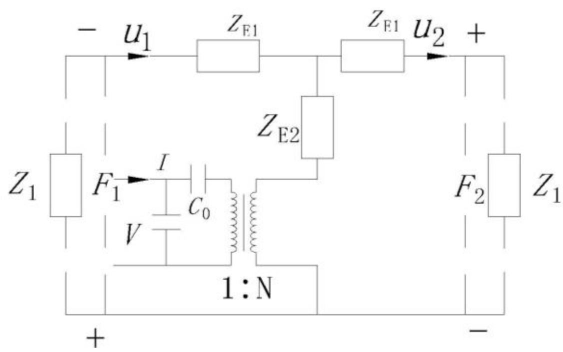

Fig. 3. The equivalent circuit figuration

In the Fig. 3, $Z_{E1}=j\rho C_t^{\mathrm{D}}\tan\frac{kt}{2}$, $Z_{E1}=\frac{-j\rho C_t^D}{\sin kt}$, $C_0=\frac{\varepsilon_{33}^S}{t}$, $N=\frac{e_{33}}{t}$, t, ρ, C_t^D, ε_{33}^S, e_{33} are thickness, density, elastic coefficient, dielectric constant, and coupling constant of the piezoelectric material, respectively. The following equations can be derived according to the electro circuit theory:

$$F_1=-(Z_{E1}+Z_{E2})u_1+Z_{E2}u_2-\frac{N}{j\omega C_0}I, \tag{15}$$

$$F_2=-(Z_{E1}+Z_{E2})u_2+Z_{E2}u_1+\frac{N}{j\omega C_0}I, \tag{16}$$

$$V=\frac{N}{j\omega C_0}u_1-\frac{N}{j\omega C_0}u_2+\frac{1}{j\omega C_0}I. \tag{17}$$

According to the Eq. (5)-(8) and the Eq. (15)-(17), the following equations yield:

$$\left(A_0+\frac{Z_{E1}+Z_{E2}}{Z_1}\right)p_i+\left(A_0-\frac{Z_{E1}+Z_{E2}}{Z_1}\right)p_r-\frac{Z_{E2}}{Z_1}p_t-\frac{N}{j\omega C_0}I=0, \tag{18}$$

$$\frac{Z_{E2}}{Z_1}p_i-\frac{Z_{E2}}{Z_1}p_r+\left(\frac{Z_{E1}+Z_{E2}}{Z_1}-A_0\right)p_t+\frac{N}{j\omega C_0}I=0, \tag{19}$$

$$\frac{N}{j\omega C_0 Z_1}p_i-\frac{N}{j\omega C_0 Z_1}p_r+\frac{N}{j\omega C_0 Z_1}p_t-V+\frac{1}{j\omega C_0}I=0. \tag{20}$$

In the Eq. (18)-(20), there are five complex amplitudes p_i, p_r, p_t, V, I. If two of these amplitudes are specified, the equations may be solved for the remaining three. For example, if p_i is set to desired incident wave amplitude and p_r is set to zero, the transformed equations read:

$$\left(A_0+\frac{Z_{E1}+Z_{E2}}{Z_1}\right)p_i=\frac{Z_{E2}}{Z_1}p_t-\frac{N}{j\omega C_0}I, \tag{21}$$

$$\left(\frac{Z_{E1}+Z_{E2}}{Z_1}-A_0\right)p_t+\frac{N}{j\omega C_0}I=-\frac{Z_{E2}}{Z_1}p_i, \tag{22}$$

$$\frac{N}{j\omega C_0 Z_1}p_t-V+\frac{1}{j\omega C_0}I=-\frac{N}{j\omega C_0 Z_1}p_i. \tag{23}$$

From the Eq. (21)-(23), the voltage V of the piezoelectric material is attained:

$$\begin{pmatrix} p_t \\ V \\ I \end{pmatrix}=\begin{pmatrix} \frac{Z_{E2}}{Z_1} & 0 & -\frac{N}{j\omega C_0} \\ \left(\frac{Z_{E1}+Z_{E2}}{Z_1}-A_0\right) & 0 & \frac{N}{j\omega C_0} \\ \frac{N}{j\omega C_0 Z_1} & -1 & \frac{1}{j\omega C_0} \end{pmatrix}^{-1}\begin{pmatrix} A_0+\frac{Z_{E1}+Z_{E2}}{Z_1} \\ -\frac{Z_{E2}}{Z_1} \\ -\frac{N}{j\omega C_0 Z_1} \end{pmatrix}p_i. \tag{24}$$

According to the Eq. (24), the voltages applied on the piezoelectric material can make the reflected wave be zero and the aim of active absorption is gained.

