The Exact Nonlinear Dynamics. New Bifurcation Groups with Chaos and Rare Attractors

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Abstract. New nonlinear models which allows to obtain exact periodic, quasi-periodic and chaotic attractors and transients processes, are proposed. The idea of "exact" models has its corner-stone in description of restoring, damping and excitation forces only by constant forces on each its linear sub-system. Examples with a system with one potential well, two potential wells and pendulum systems are discussed. We suppose that it will be possible finding the exact simple analytical formulae for periodic, quasi-periodic and chaotic dynamics in different nonlinear models.

1. Introduction

A new type of the simplest models of essentially nonlinear damped parametrically-excited or forced systems, suggested by the author, is presented. The main idea of the so-called "exact model" is using for its description a qualitatively "equivalent" piece-wise linear model with only constant forces on each its linear sub-system. For "exact" model all restoring and driven forces are changed into piece-wise constant linear forces or impulse forces. For periodic excitation in exact nonlinear dynamics may be used instant periodic impulses, if necessary. Damping in the exact models is introduced by models of impact damping or by models with dry friction. So to obtain the exact solution in a linear sub-system of the exact model, and to find its switching times, only quadratic equations are necessary. It's important to underline that the simplicity of finding switching times stays the same for systems with several or even many degrees-of-freedom.

Exact models allow finding and investigating all main nonlinear phenomena in typical nonlinear models using Poincaré ideas, well-known theory of piecewise linear systems and their stability evaluation. The main advantages of "exact" models are their simplicity, speed and exactness. Some results are qualitatively compared with known results for different non-linear systems with parametrical or external excitation. We suppose, that all known nonlinear effects may be illustrated by exact nonlinear models.

We illustrate (in this presentation) the main ideas of the exact nonlinear dynamics for three driven damped one DOF models. The first system (see Figure 1) is a bilinear relay symmetric system, for what we compare two models usual and "exact". The second model is three-well potential exact driven damped system (see Figures 2-3) where a chaotic attractor for exact model is shown. The last example describes nonlinear exact dynamics for pendulum system with rectangle restoring force, impact damping and rectangle sine excitation force.

We suppose that the exact nonlinear dynamics may be very useful for systems with several and many degrees of freedom. In particular, for chain system and pendulum system with n-degree of freedom. For such complex system it will be possible to find some new interesting unknown behavior, new bifurcation groups with rare attractors and unknown topology.

The main idea of the exact of nonlinear dynamics may be found in works [1-9] and in some other publications.

We hope that the most interesting and important result of this presentation, is the possibility of obtaining the exact simple analytical formulae for chaotic dynamics in different models.

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2. Bilinear relay system with usual and exact dynamics (see Figure 1)

