909. Low frequency and high intensity ultrasound in vascular surgery: theory, instrumentation and possibilities of clinical application

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Abstract. This paper presents a brief review of applications of ultrasound in modern surgery and results of original studies of the authors in the field of application of low frequency (24-36 kHz) high-intensity (up to 20 W/cm^2) ultrasonic vibrations for disruption of thrombi and calcified atherosclerotic plaques in blood vessels. Application of non-rigid wire ultrasonic waveguides with length up to 980 mm and diameter of working tip down to 0.3 mm enables minimally invasive surgical intervention, since a waveguide can be introduced along curved segments of blood vessels through a small incision situated at substantial distance from occlusion. Ultrasonic angioplasty can be successfully applied in combination with administration of thrombolytic drugs. The paper also considers physical mechanisms of thrombus disruption under influence of ultrasonic vibrations, particularly, effects of cavitation and acoustic streaming. We described design of ultrasonic waveguides for endovascular surgery and their manufacturing technology based on plasma-electrolytic etching. Application of finite element method and transfer matrix approach for design and model of wire waveguides is considered. Description of clinical system for ultrasonic angioplasty with automated resonance tuning of a waveguide is also provided. In addition, we report results of clinical application of ultrasonic angioplasty in patients with occlusion of iliofemoral segment.

Keywords: low-frequency ultrasonic vibrations, waveguides, finite element method, thrombolytic therapy.

Introduction

Ultrasound is widely used in medicine practice including functional diagnostics, therapy and surgery, due to its ability to produce targeted and controlled action on biological tissues. In surgical applications low-frequency ultrasound of high intensity (with frequency from 20 to 80 kHz) and high-frequency ultrasound of high intensity (with frequency from 0.5 to 4.0 MHz) also known as High-Intensity Focused Ultrasound (HIFU) are used. Low-frequency ultrasound is used to excite vibrations in surgical tools and reduce invasiveness of surgical procedures, it helps to increase efficiency of surgeon's work due to the increase in cutting speed and reduction of cutting force, provides haemostatic and antiseptic effects [1-3]. Moreover, application of ultrasound enables selective dissection of tissues. High-frequency ultrasound is often used in the form of focused beam for non-invasive ablation of soft tissues, e.g. skin or brain tumors [2, 4-6]. Frequency ranges indicated above are approximate: for example, miniature surgical tools use vibrations with frequency up to 290 kHz [7]. Clinical applications of low-frequency ultrasound include cutting of soft tissues and bones, drilling of bones, fragmentation and aspiration. Ultrasonic surgical tools are widely used in minimally invasive and endoscopic operations [8]. Recently one of other actively developing directions of ultrasound application is angio-vascular surgery. This paper is devoted to the analysis of global trends and authors' developments in this research field.

1. Ultrasound application in cardio-vascular surgery

Technique of ultrasound application for fragmentation of thrombi and atherosclerotic plaques in blood vessels (ultrasonic ablation) and dissection of endarterium affected by atherosclerosis process (endarterectomy) is attempted in cardiovascular surgery practice. Both these operations are often referred to as ultrasonic angioplasty procedure. One of the first descriptions of ultrasonic ablation presented in the United States (Patent No. 3352303) in 1967 year [9]. This patent discloses the methods of thrombi retracting in carotid artery, coronary arteries, aorta, femoral artery and brachial artery. It proposes to use a tool in the form of solid or hollow ultrasonic waveguide made of stainless steel or money metal. Waveguide according to the patent has diameter of 2.7 mm in input cross-section attached to ultrasonic transducer and tapered to the diameter of 1.6 mm at the distance 50 mm from the input section. Operating frequency of the waveguide is set within the range of 10-30 kHz. Waveguide was introduced into the blood vessel through the incision made by means of transverse arteriotomy.

Since 1974, the process of ultrasonic ablation has been actively studied in Laboratory of ultrasound of Rhine-Westphalian higher technical school (Aachen, Germany) [10-13]. Ablation of thrombi and aspiration of debris implemented by means of hollow waveguide with external diameter of 2 mm and internal diameter of 1.4 mm. The working end of the waveguide was provided with inlet opening with diameter from 0.2 to 0.5 mm. Such design prevents the occlusion of internal channel of the waveguide, since debris particles with dimensions greater than diameter of inlet opening are not able to pass into the channel. Waveguide had a length of 210 mm and operated at longitudinal resonance with amplitude of vibration 25 μm and frequency of 26.8 kHz. Waveguide was placed inside a catheter and cooled by means of saline solution, introduced through the catheter. Catheter can be also used for introduction of radiopaque substance, which is necessary for angiographic imaging of vessel.

In 1988, Siegel et al. [14] studied efficiency of ultrasonic ablation ex vivo on excised segments of human arteries affected by atherosclerosis as well as in vivo on femoral artery of a dog. The drawing of the device used in these experiments is shown in the Fig. 1.

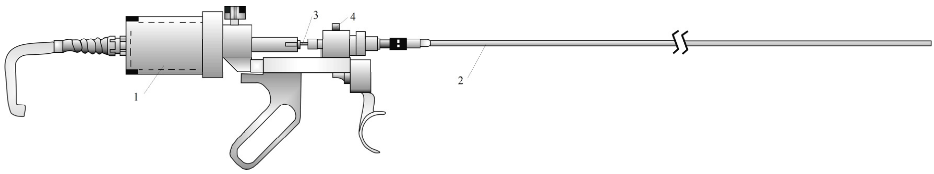


Fig. 1. Apparatus for ultrasonic ablation used in experiments by Siegel

Device consists of piezoelectric ultrasonic transducer (1), catheter (2) with diameter 2.3 mm (size 7F) and metal wire waveguide (3) with diameter of 0.89 mm (size 2.6F). Irrigation port (4) is intended to supply saline solution used for cooling of the waveguide. Operating frequency was 20 kHz; vibration amplitude of the working end (for both longitudinal and flexural vibrations) was in the range from 25 to 75 μm. Waveguide was provided with spherical working tip. Experiments demonstrated that ultrasonic ablation enabled efficient recanalization even in the case of significantly or totally occluded vessels with calcified plaques and without damage to normal tissues (selective action). In several cases operation resulted in perforation of arterial wall, however this adverse effect can be avoided by using pulsed ultrasound and limiting vibration amplitude and duration of treatment. Rosenschein et al. obtained similar results in 1990 [15].

In 1991 Rosenstein and co-workers [16] presented preliminary clinical results on ultrasonic recanalization of occluded femoral arteries. They used device consisting of aluminum waveguide with diameter of 1.6 mm, piezoelectric ultrasonic transducer and ultrasonic generator. Operating frequency was 20 kHz, amplitude – in the range from 125 to 175 μm. Generator was operating in pulsed mode with duty cycle of 50 %.

In 1994, Siegel et al. [17] successfully used ultrasonic ablation in clinical conditions for recanalization of coronary arteries.

Company Omnisonics Medical Technologies, Inc. (USA) developed device "Resolution Endovascular System" with 600 mm long waveguide consisting of three sections with diameters 0.64 mm, 0.46 mm and 0.23 mm [18]. Resonant frequency of the waveguide is 20 kHz. Operating principle of apparatus based on flexural vibrations of distal section of the waveguide with length of 87.5 mm. According to the developers, usage of flexural vibrations enables expansion of the active zone, where ablation of thrombus takes place, from the dimensions of the working tip to the full length of distal section. FDA (Food and Drug Administration, USA) approved "Resolution Endovascular System" for ablation of thrombi in synthetic haemodialysis access grafts. Unfortunately, at present there are no data on applications of this device because its manufacturer went bankrupt in 2009.

