255. Investigation of vibrations of a tape in the printing device

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Abstract. The model for the analysis of vibrations of a tape in the printing device is presented. It is assumed that a tape performs transverse vibrations with additional stiffness due to static longitudinal tension. The first eigenmodes are determined. It is shown that the eigenmodes of the analyzed tape are influenced by the static longitudinal tension. The obtained results are used in the process of design of the elements of the printing device.
Keywords: tape, eigenmode, printing device, finite elements.

Introduction

The analysis of dynamics of a printer tape and of the tapes used for the transmission of motion between various elements of the printing device is important in the design of printing devices.

The model for the analysis of vibrations of a tape in the printing device is proposed on the basis of the models for the analysis of beam bending described in [1, 2]. It is assumed that a tape performs transverse vibrations with additional stiffness due to its static longitudinal tension.

Thus the analysis consists of two stages:
1) the static problem of longitudinal motion of a bar by assuming the longitudinal displacements at the ends of the analyzed tape to be given is solved;
2) the transverse vibrations of the investigated tape as of a beam with additional stiffness due to the static tension determined in the previous stage of analysis are analyzed (the first eigenmodes are determined).

The obtained results are used in the process of design of the elements of the printing device.

Model for the analysis of vibrations of the tape

Further x, y and z denote the axes of the orthogonal Cartesian system of coordinates. First the static problem of longitudinal motion of a bar is analyzed. The element has one nodal degree of freedom: the displacement u in the direction of the x axis. The stiffness matrix has the form:

$$[K] = \int [B]^T \left[\frac{E}{1-\nu^2} h \right] [B] dx, \qquad (1)$$

where E is the modulus of elasticity, ν is the Poisson's ratio, h is the thickness of the tape and:

$$[B] = \left[\frac{dN_1}{dx} \quad \ldots \right], \qquad (2)$$

where N_i are the shape functions of the finite element.

It is assumed that the longitudinal displacements at the ends of the analyzed tape are given and they produce the loading vector $\{F\}$. Thus the vector of displacements $\{\delta\}$ is determined by solving the system of linear algebraic equations:

$$[K]\{\delta\} = \{F\}. \tag{3}$$

In the second stage of the analysis the eigenproblem of transverse vibrations of the beam with additional stiffness due to the static longitudinal tension is solved. The beam bending element has two nodal degrees of freedom: the displacement w in the direction of the z axis and the rotation Θ_y about the y axis. The displacement u in the direction of the x axis is expressed as $u=z\Theta_y$.

The mass matrix has the form:

$$[\overline{M}] = \int [N]^T \begin{bmatrix} \rho h & 0 \\ 0 & \rho \dfrac{h^3}{12} \end{bmatrix} [N] dx, \tag{4}$$

where ρ is the density of the material of the tape and:

$$[N] = \begin{bmatrix} N_1 & 0 & \ldots \\ 0 & N_1 & \ldots \end{bmatrix}. \tag{5}$$

The stiffness matrix has the form:

$$[\overline{K}] = \int \begin{pmatrix} [\overline{B}]^T \left[\dfrac{E}{1-\nu^2} \dfrac{h^3}{12} \right] [\overline{B}] + \\ + [\hat{B}]^T \left[\dfrac{E}{2(1+\nu)1.2} h \right] [\hat{B}] + \\ + [G]^T [M][G] \end{pmatrix} dx, \tag{6}$$

where:

$$[\overline{B}] = \begin{bmatrix} 0 & \dfrac{dN_1}{dx} & \ldots \end{bmatrix}, \tag{7}$$

$$[\hat{B}] = \begin{bmatrix} \dfrac{dN_1}{dx} & N_1 & \ldots \end{bmatrix}, \tag{8}$$

$$[G] = \begin{bmatrix} \dfrac{dN_1}{dx} & 0 & \ldots \end{bmatrix}, \tag{9}$$

$$[M] = \left[\frac{E}{1-\nu^2} h \right] [B]\{\delta\}. \tag{10}$$

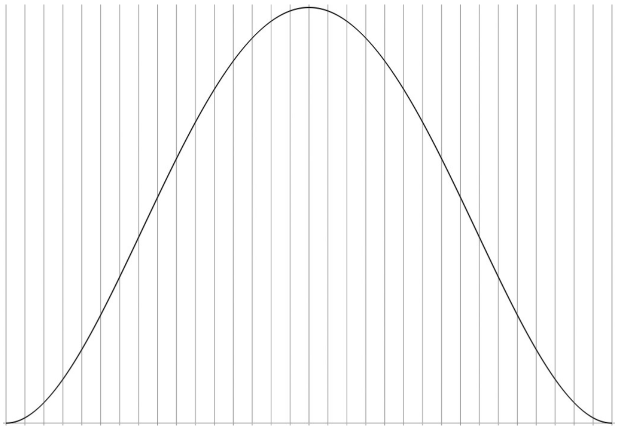
The i-th eigenfrequency ω_i and the corresponding eigenmode $\{\delta_i\}$ are determined from:

$$\left([\overline{K}] - \omega_i^2 [\overline{M}]\right)\{\delta_i\} = \{0\}. \tag{11}$$

Results of analysis of vibrations of the tape

For the static problem of longitudinal motion of a bar the following boundary conditions are assumed: at the left end it is assumed that the longitudinal displacement is equal to zero, while at the right end it is assumed to be given a prescribed positive value. For the problem of transverse vibrations at both ends both generalized displacements are assumed equal to zero.

The first eigenmode without longitudinal tension is presented in Fig. 1a and with longitudinal tension in Fig. 1b, the second eigenmode without longitudinal tension is presented in Fig. 2a and with longitudinal tension in Fig. 2b, …, the tenth eigenmode without longitudinal tension is presented in Fig. 10a and with longitudinal tension in Fig. 10b.



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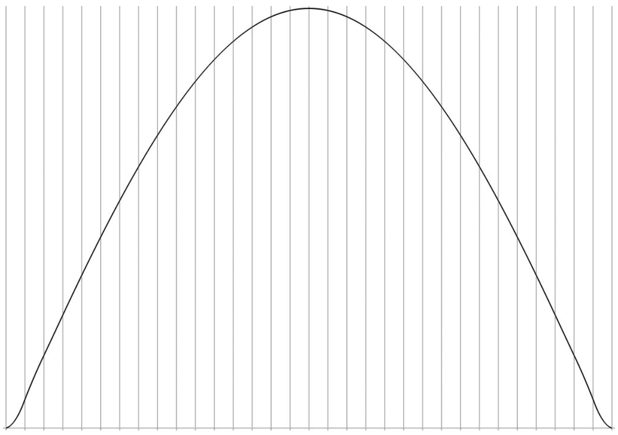
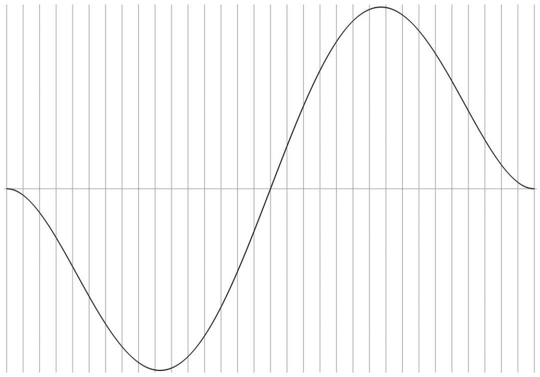


b)

Fig. 1. The first eigenmode: a) without longitudinal tension, b) with longitudinal tension



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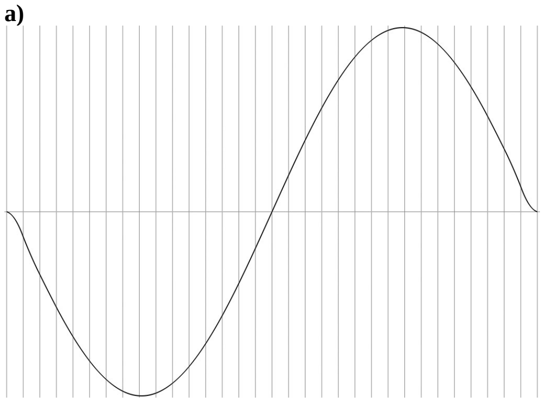

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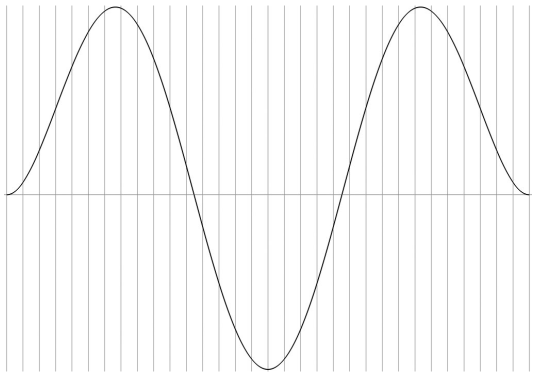
Fig. 2. The second eigenmode: a) without longitudinal tension, b) with longitudinal tension