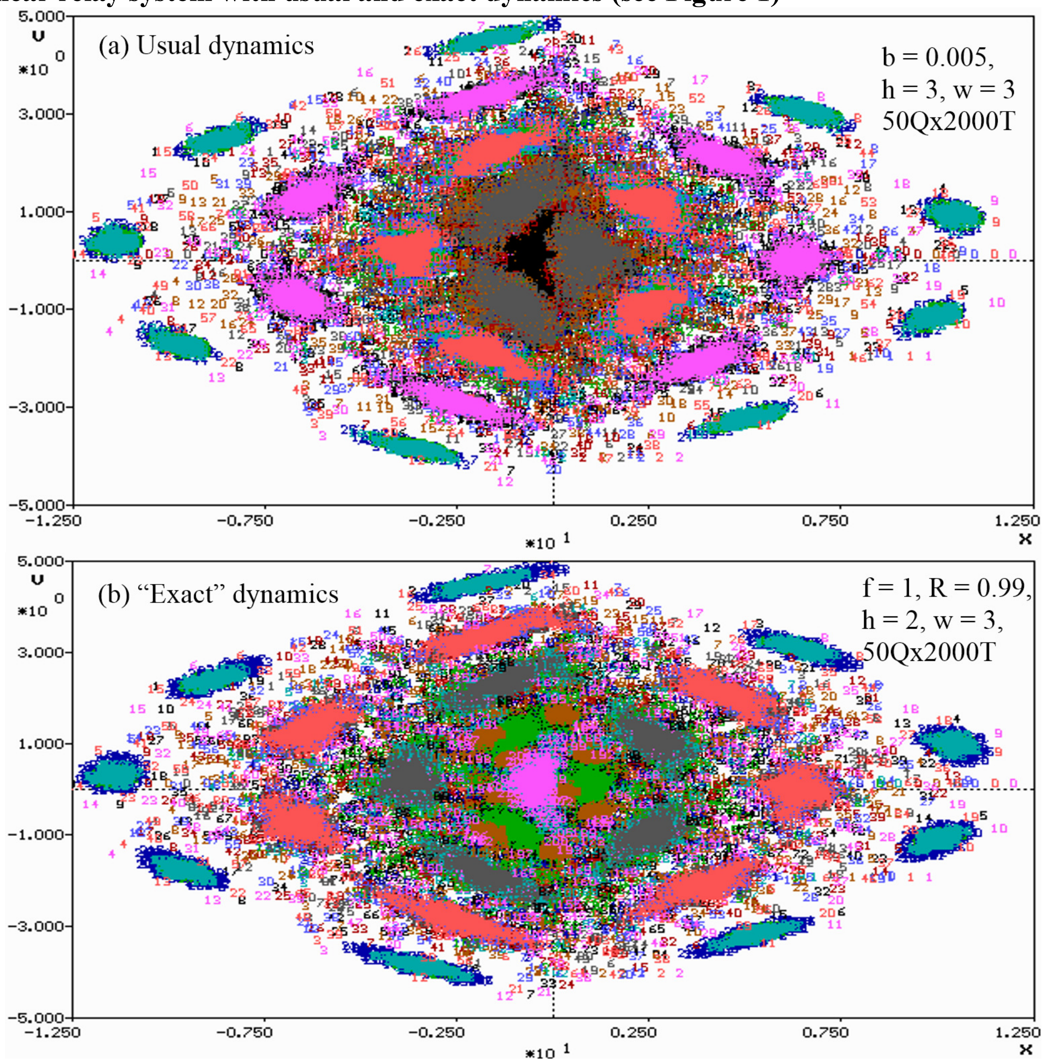
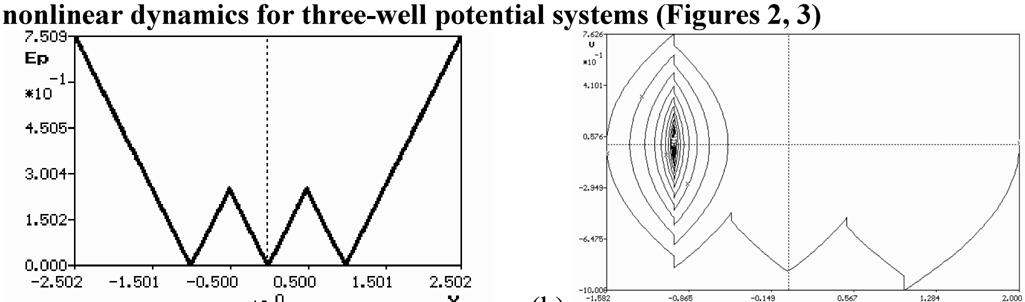


