Identification of Certain Artillery Kinetic Parameters by Limited Memory Least Square Method

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Abstract. This paper is concerned with the identification problem of certain artillery kinetic parameters during firing. An artillery's model can be simplified as three rigid body structure joined by a prismatic pair and a revolute pair. The unrestraint dynamic equation of the artillery can be obtained by using multi-rigid-body theory. In order to acquire unknown kinematics parameters, such as displacement, velocity and acceleration, a field experiment was designed. Then by choosing limited memory least square method and using the experiment results, the mass of rigid body could be identified. Finally, the calculative mass was compared to the "real" mass which was consulted in artillery specification book. The whole process shows that the artillery rigid dynamic model and the method of identification are both effective.

1. Introduction

Artillery is a weapon of war that operates by projection of munitions far beyond the effective range of personal weapons. For modern artillery's structure is complex and time-varying during the firing process, the mass matrix and the stiffness matrix of the whole system is also time-varying. So, it's difficult for researchers to describe artillery's dynamic model accurately.

At present, research field is mainly divided into two direction, forward problem and inverse problem. Based on mathematical model, forward problem is concerned with the structure's dynamic response under different excitation. Q Chen analysed a certain artillery's dynamic response by using the wavelet transform method [1]. By means of multi-body transfer matrix method, X Rui obtained the model function's accurate expression [2]. Inverse problem, namely, dynamic parameter identification problem, uses some model class to find an equivalent model upon input and output. X Liu, D Jing and W Zhuang provide many identifying methods of time-varying parameters in their papers [3-5]. Because of least square method's simple principle and fast convergence, it is widely used in identifying system parameters. As one of least square method's improved algorithms, limited memory least square method can effectively overcome "data saturation" phenomenon when it is applied to time-varying identification. X Chen and C Chen proved its effect in their work [6]. But, most of inverse problem dynamic equations lack physical significance and it's hard to examine the result of identification intuitively.

In this paper, taking certain artillery as the research object, we can identify the time-varying mass effectively when combining artillery dynamic model with limited memory least square method. Because the mass has physical significance, the result of identification can be compared to the static measurement in order to examine the method's effect on time-varying parameters.

2. Problem Formulation

2.1. Dynamic model

It is supposed that the mechanical system is moving in the vertical plane of symmetry when shooting and consists of three rigid parts, namely, recoiling part B_3, tipping part B_2 and carriage part B_1.

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Recoiling part moves linear relative to tipping part expressed as x while tipping part rotates relative to carriage part expressed as $\theta+\beta$. The whole artillery's motion is expressed as the horizontal motion ξ, the vertical movement η and the rotation α. The external force is made up of the gravity acting on each rigid-body, the breech force P_{KH} working on B_3 and the constraining force coming from C and D points on the ground. The internal force consists of the R_f between B_3 and B_2 and the M_c between B_2 and B_1. So, the dynamic model can be simplified into three-rigid-body structure and is shown in figure 1:

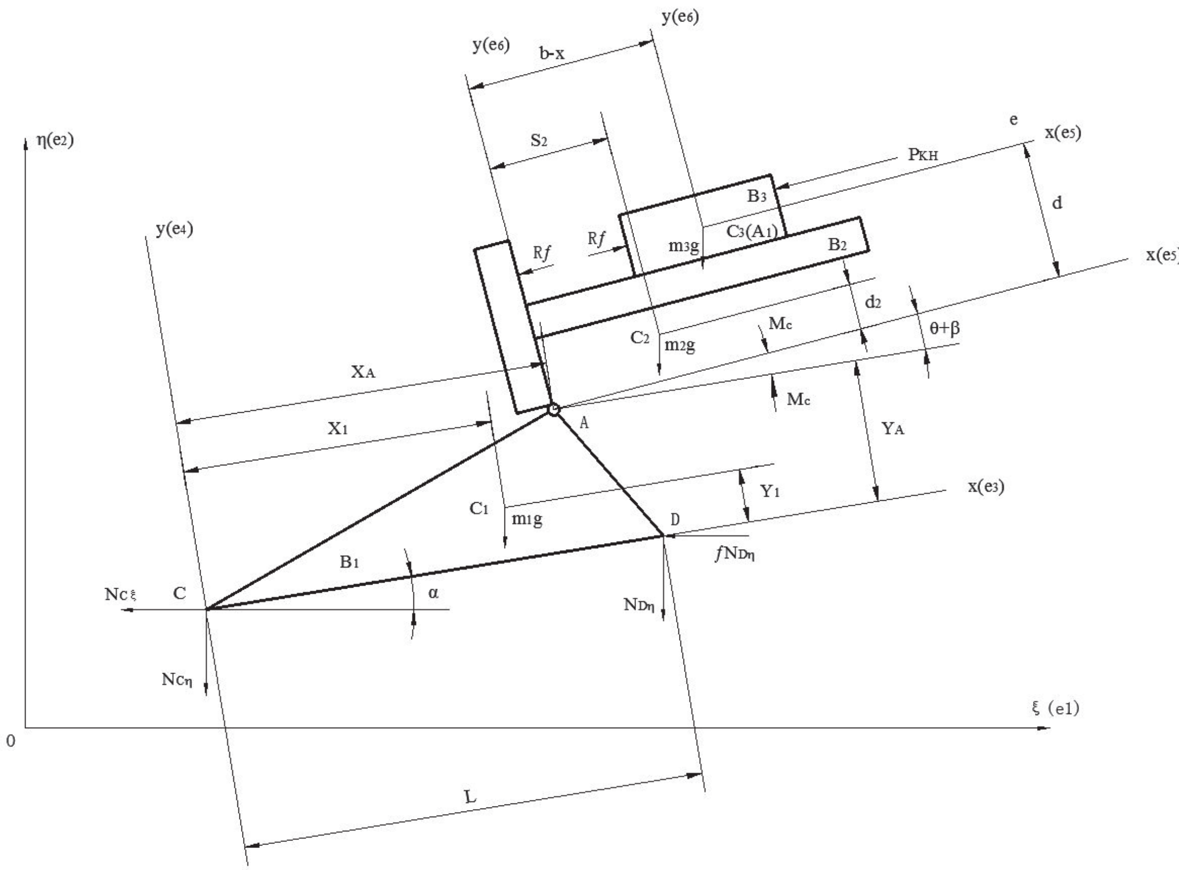

Figure 1. The dynamic model of the artillery.

In figure 1, $O\xi\eta$ is chosen as the inertial reference frame; the points (C_1, C_2, C_3) are centre of mass on the rigid body (B_1, B_2, B_3); the points (C, A, A_1) are the base points; the coordinate system (CXY, AXY, A_1XY) is built on these base points; the unit base vector is expressed as e_1, e_2, e_3, e_4, e_5, e_6 and e_7, where e_7 is established by right hand rule; m_1, m_2, m_3 represents each rigid body's mass; J_1, J_2, J_{3C} is the rotational inertia relative to C, A, A_1; the force $N_{C\xi}$ and $N_{C\eta}$ is the constraining force coming from the ground.

2.2. Dynamic equations

The whole artillery dynamic equations are established as equation 1:

$$A\ddot{q}=F \tag{1}$$

where,

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{15} \\ a_{21} & a_{22} & \cdots & a_{25} \\ \cdots & \cdots & \cdots & \cdots \\ a_{51} & a_{52} & \cdots & a_{55} \end{bmatrix} \tag{2}$$

$$q=\begin{bmatrix} \xi & \eta & \alpha & \beta & x \end{bmatrix}^T \tag{3}$$

$$F=\left[-\left(Q_1+f_1\ \right)\ \ -\left(Q_2+f_2\ \right)\ \ -\left(Q_3+f_3\ \right)\ \ -\left(Q_4+f_4\ \right)\ \ -\left(Q_5+f_5\ \right)\right]^T \tag{4}$$

