Wave Displacement

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Abstract. Here the system displaced by waves with two degrees of freedom is analyzed when one member of the system, while contacting with the working profile of the input member performing wave motion, provides motion to the output system. For the analytical investigation modification of the asymptotic method is used, which is based on the division of motion into the slow and quick motions. This method is justified when the frequencies of variation of slow motions are much smaller than the frequencies of quick motions. More complicated cases are investigated by numerical methods.

1. Introduction

The conveyance of particles and bodies by propagating waves is an important scientific and engineering problem with numerous applications [1-6]. This is important in a number of mechanical engineering applications. For the case of Rayleigh harmonic waves the investigations are performed in more detail.

2. The model of the system

The model of the system is presented in figure 1, where:

$$A_1(u,\xi),\ A_2(u,v),\ B_1(u,v+v_1),\ u=x+\eta,\ v=\xi. \tag{1}$$

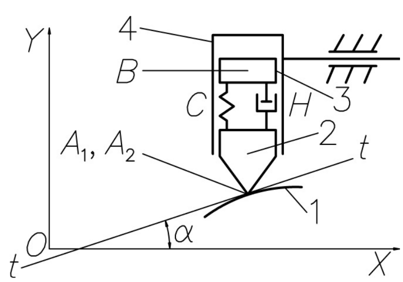

Figure 1. The model of the system:
1 – the working profile of the input member,
2 – contact member of the output system,
3 – member may move with respect to member 4 according to the coordinate *OY*.

The working profile of the input member 1 is given by:

$$\eta(x,t),\ \xi(x,t), \tag{2}$$

and the components of velocities of their contact point A_1 are:

$$\eta_t',\ \xi_t'. \tag{3}$$

The components of velocities and accelerations of contact point A_2 of member 2 read:

$$\dot{u}=\left(1+\eta_x'\right)\dot{x}+\eta_t',\ \dot{v}=\xi_x'\dot{x}+\xi_t', \tag{4}$$

$$\ddot{u}=\left(1+\eta_x'\right)\ddot{x}+\eta_{xx}''\dot{x}^2+2\eta_{xt}''\dot{x}+\eta_{tt}'',\ \ddot{v}=\xi_x'\ddot{x}+\xi_{xx}''\dot{x}^2+2\xi_{xt}''\dot{x}+\xi_{tt}'',\ \dot{}=d/dt. \tag{5}$$

The angle α of the tangent at the point of the profile A_1 with the axis *OX*:

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$$\tan\alpha = \frac{\xi'_x}{1+\eta'_x}. \tag{6}$$

The velocity of slippage of point A_2 with respect to point A_1 in the tangential direction takes the form:

$$\dot{s}_{21} = \sqrt{a}\dot{x},\ a = \left(1+\eta'_x\right)^2 + \xi'^2_x. \tag{7}$$

Differential equations of motion are:

$$\left(1+\eta'_x\right)I_u + \xi'_x I_v + \left[-\xi'_x I_u + \left(1+\eta'_x\right)I_v\right] f_0 \operatorname{sgn}\dot{s}_{21} + h\alpha\dot{x} = 0, \tag{8}$$

$$\ddot{v} + \ddot{v}_1 + g + f_v = 0, \tag{9}$$

where:

$$I_u = \ddot{u} - p_u,\ I_v = \mu_2\left(\ddot{v}+g\right) - \mu_3 f_v,\ \ p_u = \frac{P_u}{m_2+m_3+m_4},\ \mu_2 = \frac{m_2}{m_2+m_3+m_4},\ \mu_3 = \frac{m_3}{m_2+m_3+m_4},$$

$$f_v = \frac{F_v}{m_3} = h_1\dot{v}_1 + n^2 v_1,\ h_1 = \frac{H}{m_3},\ n^2 = \frac{C}{m_3},$$

m_2, m_3, m_4 are masses of the members 2, 3 and 4, g is acceleration of gravity of the earth, P_u is the external force acting to the member 4 according to the coordinate OX, f_0, f_1 are the coefficients of dry and viscous friction between the members 1 and 2,