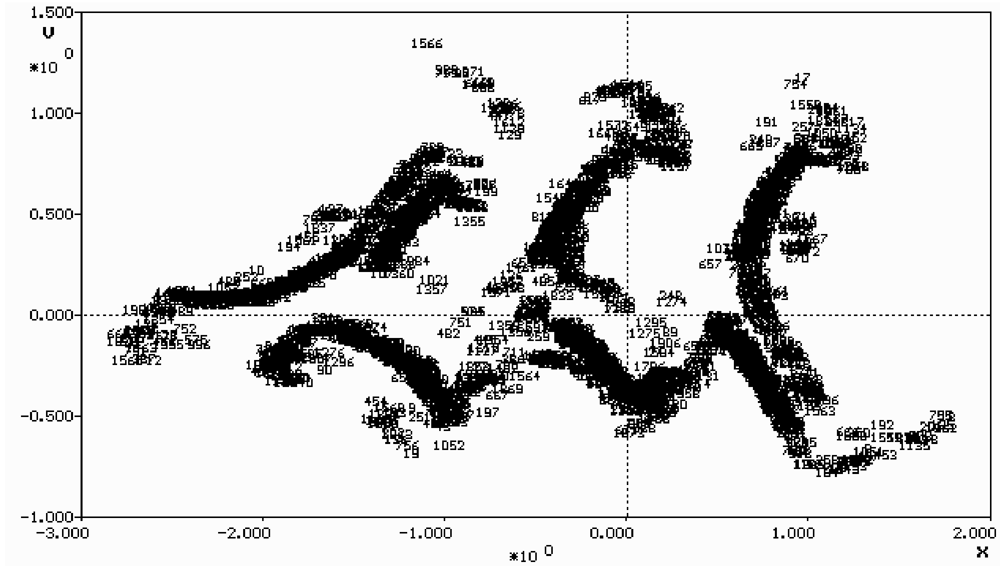
Figure 1. (a) Bilinear relay symmetric system with linear damping and harmonical excitation, Poincaré mapping 50Qx2000T. Parameters: $c = 0$, $f = 1$, $b = 0.005$, $h = 3$, $w = 3$. (b) (“exact” dynamics) bilinear relay symmetric system with impact damping and rectangle excitation. Poincaré mapping 50Qx2000T. Parameters: $c = 0$, $f = 1$, $R = 0.99$, $h = 2$, $w =3$. Systems have period P1, P3, P5, P7 and P9 attractors.

3. Exact nonlinear dynamics for three-well potential systems (Figures 2, 3)

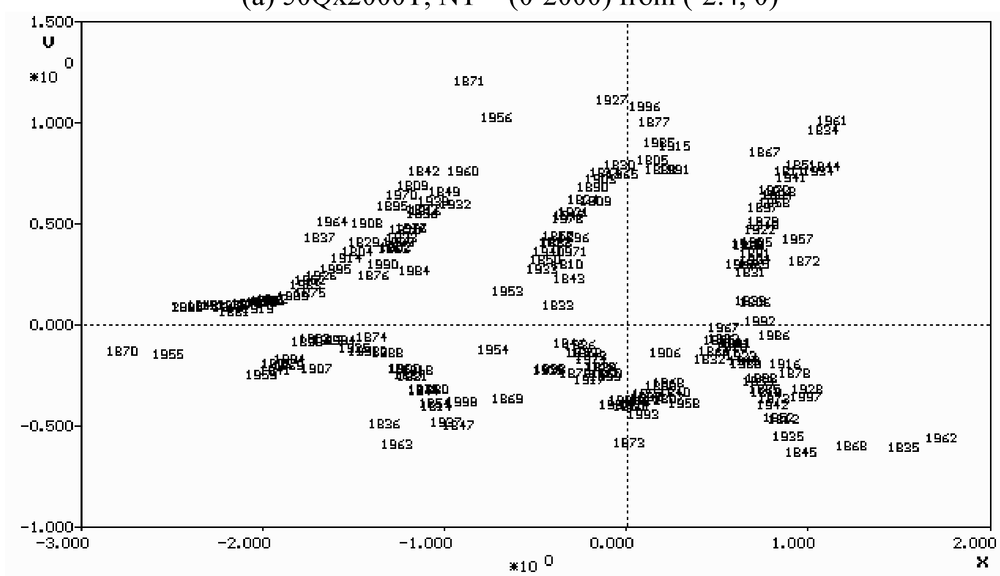


(a) (b)

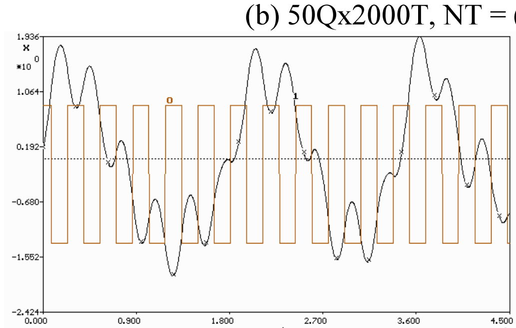
Figure 2. (a) Three-well potential system with impact damping and rectangle sine excitation; (b) free oscillation. Parameters: $d_1 = 0.5$, $d_2 = 1$, $R = 0.9$.

