Simulation Model of Polyurethane Foam for Uniaxial Dynamical Compression

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Abstract. Article deals with definition of model of mechanical properties of polyurethane foam with lumped parameters. The problem is considered from the car driver vibroisolation point of view. Model definition comprises restoring force, damping and frictional forces. It is verified for experimental results obtained from measurement of PU foam specimen with open cells. Apart from empirical way of model constants setting the optimization method of genetic algorithms is used for more simple phenomena.

1. Introduction

Polyurethane (PU) foam is a very extensively used material in vibroinsulation field and it is used largely in the field of automobile seat cushioning production. For computer simulation of statistical or dynamical behavior of seat with respect to passenger's comfort it is necessary to describe the properties of PU foam in sufficient degree which this paper deals with.

Properties of polyurethane foam were investigated on opened cells PU foam specimen of cubic shape with size (100×100×50)mm, density 55kg/m^3, made from material TDI.

The specimen was inserted into two parallel rigid plates and deformed by means of hydraulic actuator. Kinematic displacement excitation $x(t)$ has a triangle shape with constant amplitude A=19.5mm, constant mean value A_0=A=19.5mm and frequency varied in range f=(0.01÷1.28)Hz. It is given by equation (1), where T=1/f is period of excitation and n=0, 1, 2, ..., ∞ means number of periods (see figure 1).

$$x(t)=\begin{cases}\frac{4A}{T}\left(t-nT\right) for\ t\in\left\langle nT;nT+T/2\right\rangle \\ -\frac{4A}{T}\left[t-\left(n+1\right)T\right]\ for\ t\in\left\langle nT+T/2;\left(n+1\right)T\right\rangle\end{cases}. \tag{1}$$

2. Definition of PU foam model

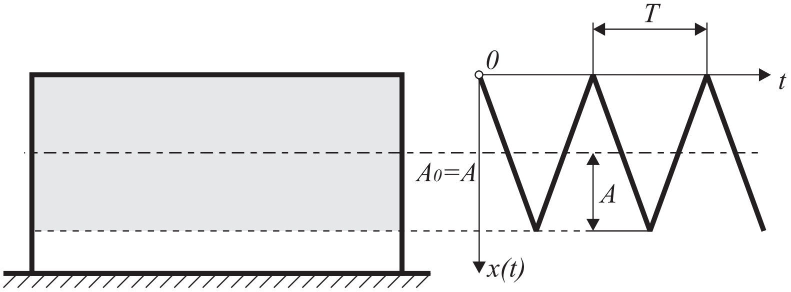

Figure 1. Triangle kinematic excitation of PU foam specimen.

Development of material model of polyurethane foam is documented by many publications. One group of authors uses approach of mechanics of continuum [1, 2] while second one, more often, uses method of models with lumped parameters or strictly mathematical design (e.g. [3, 4, 5]). The level of

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complexity in some cases differs considerably.

Herein described design of lumped parameters model uses phenomenological approach. It comprises nonlinear restoring force F_R, damping force F_d and frictional force F_f in general force response F:

$$F(x,\dot{x}) = F_R(x) + F_d(x,\dot{x}) + F_f(x,\dot{x}) \tag{2}$$

2.1. Restoring force

Pores of PU foam create a typical material structure which is up to a certain degree able to resist to pressure loading due to its buckling strength. In figure 2 there is pictured a graduating deformation of loaded cell. Undeformed shape of cell is in figure 2a), partially deformed in figure 2b), and figure 2c) corresponds with state where cells are deformed in high rate and their buckling strength marked F_{b0} has already been overcame and further stops to influent total restoring force F_R. This phase is approximated by degressive function (3). F_b is a force evoked by buckling strength of foam cells, c_b is a coefficient of the structure buckling strength.

$$F_b = F_{b0}\left(1 - e^{-c_b x}\right) \tag{3}$$

After the initial cells crush with increasing compression there comes to contact between cell walls. Force characteristics of this phase is very similar to course of force arising during compression of ideal gas in closed vessel. This is described by progressive polytrophic function (4). S_p, p_p, h_p, n_p are constants of model, where h_p means the vertical asymptote position in figure 4. Total restoring force is than given by equation (5).

$$F_P = p_P S_P \left[\left(\frac{h_P}{h_P - x} \right)^{n_P} - \left(\frac{h_P}{h_P + x} \right)^{n_P} \right] \tag{4}$$

$$F_R = F_b + F_p \tag{5}$$

2.2. Viscous damping

It can be assumed that total damping of compressed PU foam with opened cells is caused by the material damping of polyurethane and by the damping caused by air flow through the porous material structure. In which relation these two parts of damping force are it was investigated in [6]. As it follows from the article the influence of air flow to overall damping force is possible to neglect in case of specimen cut from cushioning.