Company "Bard Peripheral Vascular Inc." (USA) developed catheter Crosser that uses ultrasonic vibrations with frequency 20.5 kHz for crossing of chronic total occlusions [19, 20].

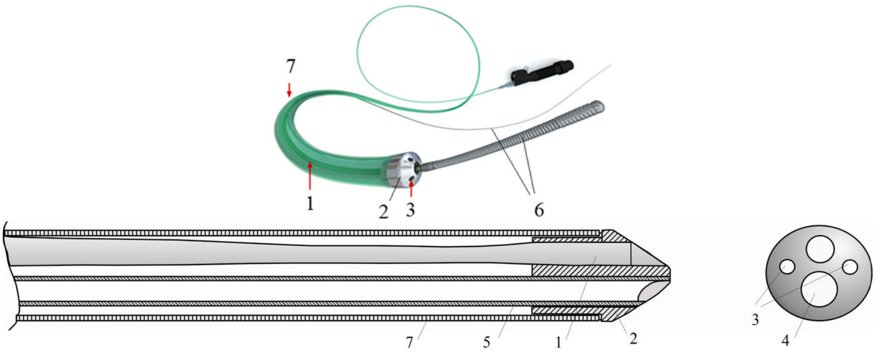


Fig. 2. Ultrasonic catheter for crossing chronic total occlusions

Design of the catheter is presented in Fig. 2. Vibrations with amplitude up to 20 μm are transmitted through nitinol wire waveguide 1 to the tip 2 made of stainless steel. Tip has openings (3) for irrigation; opening (4) for fastening of guiding tube (5) and guide wire (6). Waveguide and guiding tube situated inside the plastic sheath (7). Variable diameter of the waveguide provides amplification of vibration and reduction of stiffness of distal section (this enhances maneuverability of the catheter).

Studies in the field of ultrasonic ablation were also carried out in Singapore [21, 22]. The experimental model of ultrasonic vibratory system for angioplasty with 1000 mm long and 0.5 mm diameter wire waveguide made of titanium alloy Ti-6Al-4V has been developed in Nanyang Technological University (NTU). Operating frequency of the system is 26.7 kHz. Waveguide has a spherical or short-length cylindrical working tip with diameter of 1.5 mm.

Possibility of application of ultrasound in endarterectomy was for the first time reported by Austrian scientists in 1974 [23, 24]. They implemented open endarterectomy by means of tool vibrating at ultrasonic frequencies and came to the conclusion that application of ultrasound reduces risk of perforation of arterial wall, enables reduction of cutting force and easy isolation of even calcified atherosclerotic plaques. There is no need for the search of cutting plane, since isolation of lesion propagated along the boundary between tissues with different elastic properties (selective dissection of tissues). Histological studies carried out by optical and electron microscopy did not reveal any adverse effects on arterial wall from ultrasonic treatment. In Russia, studies in the field of ultrasonic endarterectomy started in 1974-1975. They was performed in Moscow higher technical school named after N. E. Bauman, under the supervision of prof. V. I. Loshtchilov, who was a founder of Russian scientific school on ultrasonic technologies in surgery and therapy [1]. These studies resulted in the development of ultrasonic tools for vascular surgery including tool similar to Petrovsky's blade and instruments with working parts in the form of rings, scoops and bogies (Fig. 3).

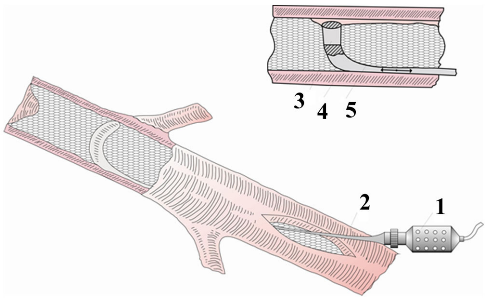


Fig. 3. Ultrasonic endarterectomy using instrument with ring-shaped working part:
1 – ultrasonic transducer; 2 – waveguide; 3 – working part; 4 – arterial wall; 5 – atherosclerotic lesion

Application of ultrasound enabled reduction of cutting force by 2.5-3.5 times and isolation of atherosclerotic lesions with the length up to 450 mm through small incisions in artery (semi-open method of endarterectomy). Waveguides for vascular surgery provided transmission of vibrations, which amplitude is not less than 30 μm at the distances up to 500 mm under conditions of sufficient flexibility and preservation of dynamic stability of the waveguide. Dynamic stability was enhanced by using specially designed waveguide consisting of several half-wavelength sections with elliptic cross-sections with major semi-axes angularly shifted relative to each other. Stability can be improved by providing waveguide with transverse grooves separated by half-wavelength distance and angularly shifted relative to each other. Positive results were obtained by Italian surgeons who performed ultrasonic endarterectomy of carotid artery by means of tools in the form of scoop and ring [25].

2. Physical mechanisms of ultrasonic ablation

The main mechanisms of ultrasonic ablation are high-frequency mechanical interaction ("hammering" action), cavitation and acoustic streaming. Cavitation consists in formation of pulsing bubbles in liquid (blood). Violent contraction (collapse) of these bubbles results in generation of shock waves with pressure up to 10 kbar and micro jets with velocities up to 100 m/s [26]. Micro jets and shock waves produce damage (cavitation erosion) of materials placed in the ultrasonic field. Cavitation was observed only at intensities of ultrasound exceeding threshold value referred to as cavitation threshold, which is proportional to frequency and for this reason cavitation is relatively easy developed in the range of low ultrasonic frequencies used in instrumental ultrasonic surgery. Acoustic pressure generated in the liquid and determining possibility of cavitation onset depends on vibration amplitude of working tip (head) of the waveguide and its geometric shape [27, 28]. For example, for diameter 1 mm, frequency 23.5 kHz and amplitude 60 μm spherically shaped head generates maximum acoustic pressure 300 kPa, flat head – 550 kPa and head with spherical depression – 1100 kPa [27].

Increase in diameter of head results in increased pressure. Fig. 4 presents diagram of dependence between reduction of thrombus mass and intensity of ultrasound for different head shapes [28].

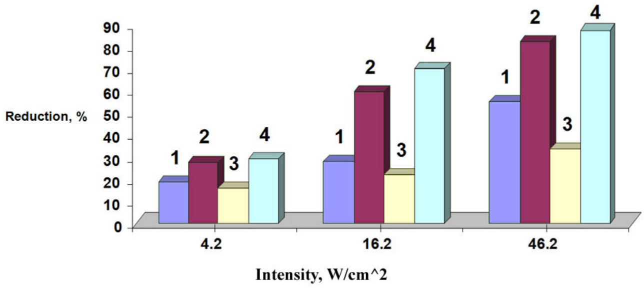


Fig. 4. Relation between reduction of thrombus mass and intensity of ultrasound for different head shapes: 1 – spherical head; 2 – flat head; 3 – spherical head with opening; 4 – flat head with opening

Results presented in the figure give possibility to arrange waveguides with different shapes of heads in decreasing order starting with the most efficient ones: flat head with opening waveguides, flat head waveguides, spherical head waveguides, and spherical head with opening waveguides. Shape of the head also affects the character of cavitational jet generated by the waveguide. Waveguide with flat head generates cylindrically shaped jet with diameter equal to the diameter of the head, and waveguide with spherical head generates jet in the form of diverging cone.

Apart from the destructive effect cavitation produces changes in the structure of fibrin protein appearing as constituent of thrombi. This provides possibility to use ultrasonic ablation in combination with administration of thrombolytic drugs [29, 30]. Elementary structural units of fibrin in the form of double-stranded chains (protofibrils) build fibers of larger diameter. Under the influence of ultrasound, these fibers were split into the smaller fragments and these results in the growth of specific surface available for binding of thrombolytic agents and acceleration of their diffusion into the material of thrombus.