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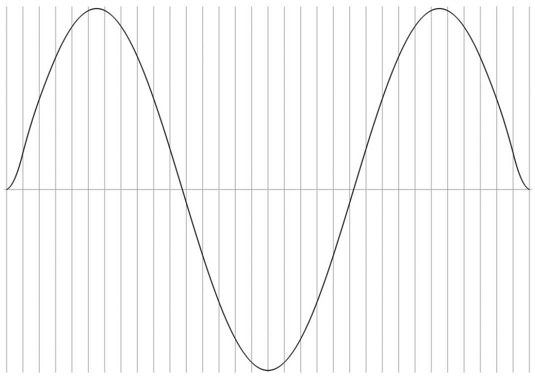
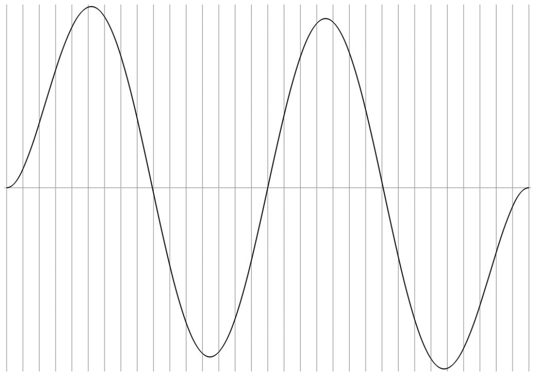


b)

Fig. 3. The third eigenmode: a) without longitudinal tension, b) with longitudinal tension



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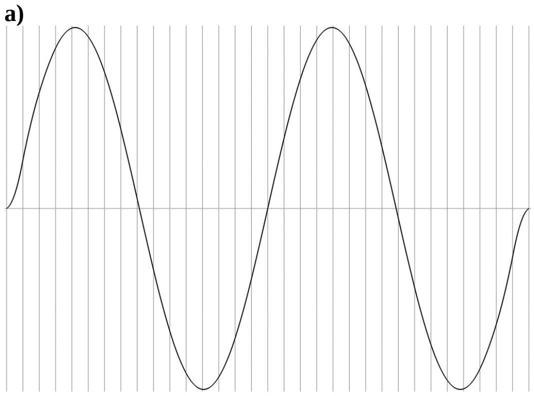
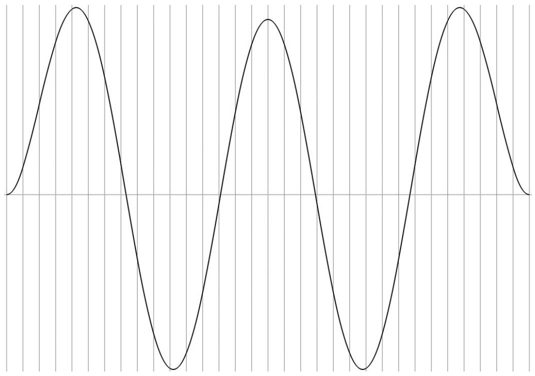


b)

Fig. 4. The fourth eigenmode: a) without longitudinal tension, b) with longitudinal tension



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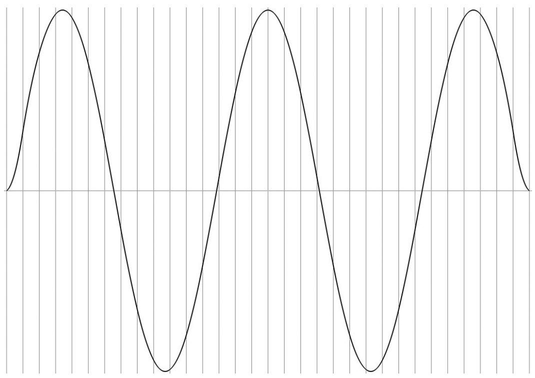
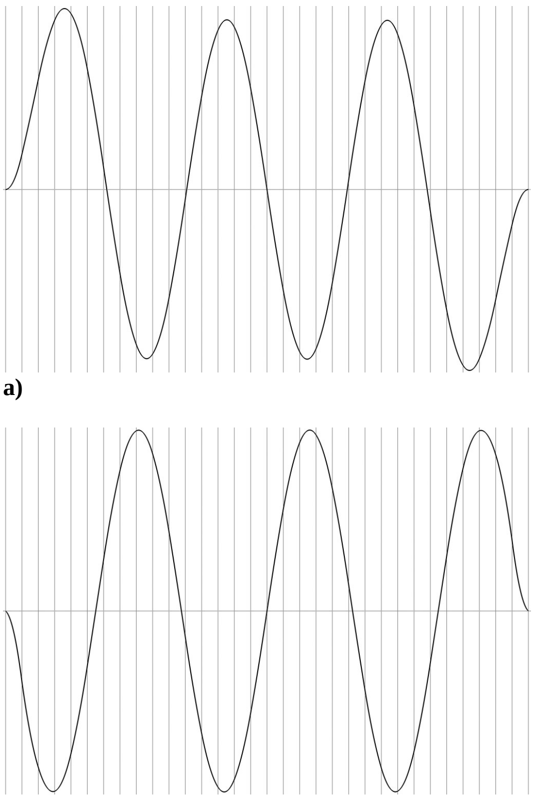