Damping of the matrix material is then described by Maxwell's viscoelastic components with nonlinear spring with polytrophic characteristics (6) with constants S_{0i}, p_{0i}, h_i, n_{0i}, and nonlinear damper with constant of damping c_i and exponent n_i defined by (7) where m=3 is a number of used Maxwell's components.

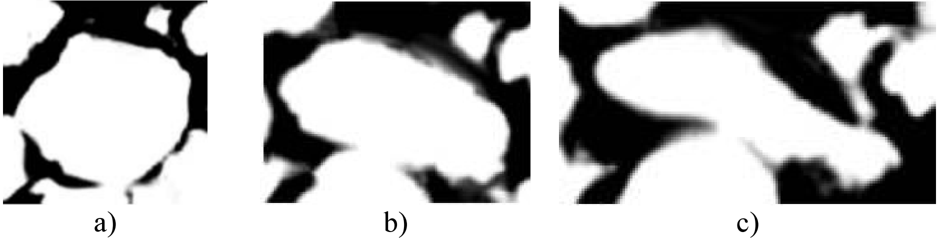

a) b) c)

Figure 2. Gradual deformation of loaded cell.

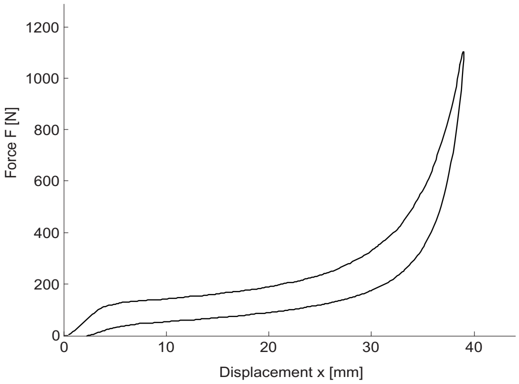
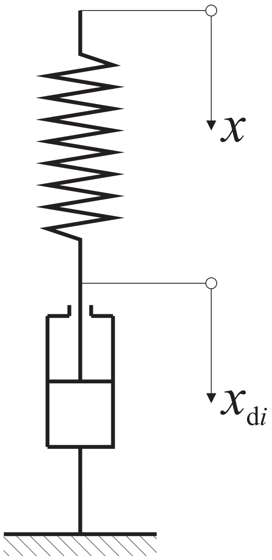


Figure 3. Typical force response to triangle kinematic excitation.



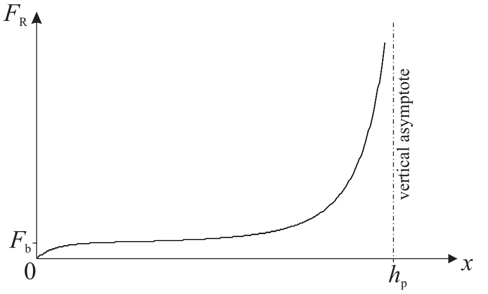


Figure 4. Course of restoring force F_R defined by (5).

Figure 5. Maxwell's component.

$$F_{di} = p_{0i} S_{0i} \left[\left(\frac{h_i}{h_i - (x - x_{di})} \right)^{n_{0i}} - \left(\frac{h_i}{h_i + (x - x_{di})} \right)^{n_{0i}} \right] \tag{6}$$

$$F_{di} = c_i \left| v_{di} \right|^{n_i} \operatorname{sgn}(v_{di}), \qquad v_{di} = \dot{x}_{di}, \qquad i = 1...m. \tag{7}$$

Differential equation of this element is given by equality of forces in serially added components:

$$p_{0i} S_{0i} \left[\left(\frac{h_i}{h_i - (x - x_{di})} \right)^{n_{0i}} - \left(\frac{h_i}{h_i + (x - x_{di})} \right)^{n_{0i}} \right] = c_i \left| \dot{x}_{di} \right|^{n_i} \operatorname{sgn}(\dot{x}_{di}) \tag{8}$$

2.3. Friction damping

With regard to the contact of cell-walls and struts of cells during compression and their mutual slipping there is logical assumption that also friction participates in PU foam damping. Friction is included in model with course of friction coefficient f_f defined in dependence on velocity $v = \dot{x}$ by function arctan in combination with power function in equation (9) which is presented in figure 6.

$$f_f = \frac{2 f_{f0}}{\pi} \arctan(k_1 v) + k_2 \left| v \right|^{k_3} \operatorname{sgn}(v) \tag{9}$$