$$h = \frac{f_1}{m_2+m_3+m_4}. \tag{10}$$

3. Investigation of dynamics

For the case of harmonic travelling waves:

$$\eta = A\cos\left(\omega t - kx\right),\ \xi = B\sin\left(\omega t - kx\right), \tag{11}$$

or by denoting:

$$\beta = k\psi,\ \beta_1 = kv_1,\ a = kA,\ b = kB,\ \psi = \omega t - \beta, \tag{12}$$

it is obtained:

$$\eta'_x = a\sin\psi,\ \xi'_x = -b\cos\psi,\ \ k\ddot{u} = -\ddot{\psi} - a\left(\ddot{\psi}\sin\psi + \dot{\psi}^2\cos\psi\right),\ k\ddot{v} = b\left(\ddot{\psi}\cos\psi - \dot{\psi}^2\sin\psi\right). \tag{13}$$

When $p_u = f_0 = 0$ by taking into account equations (8)-(13) the differential equations of motion become the following ones:

$$\begin{aligned}-\left(1+a\sin\psi\right)\left[\ddot{\psi} + a\left(\ddot{\psi}\sin\psi + \dot{\psi}^2\cos\psi\right)\right] - \left[\mu_2 b\left(\ddot{\psi}\cos\psi - \dot{\psi}^2\sin\psi\right) + \mu_2 kg - \mu_3 kf_v\right] b\cos\psi \\ + h\left[\left(1+a\sin\psi\right)^2 + b^2\cos^2\psi\right]\left(\omega - \dot{\psi}\right) = 0,\end{aligned} \tag{14}$$

$$b\left(\ddot{\psi}\cos\psi - \dot{\psi}^2\sin\psi\right) + \ddot{\beta}_1 + kg + kf_v = 0, \tag{15}$$

where:

$$kf_v = h_1\dot{\beta}_1 + n^2\beta_1. \tag{16}$$

When $b = 1.5a$, $\frac{h}{\omega} = 0.5$, $\frac{kg}{\omega} = 2$, $f_v = h_1 = n^2 = \mu_3 = 0$ slowly changing $a = a(\varepsilon\tau)$ graphical relationships have been obtained $\psi' = \psi'(a)$, $\bar{\psi}' = \bar{\psi}'(a)$, $\psi'_{max} - \psi'_{min} = \Delta\psi'$, $\psi' = \psi'(\psi'')$, which reflect the dynamical qualities of the system. The obtained results are presented in the figures 2, 3, 4.

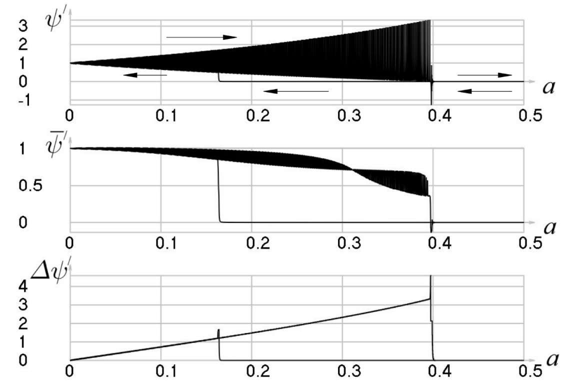

Figure 2. Steady state motions of the system.

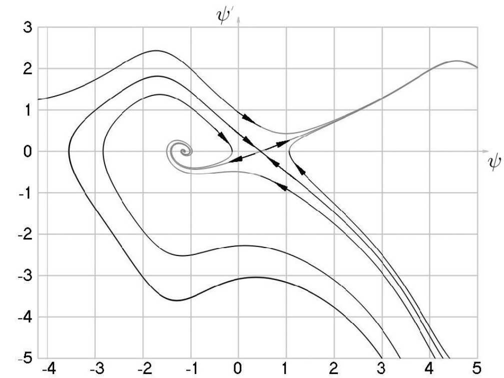

Figure 3. Transition to the steady state process when $a = 0.25$, $\frac{h}{\omega} = 0.5$, $\frac{kg}{\omega} = 2$ from the unstable point $\bar{\psi} = 1.08825$, $\Psi'_{\bar{\psi}} = 0.748257$ and from the stable point $\bar{\psi} = 4.5421$, $\Psi'_{\bar{\psi}} = -0.794566$, phase trajectories of motion and attractors.