where,

$$Q_1=-P_{KH}\cos\phi_2-N_{C\xi}-fN_{D\eta} \tag{5}$$

$$Q_2=-P_{KH}\sin\phi_2-\left(m_1+m_2+m_3\right)g-N_{C\xi}-N_{D\eta} \tag{6}$$

$$\begin{aligned} Q_3=&P_{KH}\left[e+d-X_A\sin\left(\phi_2-\phi_1\right)+Y_A\cos\left(\phi_2-\phi_1\right)\right]- \\ &m_1g\left(X_1\cos\phi_1-Y_1\sin\phi_1\right)-\left(m_2+m_3\right)g\left(X_A\cos\phi_1-Y_A\sin\phi_1\right)- \\ &m_2g\left(s_2\cos\phi_2-d_2\sin\phi_2\right)-m_3g\left[\left(b-x\right)\cos\phi_2-d\sin\phi_1\right]- \\ &N_{D\eta}L\cos\phi_1+fN_{D\eta}L\sin\phi_1 \end{aligned} \tag{7}$$

$$\begin{aligned} Q_4=&P_{KH}\left(e+d\right)-m_2g\left(s_2\cos\phi_2-d_2\sin\phi_2\right)- \\ &m_3g\left[\left(b-x\right)\cos\phi_2-d\sin\phi_1\right]-M_C \end{aligned} \tag{8}$$

$$Q_5=P_{KH}-R_f+m_3g\sin\phi_2 \tag{9}$$

$$\begin{aligned} f_1\ =&\left\{\left(m_1X_1+m_2X_A+m_3X_A\right)\cos\phi_1-\left(m_1Y_1+m_2Y_A+m_3Y_A\right)\sin\phi_1+\right. \\ &\left.\left[m_2s_2+m_3\left(b-x\right)\right]\cos\phi_2-\left(m_2d_2+m_3d\right)\sin\phi_2\right\}\dot{\alpha}^2+ \\ &\left\{\left[m_2s_2+m_3\left(b-x\right)\right]\cos\phi_2-\left(m_2d_2+m_3d\right)\sin\phi_2\right\}\left(\dot{\beta}^2+2\dot{\alpha}\dot{\beta}\right)- \\ &2m_3\dot{x}\left(\dot{\alpha}+\dot{\beta}\right)\sin\phi_2 \end{aligned} \tag{10}$$

$$\begin{aligned} f_2\ =&\left\{\left(m_1X_1+m_2X_A+m_3X_A\right)\sin\phi_1-\left(m_1Y_1+m_2Y_A+m_3Y_A\right)\cos\phi_1+\right. \\ &\left.\left[m_2s_2+m_3\left(b-x\right)\right]\sin\phi_2-\left(m_2d_2+m_3d\right)\cos\phi_2\right\}\dot{\alpha}^2+ \\ &\left\{\left[m_2s_2+m_3\left(b-x\right)\right]\sin\phi_2-\left(m_2d_2+m_3d\right)\cos\phi_2\right\}\left(\dot{\beta}^2+2\dot{\alpha}\dot{\beta}\right)- \\ &2m_3\dot{x}\left(\dot{\alpha}+\dot{\beta}\right)\cos\phi_2 \end{aligned} \tag{11}$$

$$\begin{aligned} f_3\ =&\left\{\left[m_2s_2+m_3\left(b-x\right)\right]\left[X_A\sin\left(\phi_2-\phi_1\right)-Y_A\cos\left(\phi_2-\phi_1\right)\right]+\right. \\ &\left.\left(m_2d_2+m_3d\ \right)\left[X_A\cos\left(\phi_2-\phi_1\right)+Y_A\sin\left(\phi_2-\phi_1\right)\right]\right\}\left(\dot{\beta}^2+2\dot{\alpha}\dot{\beta}\right)+ \\ &2m_3\left[X_A\cos\left(\phi_2-\phi_1\right)+Y_A\sin\left(\phi_2-\phi_1\right)+\left(b-x\right)\right]\dot{x}\left(\dot{\alpha}+\dot{\beta}\right) \end{aligned} \tag{12}$$