The base value for friction force calculation is sum of restoring force F_R and force of Maxwell's viscoelastic components F_{di}:

$$F_{Rd} = F_R + \sum_{i=1}^{m} F_{di} \tag{10}$$

Friction force then is

$$F_f = f_f F_{Rd} \tag{11}$$

Total force response of PU foam model presented in figure 7 is:

$$F = F_{Rd} + F_f \tag{12}$$

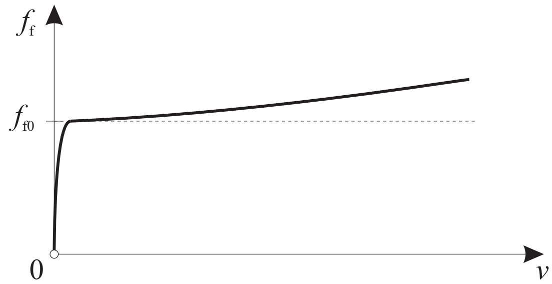


Figure 6. Dependence of friction coefficient f_f on velocity v.

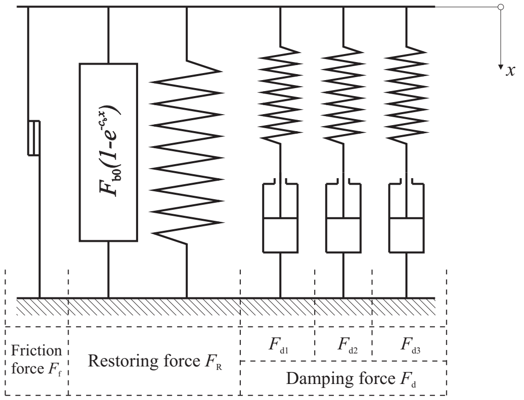


Figure 7. Scheme of PU material model for *m*=3.

Table 1. Parameters of PU foam model.

Force component	Parameter	Physical unit	Value		
F_b	F_{b0}	[N]	80		
	c_b	[N/m]	600		
F_P	S_P	[m^2]	0.0095		
	p_P	[Pa]	100		
	n_P	[1]	6.2		
	h_P	[m]	0.06		
F_{di}			*i*=1	*i*=2	*i*=3
	S_{0i}	[m^2]	1.2	0.03	0.8
	p_{0i}	[Pa]	100		
	n_{0i}	[1]	3	3.5	2
	h_i	[m]	0.05	0.05	0.18
	c_i	[-]	50	300	300
	n_i	[1]	0.2	0.2	0.2
F_f	f_{f0}	[1]	0.05		
	k_1	[s/m]	5000		
	k_2	[-]	0.2		
	k_3	[1]	1		

3. Model verification

Model with parameters mentioned in table 1 has been verified on the same courses of excitation signals as it was experimentally tested. It means triangle course of displacement $x(t)$ given by equation (1) with frequency range f=(0.01÷1.28)Hz. In figure 8 and figure 9 there is a comparison of experimentally measured total force of loaded PU foam specimen and force response of simulating model. It is possible to say that total force is simulated with satisfactory accuracy. In figure 10 and figure 11 there is measured damping force and simulated one. Also courses of damping force separated from total force are simulated in very good precision. However in case of lowest frequency f=0.01Hz simulated damping force does not match desirable values for high rates of compression. In this way depicted damping force $\overline{F}_d$ has slightly different definition from F_{di} in (6) or their summation through m, which mentioned in [7] or [8].

The problem of setting of proper material constants is very complex because of their large amount 28. Those presented in table 1 were settled empirically. Model than was exemplified as it is shown in figures 8-10. For possibility to expect better results in future or verify model for wider range of phenomena we tried to apply genetic algorithms for setting of constants. A Monte-Carlo method seems to be the only way how get a solution in a adequate time. Fitness function was defined as the area between the experimentally obtained displacement-force curve and the same curve got by the model. Genetic algorithm was implemented in C++ language with continuous variables and one parent. The offspring mutation variance was set proportional to the fitness of the parent. We obtained satisfactory results with the population of hundreds individuals in order of thousands generations. An example of experimental data and data from the model with constants found using genetic algorithm is in figure 12.

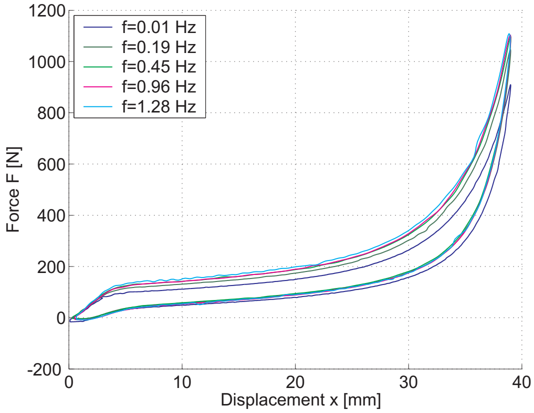


Figure 8. Measured total force F.