Ultrasonic treatment results in alteration of biomechanical properties of arterial wall simplifying consequent balloon angioplasty experimentally confirmed [31]. Particularly, elastic modulus of arterial wall after guide wire recanalization has a mean value of 424.4 kPa, and after ultrasonic recanalization – 264.5 kPa, i.e. 38 % lower. This provides possibility to reduce pressure of balloon expansion during balloon angioplasty.

Equipment for ultrasonic ablation contains the main element of the devices for ultrasonic ablation – vibratory system, which typical design is presented in Fig. 5.

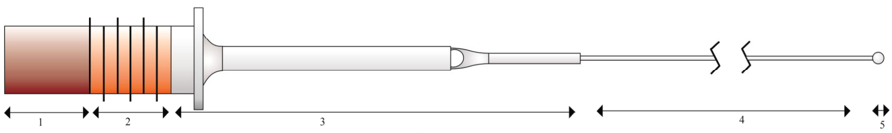


Fig. 5. Typical design of vibratory system for ultrasonic ablation:
1 – back piece; 2 – piezoceramic elements; 3 – horn-shaped working piece; 4 – wire waveguide; 5 – head

Ultrasonic vibrations are generated by means of electroacoustic transducer converting electric energy from ultrasonic generator into acoustic energy. In old designs of ultrasonic surgical tools there were used magnetostrictive transducers based on the effect of magnetostriction consisting in deformation of ferromagnetic materials in magnetic field [1]. These transducers possessed several disadvantages, particularly, because they needed water-cooling. Consequently, at present they are replaced with piezoelectric electroacoustic transducers based on polycrystalline piezoelectric ceramics, e.g. lead zirconate-titanate (PZT). The working principle of these transducers is based on the inverse piezoelectric effect consisting in deformation of piezoelectric material under the action of applied electric voltage. The most commonly used Langevin piezoelectric transducer consists of sandwiched piezoceramic elements connected in parallel to ultrasonic generator and two metal pieces: working piece and back piece. Pieces are intended for reduction of resonant frequency, since piezoceramic elements have high eigenfrequencies of vibration. Piezoelectric transducer attached to the one or several horns in the form of bars with gradually decreasing cross-sectional area intended for amplification of vibration amplitude. In some cases, working piece of transducer can be horn-shaped.

In the future medical ultrasonic transducers materials other than piezoelectric ceramics will be used. For example, research works are underway in developing transducers made of materials with giant magnetostriction (Terfenol-D [32]) and piezoelectric monocrystals (lithium niobate [33]). These transducers are characterized by more efficient electromechanical conversion and ability under equal conditions to yield higher vibration amplitudes in comparison to piezoceramic transducers.

Ultrasonic waveguide attached to the horn enables transmission of ultrasonic vibrations into the area of surgical treatment. Constructively waveguides can be designed in the form of bars with constant cross-sectional area and bars with variable cross-sectional area (horn-type waveguides). The latter design provides additional amplification of vibration amplitude. Horn-type waveguides consist of two or three cylindrical sections (steps) with different cross-sectional areas connected to each other by means of transitional sections with gradually varying cross-sectional areas (Fig. 6).

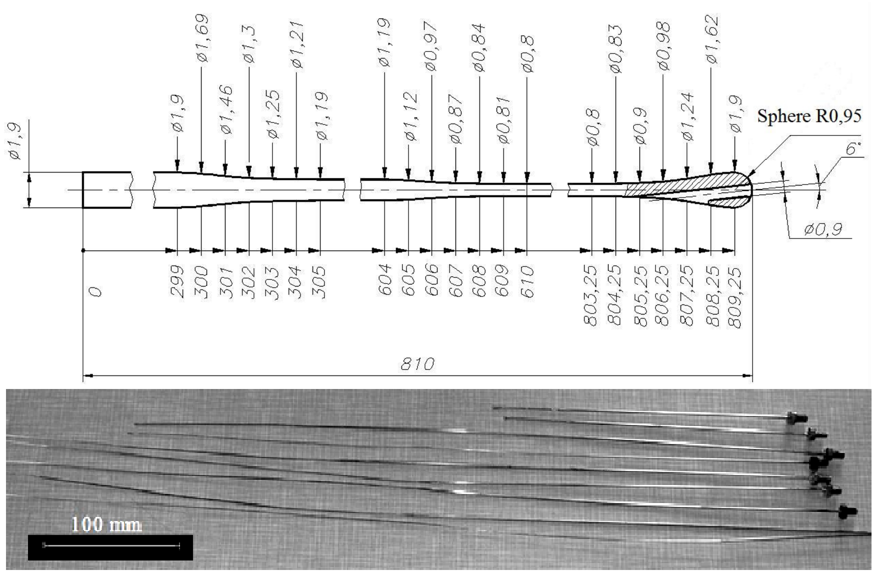


Fig. 6. Wire waveguides for ultrasonic ablation

Waveguide presented in Fig. 6 are provided with pear-shaped working tip with bore for guide wire. Waveguides can be made of stainless steels (316L), titanium or its alloys and nitinol. Material of the waveguide should be characterized by high fatigue strength in combination with minimum mechanical losses. There is information on fabrication of ultrasonic waveguides from optical fibers, e.g. fused silica [34, 35]. Application of optical fibers enables combined therapeutic treatment (so called Laser-Ultrasound Surgical Therapy – LUST) and collection of diagnostic information, e.g. by transmitting optical image of surgical treatment area through the waveguide. Due to their amorphous structure, waveguides made of optical fibers have very low mechanical losses in comparison with metal waveguides prone to strong heating in nodal planes of vibratory displacements resulting from internal intercrystalline friction.

Waveguides can be fabricated using technologies of metal forming, e.g. drawing with subsequent annealing, as well as electrophysical machining, e.g. plasma-electrolytic etching (PEE) [36]. Fig. 7 illustrates principles of PEE.

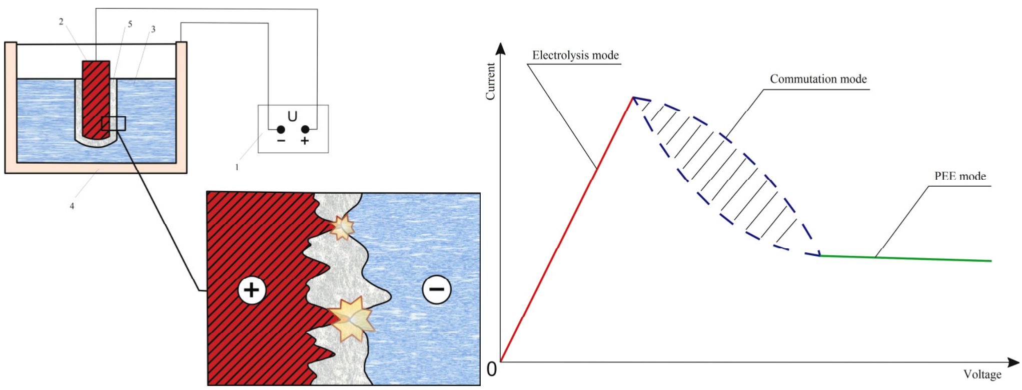


Fig. 7. Plasma-electrolytic etching of wire waveguides

Workpiece 2 is immersed into the bath 4 filled with electrolyte 3. Workpiece connected to the positive electrode of the power source 1 and bath – to the negative electrode. When potential difference between the electrodes is increased up to 60-70 V and current density is increased up to 10-16 A/cm^2, workpiece is periodically surrounded with gas-vapor film 5 (this mode is referred to as commutation mode and corresponds to the section AB of current-voltage characteristic). Application of voltage from 150 to 360-380 V results in formation of stable gas-vapor film (PEE mode corresponding to the section BC of current-voltage characteristic). Since surface area of the workpiece is substantially smaller than surface area of the bath, power density in the vicinity of the workpiece becomes sufficient for local film boiling of electrolyte resulting in formation of gas-vapor film. Vaporization of electrolyte results in the growth of electric resistance of the circuit "cathode-electrolyte-anode" and decrease of current. Applied voltage drops mostly at gas-vapor film and this results in formation of high-strength electric field and ionization breakdown of gas-vapor film arising mainly in areas corresponding to micro asperities of workpiece surface.