According to the Eq. (12)-(14) and Eq. (24), the calculated results are the same, thus denote that the methods are correct and applicable.

The density of the air is $\rho = 1.21$ kg/m^3. The propagation velocity of sound wave in the air is $c = 343$ m/s^{-1}. The density, the propagation velocity of sound wave in the piezoelectric material, and the dielectric constant of the piezoelectric material, respectively, are $\rho = 5400$ kg/m^{-3}, $c_t = 4540$ m/s, $\varepsilon_{33}^S = 1200\varepsilon_0$. The diameter of the piezoelectric material is $d = 29$ mm and the thickness is $t = 1.5$ mm. If the amplitude of the incidence wave is 1.00 Pa, the theoretical calculated amplitude and phase of the piezoelectric material, according to the Eq. (24), are shown in Fig. 4.

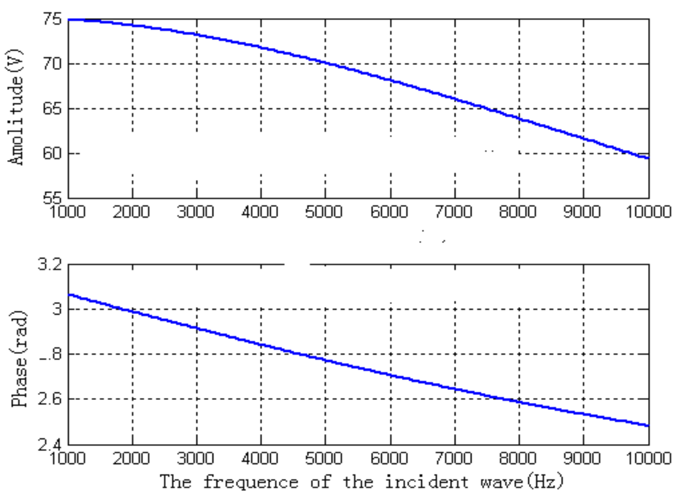

Fig. 4. The theoretical calculated amplitude and phase of the piezoelectric ceramic

For the same piezoelectric material, the thickness is $t = 2.5$ mm. If the amplitude of the incidence wave is 1.00 Pa, the theoretical calculated amplitude and phase of the piezoelectric material, according to the Eq. (24), are shown in Fig. (5).

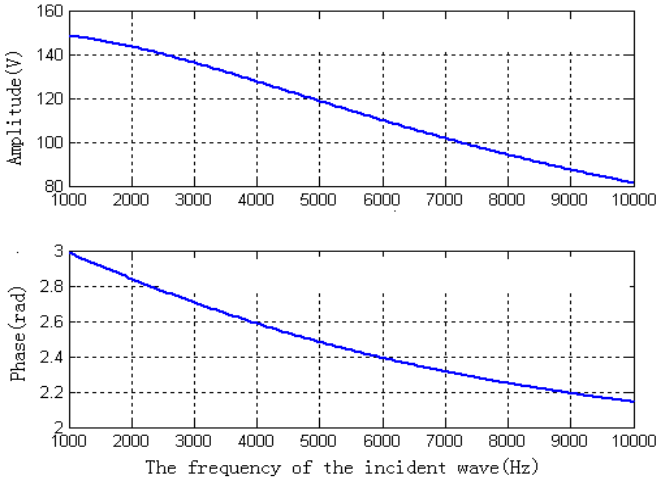

Fig. 5. The theoretical calculated amplitude and phase of the piezoelectric ceramic

For the same piezoelectric material, the thickness is $t = 3.5$ mm. If the amplitude of the incidence wave is 1.00 Pa, the theoretical calculated amplitude and phase of the piezoelectric material according to the Eq. (24) are shown in Fig. (6).