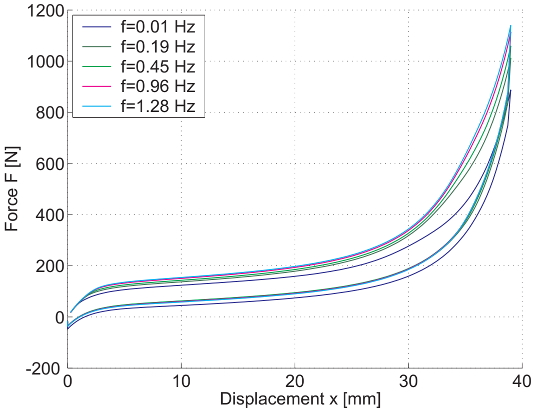


Figure 9. Simulated total force F.

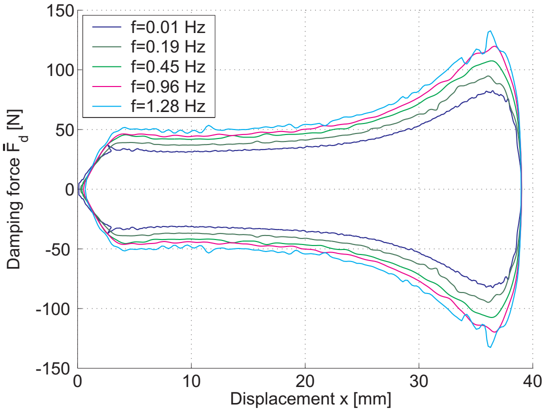


Figure 10. Measured damping force $\overline{F}_d$.

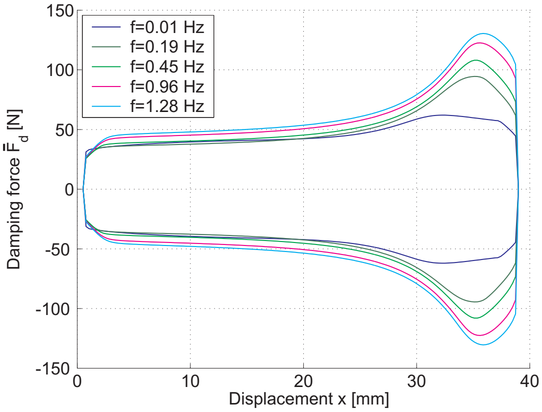


Figure 11. Simulated damping force $\overline{F}_d$.

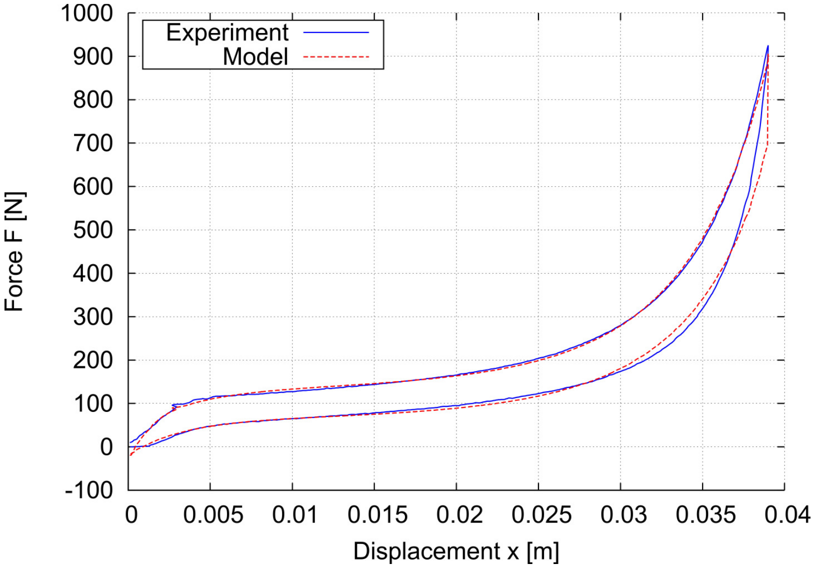


Figure 12. Simulation result obtained by means of genetic algorithms.

4. Conclusion

Rheological model of polyurethane foam for uniaxial dynamical compression was derived in this article. It was verified for triangle kinematic excitation with constant mean and amplitude and varying frequency. Model constants presented in article has been set empirically. How verification shows this kind of model definition leads to satisfactory results. However more sophisticated procedure for setting of constants is being prepared using method of genetic optimization as it exemplified in one of experimental cases.

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