Drawing and photograph of device for ultrasonic angioplasty developed under the supervision of Prof. V. T. Minchenya and manufactured by scientific and technological park "Polytechnic" of the Belarusian National Technical University are presented in Fig. 8.

Device consists of piezoelectric Langevin transducer 2 mounted in the housing 1. Vibration from the transducer is transmitted through the horn 3 to the waveguide 4. Waveguide 4 is situated inside the catheter 7 connected to the device for aspiration of fragments of thrombus via connector 6. Transducer 2 is electrically connected to ultrasonic generator, which is controlled from computer with account for the signals received from sensor 5 of amplitude and frequency of vibrations. Computer control provides possibility to maintain constant value of vibration amplitude during shift of the resonant frequency of the waveguide resulting from its bending inside blood vessels. Measurement of amplitude by means of laser Doppler vibrometer shows that bending of the waveguide at the angle 60° results in decrease of vibration amplitude of its working tip by 10-15 % [37].

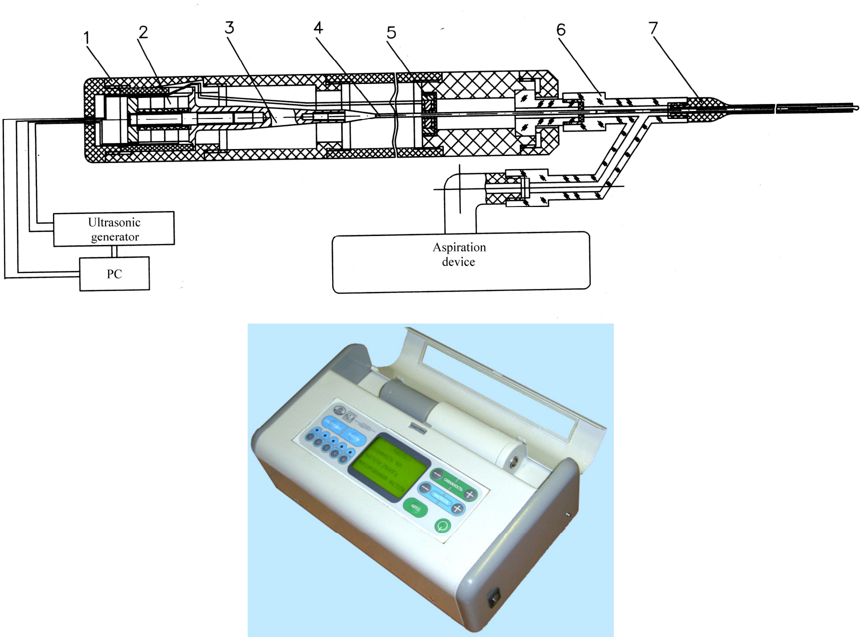


Fig. 8. Apparatus for ultrasonic angioplasty developed by Minchenya

Ultrasonic generator has discrete tuning of frequency with 15 Hz increment and discrete tuning of duty cycle with 5 % increment. Output power has 5 levels of tuning (from 20 to 120 W).

Vibration sensor (Fig. 9a) consists of plastic housing 3 with annular permanent magnet 4 mounted inside.

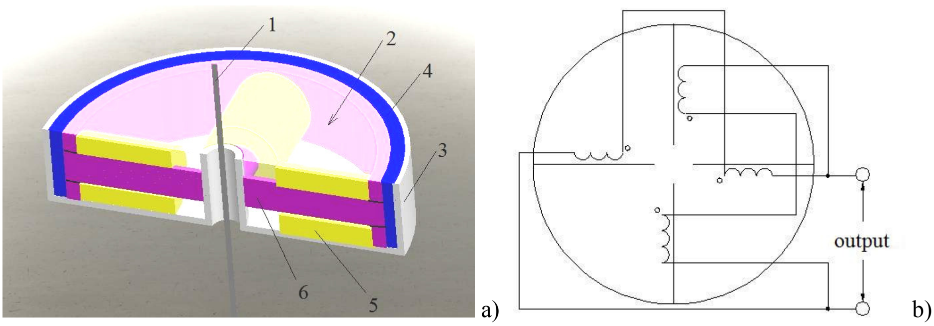


Fig. 9. Sensor of amplitude and frequency of ultrasonic vibrations

Waveguide 1 is introduced into the central opening of the housing 3. Magnetic flux generated by the magnet 4 passes through the pole pieces 6 made of magnetically soft ferrite and closes through the waveguide 1. Value of the magnetic flux will depend on the value of air gap between the pole pieces and the waveguide, which will change during flexural vibrations of the waveguide and because of the waveguide diameter variation during longitudinal vibrations (Poisson phenomenon). Alternating magnetic flux will induce electromotive force (EMF) in the coils 5 winded onto the pole pieces. Coils winded onto the opposite pole pieces are serially opposite connected in order to eliminate component of EMF related to the flexural vibrations of the waveguide (Fig. 9b). All components of sensor are sealed with epoxy compound 2.

3. Design of ultrasonic waveguides for cardiovascular surgery

Design of ultrasonic waveguides for cardiovascular surgery can be implemented by using experimental methods as well as by means of computer modeling. The simplest and the universal methods of ultrasonic waveguides design are method of transfer matrices and finite element method (FEM). Method of transfer matrices was widely used for design of ultrasonic vibratory systems by the group of medical acoustics under direction of Prof. S. E. Kvashnin (Moscow State Technical University after N. E. Bauman) [38] as well as by the authors of this chapter [39, 40]. To obtain the main relations of this method let us consider element of the waveguide in the form of bar with input cross-section coinciding with origin of coordinate system *Ox* (Fig. 10).

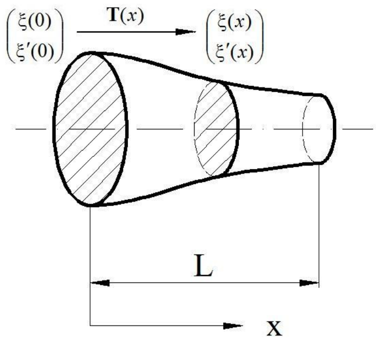


Fig. 10. Graphical explanation of transfer matrix approach

Longitudinal vibration of the bar will be described by the well-known Webster equation [41]:

$$\xi'' + \frac{2d'(x)}{d(x)}\xi' + k^2\xi = 0, \tag{1}$$

where $\xi(x)$ is amplitude of longitudinal displacements, $d(x)$ is diameter of the bar cross-section, $k = 2\pi f/c$ is wave number for longitudinal vibration, f is vibration frequency, c is velocity of longitudinal elastic waves in the bar material.