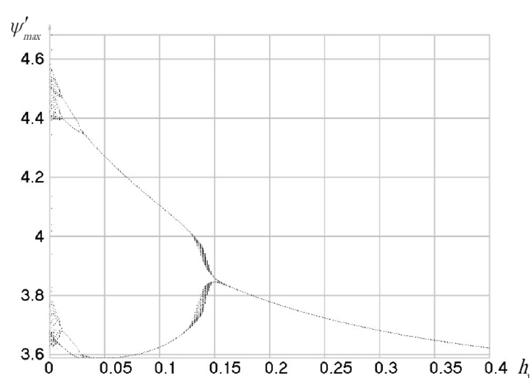

Figure 4. Bifurcation diagram when
$a = b = 0.6$,
$h = 0.4$,
$n^2 = 1$,
$\mu_2 = \mu_3 = 0.4$,
$\omega = 1$,
$kg = 0.98$.

Case 1: system displaced at the velocity of the wave, that is:

$$\omega - \dot{\beta} = 0, \text{ or } \psi = \bar{\psi}, \tag{17}$$

where $\bar{\psi}$ is a slowly varying quantity, in case of stationary motion $\bar{\psi} = \text{const}$.

From the equations (14)-(16) it is obtained:

$$-kg(\mu_2 + m_3)b\cos\bar{\psi} + h\omega\left[(1 + a\sin\bar{\psi})^2 + b^2\cos^2\bar{\psi}\right] = 0. \tag{18}$$

According to the equations (17), (14) it is determined that there may exist 0 or 2 or 4 stationary regimes. From the latter ones half of them are stable and half are unstable.

Case 2: the system is displaced by a very slow velocity if compared with the velocity of the exciting wave, that is:

$$\psi = (\omega - \varepsilon)t + \bar{\psi} + \tilde{\psi}, \tag{19}$$

where $\varepsilon \ll \omega$, $\bar{\psi}$ is a slowly varying quantity and $\tilde{\psi}$ is a quickly varying quantity. Also the system is far from parametric resonances.

For the determination of quick motions linear differential equations with constant coefficients are obtained. The latter are obtained from the main equations (14), (15), (19) by assuming in them that the dash over separate members denotes averaging with respect to time and the wave over the members means that their average values with respect to the period of time are equal to zero. Nonlinear autonomous differential equations are obtained for the determination of slow motions by assuming in the initial equations (14), (15) ψ and performing linearization over one time period.

Case 3: the system moves by any average velocity $\bar{\dot{\beta}} = 0 \div \omega$.

Case 4:

$$\eta = 0, \ \xi = B \sin kx. \tag{20}$$

In this case the differential equations of motion on the basis of the equations (20), (8), (9) are the following ones:

$$\ddot{\beta} - kp_u + \left[\mu_2 b\left(\ddot{\beta}\cos\beta - \dot{\beta}^2 \sin\beta\right) + \mu_2 g - \mu_3 f_v\right] b\cos\beta + hb^2 \dot{\beta}\cos^2\beta = 0, \tag{21}$$

$$b\left(\ddot{\beta}\cos\beta - \dot{\beta}^2 \sin\beta\right) + \ddot{\beta}_1 + k\left(g - f_v\right) = 0, \ \ kp_u = D - E\dot{\beta}, \tag{22}$$

where kf_v is determined according to the equation (16).

Stationary combined synchronous regimes are determined analytically. Limits of regions of their existence are determined by numerical methods, because they are described by nonlinear differential equations with parametric excitation.

4. Conclusions

Approximate analytical modified asymptotic method is used for the investigation of the stationary motions of the output system, which is based on the division of motion into the slow and quick motions.

Bifurcations, complicated motions and control of the system as well as limits of existence of stationary regimes have been determined by using analytical and numerical methods.

References

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