$$f_4 = -\left\{\left[m_2 s_2 + m_3(b-x)\right]\left[X_A \sin(\phi_2-\phi_1) - Y_A\cos(\phi_2-\phi_1)\right] + \left(m_2 d_2 + m_3 d\right)\left[X_A\cos(\phi_2-\phi_1) + Y_A\sin(\phi_2-\phi_1)\right]\right\}\dot{\alpha}^2 + 2m_3(b-x)\dot{x}\left(\dot{\alpha}+\dot{\beta}\right) \tag{13}$$

$$f_5 = -m_3\left[X_A\cos(\phi_2-\phi_1) + Y_A\sin(\phi_2-\phi_1)\right]\dot{\alpha}^2 - m_3(b-x)\left(\dot{\alpha}+\dot{\beta}\right) \tag{14}$$

3. Field Experiment

In order to identify the time-varying system parameters, like time-varying mass, a field experiment is designed and conducted.

3.1. Experimental equipment

Experiment Equipment mainly consists of two parts, the data collection system and the high-speed photography system.

The data collection system consists of acceleration sensors, angular acceleration sensors, pressure sensors, charge amplifiers, Dewetron data collector, Dewetron software and so on.

The high-speed photography system consists of an IDT Y3-S2 high-speed camera, a high-light LED light, a nikkor 50mm/1.2D camera lens, a nikkor 400mm/2.8D camera lens, ProAnalyst dynamic target capture software and some other ancillary equipment.

3.2. Experiment results

The angular velocity of the recoiling part is shown in figure 2. The active force of the artillery system is shown in figure 3.

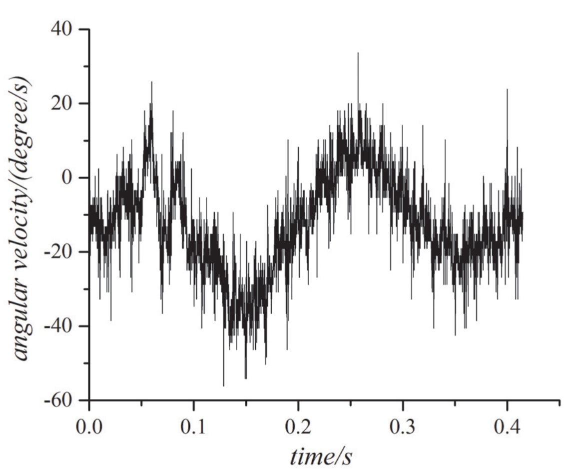

Figure 2. The angular velocity of the recoiling part.

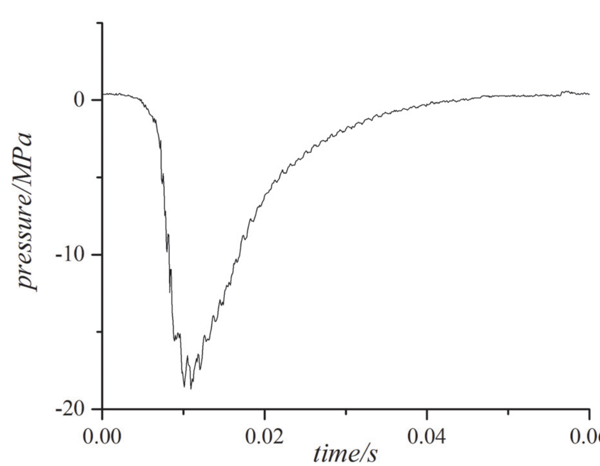

Figure 3. The active force of the artillery system.