Solution of the Eq. (1) can be uniquely found when boundary conditions of the form $\xi(0) = \xi_0$, $\xi'(0) = \xi_0'$ (Cauchy problem) are set. This implies possibility to determine variables $\xi(x)$ and $\xi'(x)$ in the arbitrary cross-section of the bar from the known values $\xi(0)$ and $\xi'(0)$ of these variables in the input cross-section $x = 0$. This can be expressed by means of certain linear operator $\mathbf{T}(x)$:

$$\begin{pmatrix} \xi(x) \\ \xi'(x) \end{pmatrix} = \mathbf{T}(x)\begin{pmatrix} \xi(0) \\ \xi'(0) \end{pmatrix}. \tag{2}$$

Operator $\mathbf{T}(x)$ can be represented in the form of matrix with dimension 2×2, which is called as transfer matrix in respect to variables $\xi(x)$ and $\xi'(x)$. It is also possible to use transfer matrices in respect to any linearly independent combinations of these variables, e.g.

$\xi(x)$ and $\sigma(x)$, where $\sigma(x) = E\xi'(x)$ is amplitude of normal mechanical stresses, E is Young modulus of the bar material.

For determination of transfer matrix, $\mathbf{T}(x)$ let us represent Webster equation (1) in the form of equivalent vector differential equation of the first order:

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}x} = \mathbf{A}(x)\mathbf{u}, \tag{3}$$

where matrix $\mathbf{A}(x)$ and vector $\mathbf{u}(x)$ are determined by expressions:

$$\mathbf{A}(x) = \begin{pmatrix} 0 & 1 \\ -k^2 & -2d'(x)/d(x) \end{pmatrix}, \quad \mathbf{u}(x) = \begin{pmatrix} \xi(x) \\ \xi'(x) \end{pmatrix}.$$

After substitution of the expr. (2) into the Eq. (3) we will receive for the transfer matrix $\mathbf{T}(x)$ the following matrix differential equation:

$$\frac{d\mathbf{T}}{dx} = \mathbf{A}(x)\mathbf{T}. \tag{4}$$

This equation should be supplemented with the boundary condition $\mathbf{T}(0) = \mathbf{I}$, where $\mathbf{I}$ is identity matrix.

Although scalar counterpart of the Eq. (4) has simple analytic solution in the form of exponential function, matrix equation (4) in the general case cannot be integrated in the closed form. This corresponds to the limited set of functions $d(x)$ for which analytic solution of the Eq. (1) exists. Solution of the Eq. (4) can be represent in the form of matrix exponential function $\mathbf{T}(x) = \exp(\mathbf{A}(x)x)$ only in the case when Lappo-Danilevskij condition is satisfied [42]:

$$\mathbf{A}(x)\int_0^x \mathbf{A}(t)\mathrm{d}t = \int_0^x \mathbf{A}(t)\mathrm{d}t \cdot \mathbf{A}(x). \tag{5}$$

Condition (5) will be automatically satisfied for the constant matrix $\mathbf{A}$, i.e. in the case $d'(x) = -\alpha d(x)$, where α is arbitrary constant. This case corresponds to exponential variation law of the bar cross-sectional diameter $d(x) = d(0)\exp(-\alpha x)$, where α is shape factor of the bar. In the special case when $\alpha = 0$ we will obtain a bar with constant cross-sectional diameter $d(x) = d(0)$. Matrix $\mathbf{A}$ for the case under consideration will take the form $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -k^2 & 2\alpha \end{pmatrix}$.

Since matrix $\mathbf{A}(x)$ has eigenvalues $\lambda_1 = \alpha x + \mathrm{j}x\sqrt{k^2 - \alpha^2} = \alpha x + \mathrm{j}k_\alpha x$ and $\lambda_2 = \alpha x - \mathrm{j}k_\alpha x$, where j is imaginary unit, then according to the theorem about spectral decomposition (Lagrange-Sylvester formula) [42] solution of the Eq. (4) can be represented in the form:

$$\mathbf{T}(x) = \exp(\mathbf{A}x) = \frac{\mathbf{A}x - \lambda_2\mathbf{I}}{\lambda_1 - \lambda_2}\exp(\lambda_1) + \frac{\mathbf{A}x - \lambda_1\mathbf{I}}{\lambda_2 - \lambda_1}\exp(\lambda_2) =$$

$$= \frac{\exp(\alpha x)}{2\mathrm{j}k_\alpha}((\mathbf{A} + (\mathrm{j}k_\alpha - \alpha)\mathbf{I})\exp(\mathrm{j}k_\alpha x) - (\mathbf{A} - (\mathrm{j}k_\alpha + \alpha)\mathbf{I})\exp(-\mathrm{j}k_\alpha x)) =$$

$$= \exp(\alpha x)\left((\mathbf{A} - \alpha\mathbf{I})\frac{\sin(k_\alpha x)}{k_\alpha} + \mathbf{I}\cos(k_\alpha x)\right) =$$

$$= \exp(\alpha x)\begin{pmatrix} -\alpha \sin(k_\alpha x)/k_\alpha + \cos(k_\alpha x) & \sin(k_\alpha x)/k_\alpha \\ -(k_\alpha^2 + \alpha^2)\sin(k_\alpha x)/k_\alpha & \alpha \sin(k_\alpha x)/k_\alpha + \cos(k_\alpha x) \end{pmatrix}.$$

In a special case when $\alpha = 0$ we will obtain transfer matrix for the bar with constant cross-sectional diameter:

$$\mathbf{T}(x) = \begin{pmatrix} \cos(kx) & \sin(kx)/k \\ -k\sin(kx) & \cos(kx) \end{pmatrix}. \tag{6}$$

In the case of the waveguides with "standard" profile of the longitudinal section (exponential, catenoidal or conical) Eq. (1) can be reduced to the normal form of Liouville containing no first derivative by means of substitution $\xi(x) = U(x)/d(x)$ and takes the form:

$$U'' + k_\alpha^2 U = 0, \tag{7}$$

where $k_\alpha = \sqrt{k^2 - \alpha^2}$ is wave number for longitudinal vibration with account for the waveguide shape factor, α is waveguide shape factor (see Table 1).

Table 1. Matrix of variable transformation for various shapes of the waveguides

Type of the waveguide and shape of its longitudinal section	Matrix of variable transformation
Exponential $d(x) = d(0)\exp(-\alpha x)$	$\exp(-\alpha x)\begin{pmatrix} 1 & 0 \\ -\alpha & 1 \end{pmatrix}$
Catenoidal $d(x) = d(L)\,\mathrm{ch}(\alpha(L-x))$	$\mathrm{ch}(\alpha(L-x))\begin{pmatrix} 1 & 0 \\ -\alpha\,\mathrm{th}(\alpha(L-x)) & 1 \end{pmatrix}$
Conical $d(x) = d(0)(1-\alpha x)$	$\begin{pmatrix} 1-\alpha x & 0 \\ -\alpha & 1-\alpha x \end{pmatrix}$

Since Eq. (7) coincides in its form with equation of vibration for the waveguide with constant cross-sectional diameter, then the following condition is satisfied:

$$\begin{pmatrix} U(x) \\ U'(x) \end{pmatrix} = \mathbf{T}_\alpha(x)\begin{pmatrix} U(0) \\ U'(0) \end{pmatrix}, \tag{8}$$

where transfer matrix $\mathbf{T}_\alpha(x)$ is obtained from the matrix (6) by means of substitution $k \to k_\alpha$.

If we determine matrix $\mathbf{M}(x)$ of variable transformation by means of relation:

$$\begin{pmatrix} U(x) \\ U'(x) \end{pmatrix} = \mathbf{M}(x)\begin{pmatrix} \xi(x) \\ \xi'(x) \end{pmatrix},$$

then Eq. (8) can be represented in the form $\mathbf{M}(x)\begin{pmatrix} \xi(x) \\ \xi'(x) \end{pmatrix} = \mathbf{T}_\alpha(x)\mathbf{M}(0)\begin{pmatrix} \xi(0) \\ \xi'(0) \end{pmatrix}$.