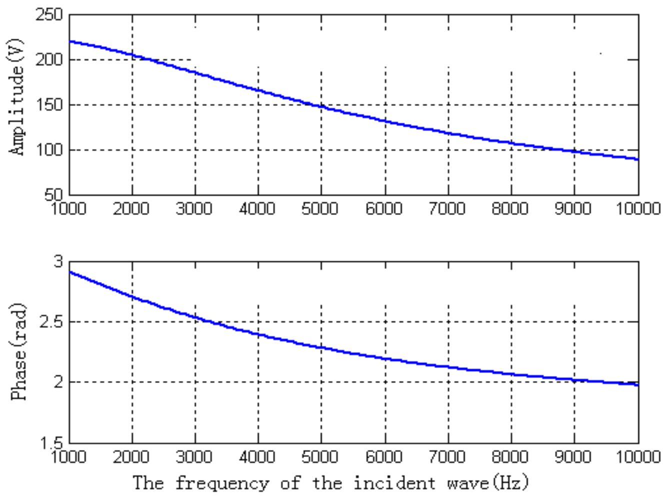

Fig. 6. The theoretical calculated amplitude and phase of the piezoelectric ceramic

According to the Fig. 4-6, the conclusion can be drawn:

(1) For the piezoelectric material of the same thickness, the amplitude and the phase of the voltage are diminished when the incident frequency is added.

(2) For the same frequency of the incident wave, the amplitude of the voltage is added and the phase is diminished when the thickness of the piezoelectric material is added.

3. Double layer piezoelectric materials

The arrangement of the double layer piezoelectric materials is shown in Fig. 7.

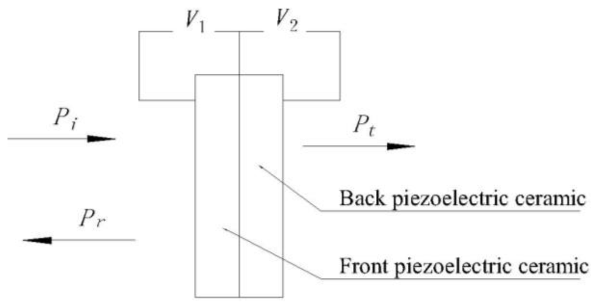

Fig. 7. The arrangement of the double layer piezoelectric materials

3.1. Theoretical method

We now consider the piezoelectric ceramic, which consists a sandwich of two piezoelectric layers of the type described above, with face 2 of one layer fixed to face 1 of the other layer. For each layer a set of equations equivalent to Eq. (1)-(3) may be written. And yet, in order to allow for different material properties and thicknesses, we subscript the characteristic quantities of the layer "1" and "2". We now have the additional boundary conditions and the forces and velocities of the two surfaces in contact are equal. The result of this boundary condition is a set of equations relating seven complex amplitudes, i.e.:

$$D_{11}P_i + D_{12}P_r + D_{13}P_t + D_{15}J_1 + D_{17}J_2 = 0, \quad (25)$$
$$D_{21}P_i + D_{22}P_r + D_{23}P_t + D_{25}J_1 + D_{27}J_2 = 0, \quad (26)$$
$$D_{31}P_i + D_{32}P_r - V_1 + D_{35}J_1 = 0, \quad (27)$$
$$D_{31}P_i + D_{32}P_r - V_1 + D_{35}J_1 = 0. \quad (28)$$

Expressions for the coefficients D_{ij} are given in the Appendix.

Equations (25)-(28) play a similar role for the double piezoelectric ceramic as Eqs. (9)-(11) for the single layer, i.e., they relate the complex amplitudes P_i, P_r, P_t, V_1, J_1, V_2 and J_2 to the complex coefficients D_{ij}. In this case, for example, if we set P_i to a desired incident wave amplitude and $P_r = P_t = 0$, according to the Equation (1)-(3) and the Equation (5)-(8), the following formula can be obtained:

$$V_1 = \frac{\left[P_i - \frac{P_i}{Z_1}Z_{E11} + \left(\frac{P_i - \frac{P_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}} - \frac{P_i}{Z_1}\right)\left(Z_{E21} - \frac{N_1^2}{j\omega C_1}\right)\right]}{N_1}, \quad (29)$$

$$I_1 = \left(\frac{P_i - \frac{p_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}} - \frac{P_i}{Z_1}\right)N_1 + j\omega C_1 V_1, \quad (30)$$

$$V_2 = -\frac{P_i - \frac{P_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}}\left(\frac{N_2}{j\omega C_2} - \frac{Z_{E22}}{N_2}\right), \quad (31)$$

$$I_2 = j\omega C_2 V_2 + \frac{P_i - \frac{P_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}}N_2, \quad (32)$$