4. Numerical calculation

4.1. Identification method

The model can be expressed in equation 15:

$$A\left(z^{-1}\right)y(k) = B\left(z^{-1}\right)y(k-d) + \xi(k) \tag{15}$$

where $\xi(k)$ is white noise, and the structure parameters m, n, d are known.

$$\begin{cases} A\left(z^{-1}\right) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m} \\ B\left(z^{-1}\right) = b_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n} \end{cases} \tag{16}$$

The target of identification is to establish m+n+1parameters according to the measurable input and output. The model can be described in least square method and is expressed in equation 17:

$$\begin{aligned} y(k) &= -a_1 y(k-1) - \cdots - a_{n_a} y(k-n_a) + b_0 u(k-d) + \cdots + b_{n_b} u(k-d-n_b) + \xi(k) \\ &= \varphi^T(k)\theta + \xi(k) \end{aligned} \tag{17}$$

where $\varphi(k)$ is the data vector, θ is the parameter need to be identified.

$$\begin{cases} \varphi(k) = \left[-y(k-1), \cdots, -y(k-n_a), u(k-d), \cdots, u(k-d-n_b)\right]^T \in R^{(n_a+n_b+1)\times 1} \\ \theta \;\; = \left[a_1, \cdots, a_{n_a}, b_0, \cdots, b_{n_b}\right]^T \in R^{(n_a+n_b+1)\times 1} \end{cases} \tag{18}$$

So, the identification equation of the rigid body artillery model can be expressed in equation 19:

$$m_3 \ddot{q}' = F' \tag{19}$$

4.2. Identification results

The time-varying identified mass of the recoiling part is illustrated in figure 4:

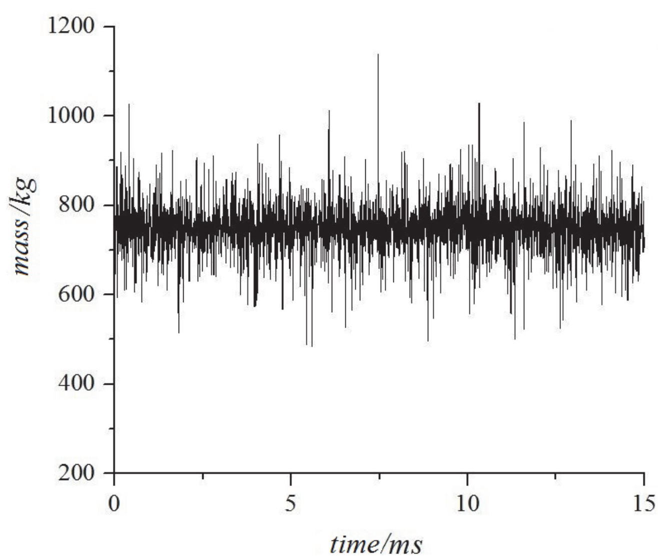

Figure 4. The time-varying identified mass of the recoiling part.

A relative percentage error (RPE) is defined to compare the result of the identification which is expressed in equation 20:

$$RPE = \frac{\Sigma\left|m_{true} - m_{identify}\right|}{\Sigma\left|m_{true}\right|} \times 100\% \tag{20}$$

In figure 4, the identified mass of the recoiling part is about 740kg. After comparing with the “real” mass which was consulted in artillery specification book, the *RPE* is finally established at about 5.4%.

5. Conclusions

In this paper, the multi-rigid-body theory and the limited memory least square method is applied to identify the time-varying mass of the recoiling part.

The mass of the recoiling part has physical significance so that the result of identification can be compared to the static measurement which is consulted in artillery specification book. Limited memory least square method can overcome “data saturation” phenomenon effectively and it can be applied to time-varying identification in this situation. The result reflects that limited memory least square method and the process of establishing the artillery’s dynamic equations are accurate in identifying the time-varying mass. At the same time, a new kind of identification method is provided to make the parameters more intuitive and can be applied in identifying dynamic loads which is difficult to measure.

At the present stage, the dynamic model is described as rigid body model. In the future, this model would be instead of flexible multi-body model to make the equations more accurately. And more parameters would be identified such as the mass and the stiffness of the structure, the active force from the propellant powder, the inertia force of the artillery.

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