From the last relation we can find transfer matrix of the waveguide:

$\mathbf{T}(x) = \mathbf{M}^{-1}(x)\mathbf{T}_\alpha(x)\mathbf{M}(0)$.

Expressions for the matrix of variable transformation for various shapes of the waveguides are given in the Table 1.

In the general case, solution of the Eq. (4) should be searched by means of numerical methods. For this purpose let us consider vector-functions of the form $\tilde{\mathbf{u}}^{(i)}(x) = \mathbf{u}^{(i)}(x)/u_i^{(i)}(0)$, where $i = 1...2$ and vector-functions $\mathbf{u}^{(i)}(x) = \left(u_1^{(i)}(x) \quad u_2^{(i)}(x)\right)^{\mathrm{T}}$ are particular solutions of the Eq. (3) for the boundary conditions $\mathbf{u}^{(i)}(0) = \left(\delta_{i1} \quad \delta_{i2}\right)^{\mathrm{T}}$, where δ_{ij} is Kronecker symbol. Elements of vectors describing boundary conditions are assumed to have dimensions corresponding to the dimensions of elements of the vector-function $\mathbf{u}(x)$.

Let us show that system of vector-functions $\mathbf{u}^{(i)}(x)$ is linearly independent, i.e. this is fundamental system of solutions for the Eq. (3). To prove this let us consider Wronskian of the system:

$$W(x) = \det\begin{pmatrix} u_1^{(1)}(x) & u_1^{(2)}(x) \\ u_2^{(1)}(x) & u_2^{(2)}(x) \end{pmatrix}.$$

Wronskian satisfies Jacoby-Liouvulle identity [43]:

$$W(x) = W(0)\exp\left\{\int_0^x \mathrm{Sp}(\boldsymbol{A}(x))\mathrm{d}x\right\} = W(0)\exp\left\{-2\int_0^x \frac{d'(x)}{d(x)}\mathrm{d}x\right\} = W(0)\left(\frac{d(0)}{d(x)}\right)^2,$$

where Sp denotes trace of matrix.

As it follows from Jacoby-Liouvulle identity, Wronskian is either identically zero or everywhere non-zero. In the first case, system of vector-functions will be linearly dependent and in the second case, it will be linearly independent. Because of boundary conditions accepted during definition of the system of vector-functions $\mathbf{u}^{(i)}(x)$ Wronskian at the point $x = 0$ will be determinant of identity matrix and will satisfy equation $W(0) = 1$. This proves linear independence of the system.

Since vector-functions $\tilde{\mathbf{u}}^{(i)}(x)$ are proportional to the vector-functions $\mathbf{u}^{(i)}(x)$, then they are also linearly independent and particular solution $\mathbf{u}(x)$ of the Eq. (3) for the arbitrary boundary conditions $u_1(0)$, $u_2(0)$ can be represented in the form of expansion:

$$\mathbf{u}(x) = u_1(0)\tilde{\mathbf{u}}^{(1)}(x) + u_2(0)\tilde{\mathbf{u}}^{(2)}(x),$$

which can be written in matrix form:

$$\mathbf{u}(x) = [\tilde{\mathbf{u}}^{(1)}(x) \quad \tilde{\mathbf{u}}^{(2)}(x)]\mathbf{u}(0). \tag{9}$$

Comparison of exprs. (9) and (2) leads to the formula expressing transfer matrix $\mathbf{T}(x)$ in terms of fundamental system of solutions of the Eq. (3):

$$\mathbf{T}(x) = [\tilde{\mathbf{u}}^{(1)}(x) \quad \tilde{\mathbf{u}}^{(2)}(x)]. \tag{10}$$

Fundamental system of solutions can be in turn determined by standard numerical methods of solution of ordinary differential equations, e.g. Runge-Kutta method.

Transfer matrix of the waveguide system in respect to any linearly independent variables continuous over coordinate *x* can be determined as product of the transfer matrices $\mathbf{T}_i(L_i, f)$ of its elements in respect to the same variables, i.e.:

$$\mathbf{T}(L_1, L_2, ..., L_N, f) = \mathbf{T}_N(L_N, f)\mathbf{T}_{N-1}(L_{N-1}, f)\cdot ... \cdot \mathbf{T}_1(L_1, f), \tag{11}$$

where transfer matrix $\mathbf{T}_1(L_1, f)$ corresponds to the input element of the system, L_i is length of the *i*-th element of the system $(i = 1, ..., N)$.

In the case of longitudinal vibration of the uniform complex waveguide system with cross-sectional diameter $d(x)$, which is continuous function of the variable *x*, we can select as variables $\xi(x)$ and $\xi'(x)$. In a more general case, e.g. for the non-uniform system consisting of elements made of different materials or for the system with cross-sectional diameter $d(x)$ which is discontinuous function, Eq. (11) will be true for the transfer matrices in respect to variables $\xi(x)$ and $F(x)$.

Transfer matrix can be applied for determination of resonant conditions and gain factor of the waveguide system. In the case of longitudinal vibration resonance, it is necessary to satisfy boundary conditions $\xi'(0)=\xi'(L)=0$ and this gives possibility to write Expr. (2) in the form:

$$\begin{pmatrix} \xi(L,f) \\ 0 \end{pmatrix} = \begin{pmatrix} T_{11}(L,f) & T_{12}(L,f) \\ T_{21}(L,f) & T_{22}(L,f) \end{pmatrix} \begin{pmatrix} \xi(0) \\ 0 \end{pmatrix}. \qquad (12)$$

Values appearing in Eqs. (11) and (12) are considered as functions of geometric parameters of the waveguide system (lengths L and L_i) and vibration frequency f that corresponds in practice to two kinds of problems:

1. Problem of analysis: we need to determine resonant frequencies of the waveguide system for the given geometric parameters.

2. Problem of synthesis: we need to determine geometric parameters of the waveguide system providing resonance of its longitudinal vibration at the given frequency.

Second equation of the system (12) results in condition of resonance $T_{21}(L,f)=0$.

Amplitude of vibration $\xi(0)$ for the input cross-section of the waveguide can be set to arbitrary value. Amplitude of vibration $\xi(L,f)$ for the output cross-section of the waveguide is relating to the amplitude $\xi(0)$ by the first equation of the system (12):

$$\xi(L,f)=T_{11}(L,f)\xi(0).$$

This equation clarifies physical meaning of the element $T_{11}(L,f)$ of the transfer matrix: it represents gain factor of vibration amplitude for the case of resonance.

As an example, Figure 11a shows resonant surfaces of three-step waveguide calculated using method of transfer matrices [40].