b)

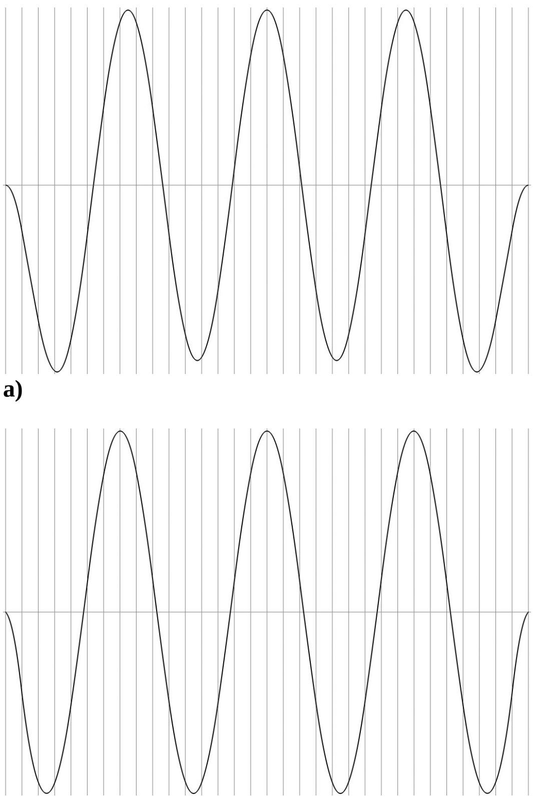
Fig. 5. The fifth eigenmode: a) without longitudinal tension, b) with longitudinal tension



a)

b)

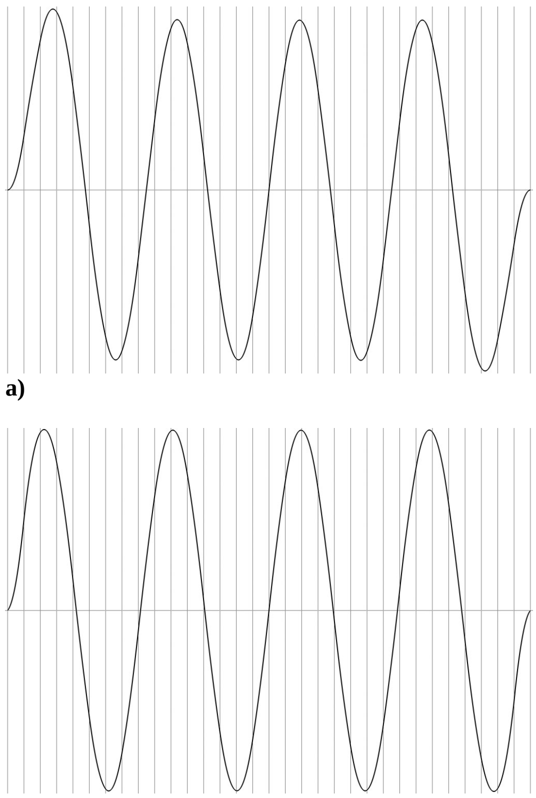
Fig. 6. The sixth eigenmode: a) without longitudinal tension, b) with longitudinal tension



a)

b)