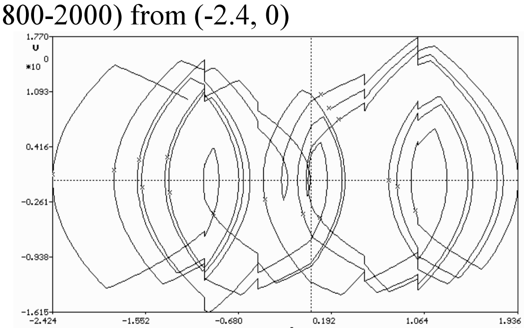


(a) 50Qx2000T, NT = (0-2000) from (-2.4, 0)



(b) 50Qx2000T, NT = (1800-2000) from (-2.4, 0)

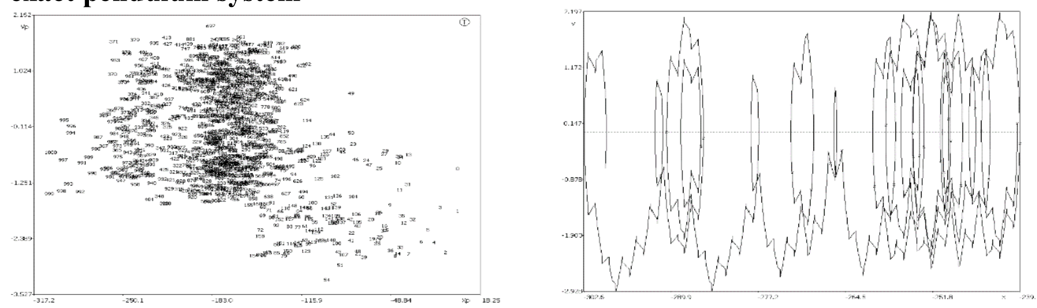




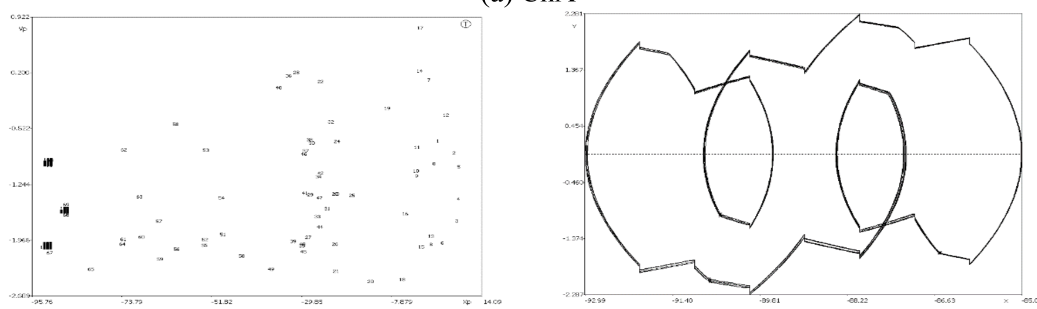
(c) (d)

Figure 3. Pendulum-like symmetrical driven system permitted exact solutions. Restoring symmetrical piece-wise $f(x)$ has $c_i = 0$ and driven rectangle sine periodical force $H(wt)$. Damping are introduced by impacts with coefficients of restitution R. Parameters: $d_1 = 0.5$, $d_2 = 1$, $R = 0.9$, $\omega = 2$.

4. The exact pendulum system



(a) ChA



(b) P3 (3/3)

Figure 4. Poincaré mappings and phase diagrams for the "exact" pendulum system with rectangle restoring force, impact damping and rectangle sine excitation. Parameters and initial conditions: $R = 0.95$, $f_1 = 1$, $l = 1$, $h_1 = 0.7$, $x_0 = 3.0$, $v_0 = -1.0$; (a) chaotic attractor ChA, $\omega = 0.7$, (b) period-three P3 (3/3) subharmonic attractor for $\omega = 1.0$.

5. Conclusions

New nonlinear models which allows to obtain exact periodic, quasi-periodic and chaotic attractors and transients processes, are proposed. The idea of "exact" models has its corner-stone in description of restoring, damping and excitation forces only by constant forces on each its linear sub-system. Examples with a system with one potential well, two potential wells and pendulum systems are discussed. We suppose that it will be possible finding the exact simple analytical formulae for periodic, quasi-periodic and chaotic dynamics in different strongly nonlinear models.

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