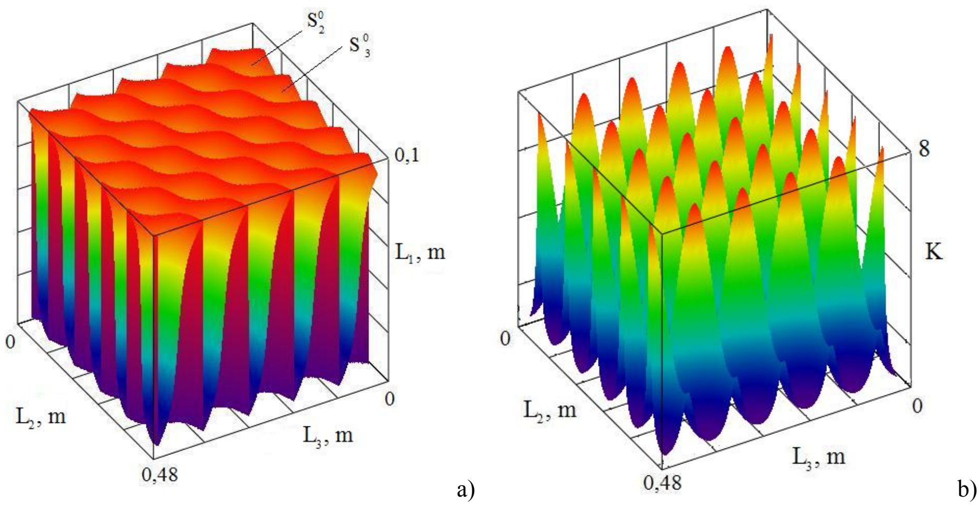


Fig. 11. Resonant surfaces (a) and gain factor (b) of three-step waveguide calculated using method of transfer matrices

Calculation was performed using MathCad software for the following values of parameters: diameters of steps of the waveguide $D_1=1.5$ mm, $D_2=0.89$ mm, $D_3=0.5$ mm; length of transitional sections $\Delta L = 6$ mm; radius of spherical part of the head $r_h = 0.75$ mm; elastic modulus of material of the waveguide $E = 187.3$ GPa; density of material $\rho = 7800\ \mathrm{kg/m^3}$; frequency of vibration $f = 25$ kHz. For each resonant surface S_n corresponding to the n-th mode of longitudinal vibration Figure 11a shows only its part S_n^0 satisfying condition $L_1 < \pi/k$. Full resonant surface S_n can be build by using expression:

$$S_n = \bigcup_{m=0}^{n-1} S_{n-m}^{\pi m/k},$$

where $\bigcup$ is union of the surfaces, $S_{n-m}^{\pi m/k}$ is surface obtained from the surface S_{n-m}^0 by means of parallel translation in direction of axis L_1 on the distance $\pi m/k$.

Resonant surfaces give possibility to determine length L_1 providing resonance of longitudinal vibration at the given frequency f for the given lengths L_2 and L_3.

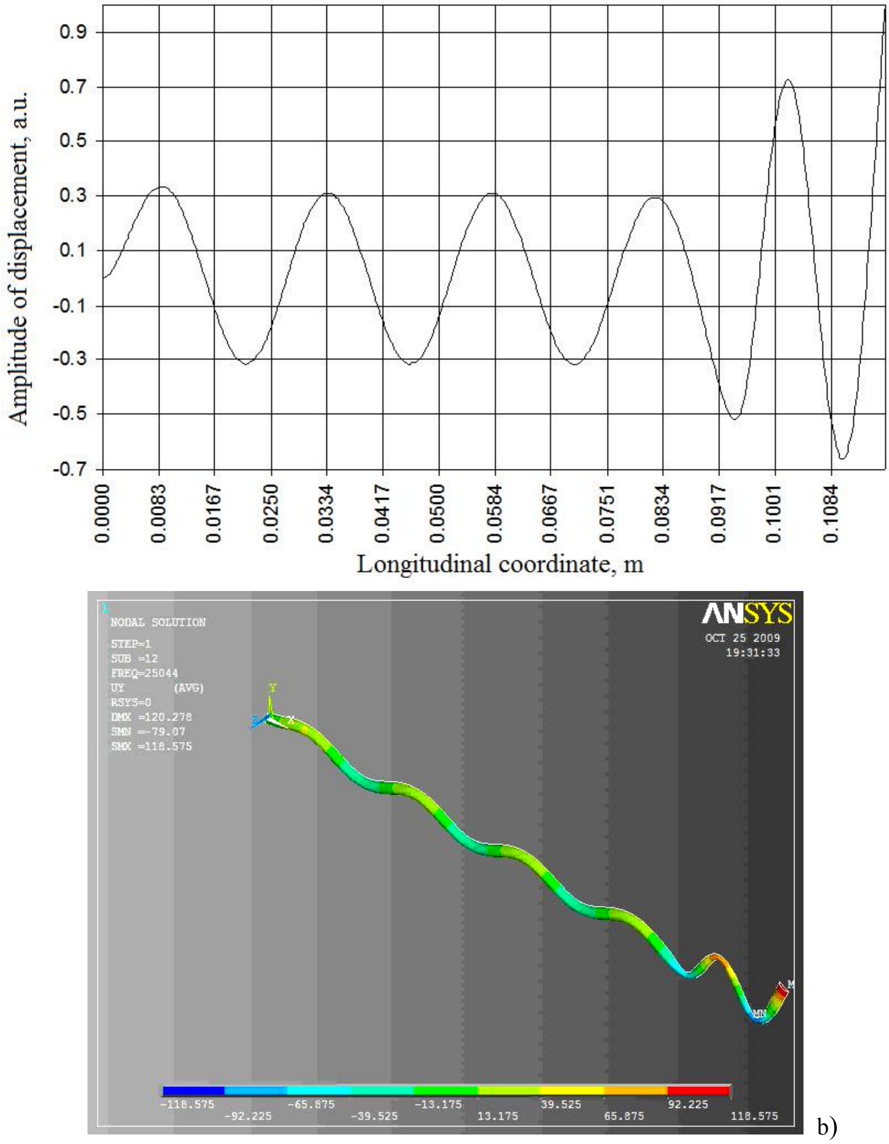
Figure 11b presents dependence between gain factor K of the waveguide and lengths L_2 and L_3. Dependence is characterized by the presence of well-defined maxima of gain factor and this gives possibility to use presented plot for optimization of waveguides in terms of their gain factor.

Vibrations of ultrasonic waveguide systems are also successfully studied by means of modal analysis with the aid of commercially available FEM software like ANSYS [7, 27, 39, 44]. Modeling of waveguides using ANSYS could be automated by means of built-in APDL programming language (ANSYS Parametric Design Language). In the case of multi-parameter optimization, automation of modeling eliminates laborious work of user on building multiple geometric models of waveguides with different parameters. APDL describes the main analysis stages with a list of commands depending on parameters of the waveguide such as material properties, geometric parameters etc. List of commands is saved as a file with .txt extension and executed with the aid of command "Read Input From …" accessible from the main menu of graphical user interface (GUI). User is prompted for all necessary parameters via GUI. The developed APDL routine can be use for analysis of longitudinal and flexural vibrations of two- and three-stepped waveguides and automatically performs the following operations:

1) building of the waveguide geometric model;
2) application of boundary conditions;
3) meshing of the model;
4) modal analysis.

Natural frequencies of longitudinal vibration are calculated using geometric model in the form of a quarter-waveguide with application of symmetric boundary conditions on cutting planes. Modal analysis of flexural vibrations is implemented using half-model with application of symmetric boundary conditions on cutting plane and constraining input cross-section in all degrees of freedom.

Analysis of the natural frequencies and modes of vibrations is implemented by user with the aid of general postprocessor (POST1). Amplitudes of transverse displacements of the waveguide axis and rotation angles of cross-sections can be displaying in the form of plots using interpolation of the nodal data along user-specified paths. The paths are defined automatically and include longitudinal axis of the waveguide as a path for plotting amplitude of transverse displacements vs. longitudinal coordinate and generatrix of the waveguide as a path for plotting rotation angles. Plots are displayed via POST1 with the aid of standard methods of the work with path variables. Values of the path variables can be stored in the file with .lis extension, which can be used after deletion of comments and changing extension to .prn for import of the data into external programs, e.g. Excel, MathCad etc. The example demonstrated in Fig. 12a shows distribution of amplitude of the transverse displacements along the waveguide axis plotted in Excel.



a) b)

Fig. 12. Distribution of amplitude of the transverse displacements along the waveguide plotted using Excel (a) and ANSYS (b)

Fig. 12b presents vizualization of calculation results for the same waveguide in graphical window of ANSYS software.