where $Z_{E11} = j\rho_1 C_{1t}^{\mathrm{D}} \tan\frac{k_1 t_1}{2}$, $Z_{E21} = \frac{-j\rho_1 C_{1t}^D}{\sin k_1 t_1}$, $C_1 = \frac{\varepsilon_{133}^S}{t_1}$, $N_1 = \frac{e_{133}}{t_1}$, $Z_{E12} = j\rho_2 C_{2t}^{\mathrm{D}} \tan\frac{k_2 t_2}{2}$, $Z_{E22} = \frac{-j\rho_2 C_{2t}^D}{\sin k_2 t_2}$, $Z_{E22} = \frac{-j\rho_2 C_{2t}^D}{\sin k_2 t_2}$, $C_2 = \frac{\varepsilon_{233}^S}{t_2}$, $N_2 = \frac{e_{233}}{t_2}$.

3.2. Electro-acoustic analogy method

According to the structure of the piezoelectric ceramic, the equivalent circuit figuration of the front piezoelectric ceramic is attained and shown in Fig. 8.

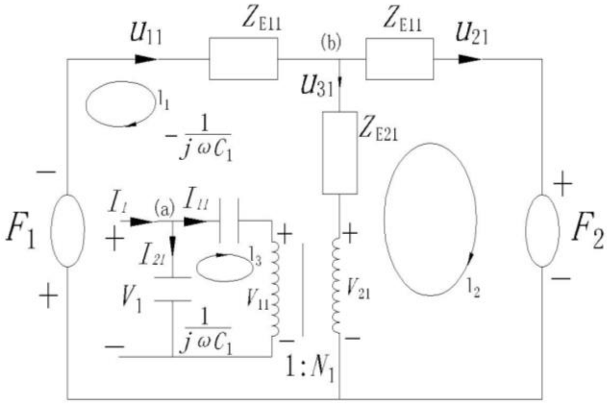

Fig. 8. The equivalent circuit figuration of the front piezoelectric ceramic

In the Fig. 8, $Z_{E11} = j\rho_1 C_{1t}^{\mathrm{D}} \tan\frac{k_1 t_1}{2}$, $Z_{E21} = \frac{-j\rho_1 C_{1t}^{D}}{\sin k_1 t_1}$, $C_1 = \frac{\varepsilon_{133}^{S}}{t_1}$, $N_1 = \frac{e_{133}}{t_1}$, t_1, ρ_1, C_{1t}^{D}, ε_{133}^{S}, e_{133} are thickness, density, elastic coefficient, dielectric constant, and coupling constant of the front piezoelectric ceramic, respectively. The following equations can be derived according to the electro circuit theory:

$$F_1 = -(Z_{E11} + Z_{E21})u_{11} + Z_{E21}u_{21} - \frac{N_1}{j\omega C_1} I_1, \tag{33}$$

$$F_2 = -(Z_{E11} + Z_{E21})u_{21} + Z_{E21}u_{11} + \frac{N_1}{j\omega C_1} I_1, \tag{34}$$

$$V_1 = \frac{N_1}{j\omega C_1} u_{11} - \frac{N_1}{j\omega C_1} u_{21} + \frac{1}{j\omega C_1} I_1. \tag{35}$$

The equivalent circuit figuration of the back piezoelectric ceramic is shown in the Fig. 9.

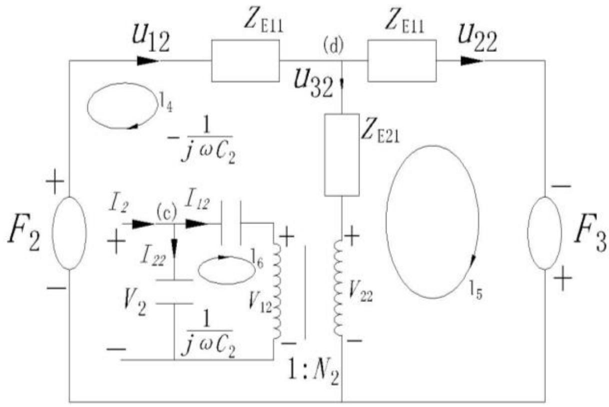

Fig. 9. The equivalent circuit figuration of the back piezoelectric ceramic

In the Fig. 9, $Z_{E12} = j\rho_2 C_{2t}^{\mathrm{D}} \tan\frac{k_2 t_2}{2}$, $Z_{E22} = \frac{-j\rho_2 C_{2t}^{D}}{\sin k_2 t_2}$, $C_2 = \frac{\varepsilon_{233}^{S}}{t_2}$, $N_1 = \frac{e_{233}}{t_2}$, t_2, ρ_2, C_{2t}^{D}, ε_{233}^{S},