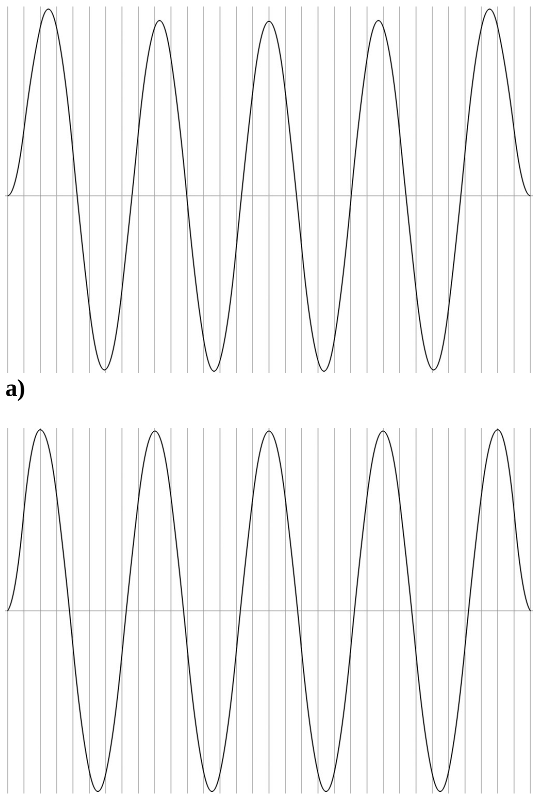
Fig. 7. The seventh eigenmode: a) without longitudinal tension, b) with longitudinal tension



a)

b)

Fig. 8. The eighth eigenmode: a) without longitudinal tension, b) with longitudinal tension



a)

b)

Fig. 9. The ninth eigenmode: a) without longitudinal tension, b) with longitudinal tension

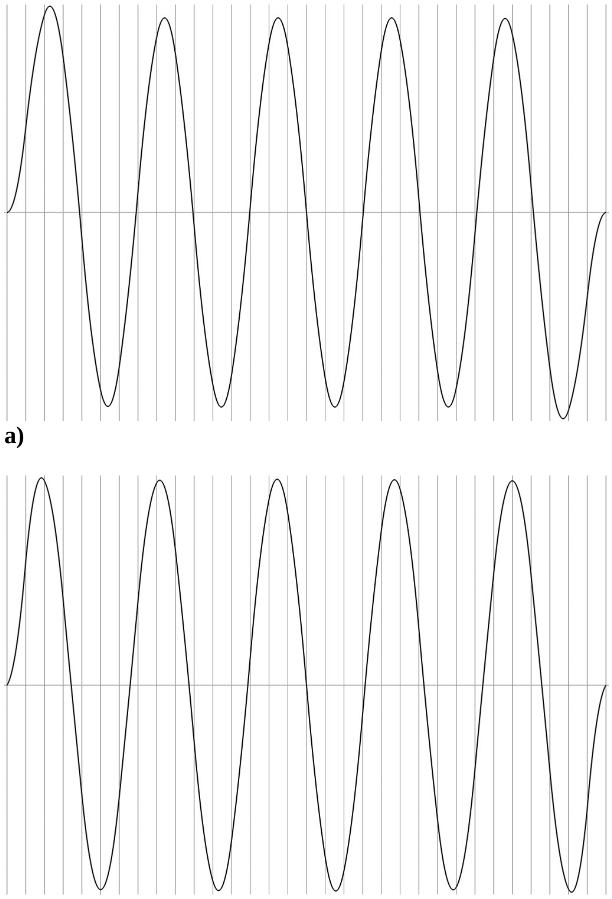

a)
b)

Fig. 10. The tenth eigenmode: a) without longitudinal tension, b) with longitudinal tension

From the presented results it is seen that in the higher eigenmodes the peak nearest to the ends of the beam in the problem without longitudinal tension is greater than the other peaks. On the basis of this fact one can judge about the tension of the tape from the shape of the corresponding eigenmode.

Conclusions

The proposed model for the analysis of vibrations of a tape in the printing device is based on the assumption that a tape performs transverse vibrations as a beam having additional stiffness due to its static longitudinal tension. The static problem of longitudinal motion of a bar by assuming the displacements at the ends of the analyzed tape to be given is solved. Then the transverse vibrations of the investigated tape as of a beam with additional stiffness due to the static tension determined previously are analyzed.

From the presented results it is seen that in the higher eigenmodes the peak nearest to the ends of the tape in the problem without longitudinal tension is greater than the other peaks. On the basis of this fact it is possible to judge about the tension of the tape from the shape of the corresponding eigenmode.

The obtained results are used in the process of design of the elements of the printing device.

References

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