e_{233} are thickness, density, elastic coefficient, dielectric constant, and coupling constant of the back piezoelectric ceramic, respectively. The following equations can be derived according to the electro circuit theory:

$$F_2 = -(Z_{E12} + Z_{E22})u_{12} + Z_{E22}u_{22} - \frac{N_2}{j\omega C_2}I_2, \tag{36}$$

$$F_3 = -(Z_{E12} + Z_{E22})u_{22} + Z_{E22}u_{12} + \frac{N_2}{j\omega C_2}I_2, \tag{37}$$

$$V_2 = \frac{N_2}{j\omega C_2}u_{12} - \frac{N_2}{j\omega C_2}u_{22} + \frac{1}{j\omega C_2}I_2. \tag{38}$$

According to the Equations (36)-(38), the following equations yield:

$$V_1 = \frac{\left[p_i - \frac{p_i}{Z_1}Z_{E11} + \left(\frac{p_i - \frac{p_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}} - \frac{p_i}{Z_1}\right)\left(Z_{E21} - \frac{N_1^2}{j\omega C_1}\right)\right]}{N_1}, \tag{39}$$

$$I_1 = \left(\frac{p_i - \frac{p_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}} - \frac{p_i}{Z_1}\right)N_1 + j\omega C_1 V_1, \tag{40}$$

$$V_2 = -\frac{p_i - \frac{p_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}}\left(\frac{N_2}{j\omega C_2} - \frac{Z_{E22}}{N_2}\right), \tag{41}$$

$$I_2 = j\omega C_2 V_2 + \frac{p_i - \frac{p_i}{Z_1}Z_{E11}}{Z_{E11} + Z_{E12}}N_2. \tag{42}$$

According to the Eq. (39)-(42), the voltages applied on the piezoelectric ceramics can make the reflected wave and transmission wave be zero. The aim of active absorption and active isolation is gained.

According to the Eq. (29)-(32) and Eq. (39)-(42), the calculated results are the same, thus denote that the methods are correct and applicable.

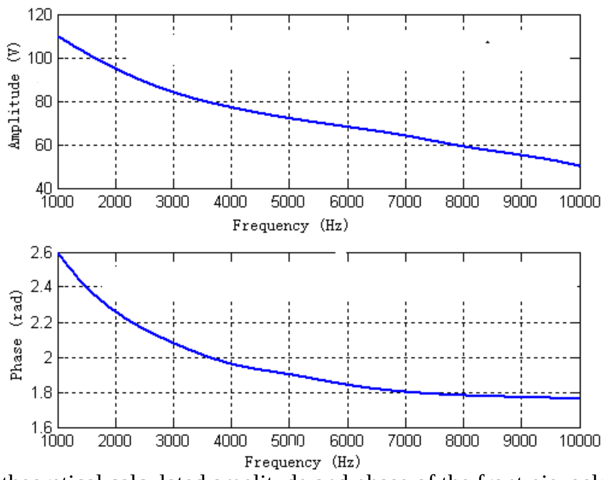

Fig. 10. The theoretical calculated amplitude and phase of the front piezoelectric ceramic

The front and back of piezoelectric ceramic are the same. The density, the acoustic impedance, the dielectric constant, and coupling constant of the piezoelectric ceramics, respectively, are

$\rho = 2430$ kg/m^{-3}, $\rho C_t^D = 7.6\times10^6$ kg(m^2/s)$^{-1}$, $\varepsilon_{33}^S = 174.3\varepsilon_0$, $e_{33} = 3.95$ C/m^{-2}. The diameter of the piezoelectric ceramic is $d = 100$ mm and the thickness is $t = 1.5$ mm. If the amplitude of the incidence wave is 1.00 Pa, the theoretical calculated amplitude and phase of the front piezoelectric ceramic according to the Eq. (41)-(44) are shown in Fig. 10.

The theoretical calculated amplitude and phase of the back piezoelectric ceramic according to the Eq. (41)-(42) are shown in Fig. 11.

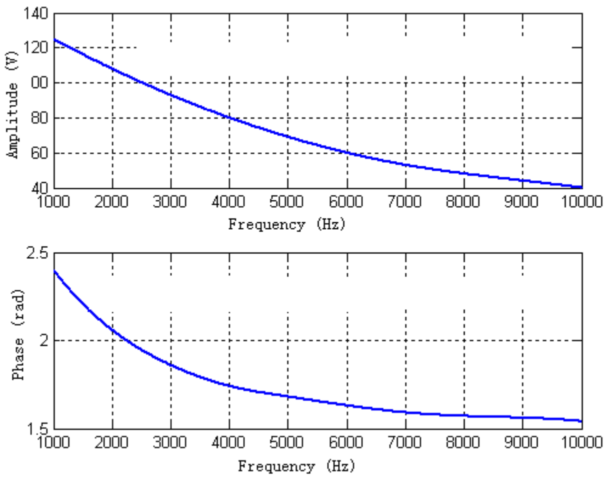

Fig. 11. The theoretical calculated amplitude and phase of the back piezoelectric ceramic

4. Conclusion

In this paper, two methods – the theoretical method and the electro-acoustic analogy method – are used in order to compare the driving voltage applied across the single and the double layer of active sound surfaces of piezoelectric material. Computational results indicate that the proposed theoretical models are correct and applicable in practical implementations.

Acknowledgment

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Appendix

$$D_{11} = -\frac{cos\alpha}{R_1} + j\frac{sin\alpha}{R_a},$$
$$D_{12} = \frac{\cos\alpha}{R_1} + j\frac{\sin\alpha}{R_a},$$
$$D_{13} = \frac{\cos\beta}{R_2} + j\frac{\sin\beta}{R_b},$$
$$D_{15} = -\frac{e_{33a}}{\omega R_a}\sin\alpha,$$
$$D_{17} = -\frac{e_{33b}}{\omega R_b}\sin\beta,$$
$$D_{21} = -\left(\frac{\cos\alpha\sin\beta}{R_a R_b} + \frac{\sin\alpha\cos\beta}{R_a^2}\right) + j\left(\frac{\sin\alpha\sin\beta}{R_1 R_b} - \frac{\cos\alpha\cos\beta}{R_a R_1}\right),$$
$$D_{22} = -\left(\frac{\cos\alpha\sin\beta}{R_a R_b} + \frac{\sin\alpha\cos\beta}{R_a^2}\right) - j\left(\frac{\sin\alpha\sin\beta}{R_1 R_b} - \frac{\cos\alpha\cos\beta}{R_a R_1}\right),$$
$$D_{23} = j\frac{1}{R_a R_b},$$
$$D_{25} = j\frac{e_{33a}}{\omega R_a}\left(\frac{(1-\cos\alpha)\sin\beta}{R_b} - \frac{\sin\alpha\cos\beta}{R_a}\right),$$
$$D_{27} = -j\frac{e_{33b}\sin\beta}{\omega R_a R_b},$$
$$D_{31} = \frac{e_{33a}}{\omega}\left(\frac{\sin\alpha}{R_a} - j\frac{1-\cos\alpha}{R_1}\right),$$
$$D_{32} = \frac{e_{33a}}{\omega}\left(\frac{\sin\alpha}{R_a} + j\frac{1-\cos\alpha}{R_1}\right),$$
$$D_{33} = j\left(\frac{e_{33a}^2\sin\alpha}{\omega^2 R_a} - \frac{\varepsilon_{33a}^S t_1}{\omega}\right),$$
$$D_{41} = \frac{e_{33b}}{\omega}\left(\frac{\sin\alpha}{R_a} + j\frac{\cos\alpha}{R_1}\right),$$
$$D_{42} = \frac{e_{33b}}{\omega}\left(\frac{\sin\alpha}{R_a} - j\frac{\cos\alpha}{R_1}\right),$$
$$D_{43} = -j\frac{e_{33b}}{\omega R_2},$$
$$D_{45} = j\frac{e_{33a}e_{33b}\sin\alpha}{\omega^2 R_a},$$
$$D_{47} = j\frac{\varepsilon_{33b}^S t_2}{\omega}.$$

With the substitutions $\alpha = k_1 t_1$ and $\beta = k_2 t_2$.