Calculation was performed for the two-stepped waveguide with the following parameters:

1) lengths of the steps $L_1 = 89.5$ mm and $L_2 = 20.9$ mm;

2) diameters of the steps $D_1 = 2$ mm and $D_2 = 0.9$ mm.

Waveguide steps are connected by smooth transitional sections described in the geometric model by means of B-splines. Amplitude of displacements is normalized by dividing on the maximum value of its modulus.

Clinical applications of ultrasonic ablation. Clinical applications of ultrasonic angioplasty will be illustrated by the following example. Patient D. with occlusion of right superficial femoral artery was operated on Regional Clinical Hospital of Minsk. Patient was suffering from intermittent claudication for 3 years and felt pain in the right shin during walking on the distance 30-50 m. Stages of operation are present in Fig. 13.

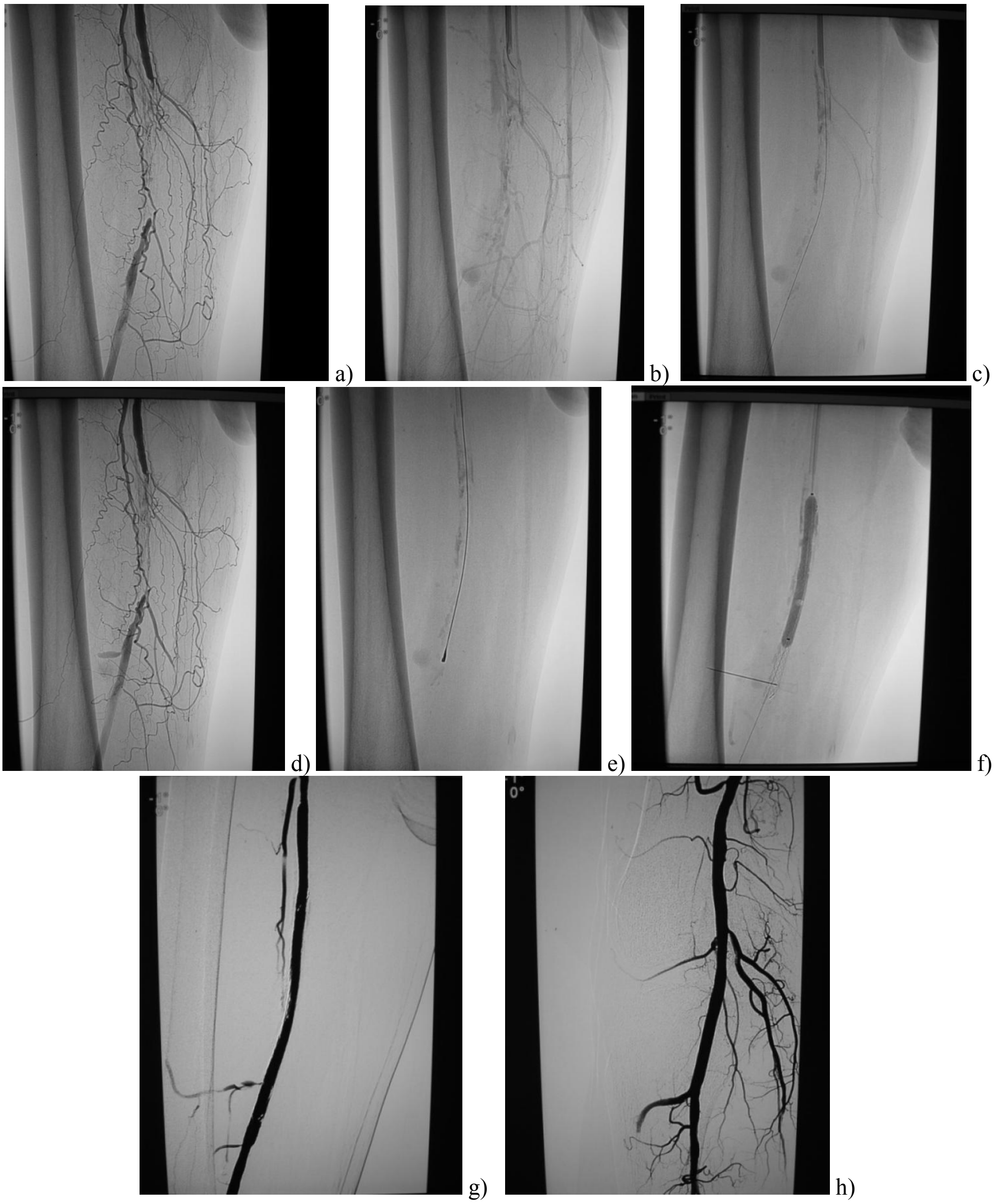


Fig. 13. Stages of recanalization of femoral artery

Figure 13a presents angiogram of occluded artery before operation. This is occlusion with length 50.5 mm on the border of lower one-third and middle one-third of the shin. Attempts to cross occlusion by means of guide wires with diameters 0.014 and 0.035 inches were unsuccessful (Fig. 13b). Ultrasonic ablation was performed using ultrasonic waveguide with length 835 mm and resonant frequency of 25960 Hz introduced without guide wire through the catheter. Intensity of ultrasound was 22.6 W/cm^2, duty cycle – 45 %, duration of treatment – 256 s. Risk of damage of arterial wall ultrasonic treatment can be implemented with localization of the waveguide head inside the catheter (at the distance 2-3 mm from its tip) with continuous flushing of the catheter with saline at flow rate 5 ml/s. Ultrasonic treatment resulted in formation of initial channel in the occlusion sufficient for its crossing by means of guide wire. After removal of the waveguide medium-stiffness coronary guide wire with diameter 0.014 inches, introduced in the channel (Fig. 13c). Angiography showed presence of thin stream of contrast medium through the occlusion with satisfactory filling of the artery, i.e. partial recanalization of artery (Fig. 13d). Improvement of biomechanical properties of recanalyzed segment of artery and prevention of vasospasm was achieved when waveguide once again was introduced into the zone of occlusion along the guide wire. Under angiographic control, waveguide advanced three times in forward and backward direction along the occlusion (Fig. 13e). Duration of treatment was 48 s, intensity of ultrasound – 6.5 W/cm^2. After ultrasonic treatment, balloon angioplasty was carried out. Angiography revealed complete bloodstream restoration in previously occluded artery. However, in the zone of occlusion dissection of intima was observed, which gives indications for stenting. Balloon-expandable stent with dimensions 6×10 mm was implanted into the zone of dissection (Fig. 13f). Angiography showed possibility of stent without signs of dissection (Fig. 13g). Degree of residual stenosis was 25 %. Popliteal and tibia arteries had good filling and possibility up to ankle joint (Fig. 13h). Distance walking by the patient without pain increased up to 1000 m, and after 6 months after operation – up to 1500 m.

Conclusions

Results of clinical studies presented in the chapter indicate great potential of application of ultrasonic technologies in cardiovascular surgery. Theoretical and technological basics of fabrication, design and modeling of ultrasonic vibratory systems, which are also considered in this paper, may be used for development of novel ultrasonic equipment for minimally-invasive surgery. There are perspectives of further wider introduction of the described devices and technologies in clinical practice as well as implementation of further theoretical and experimental studies of vibratory systems for ultrasonic surgery.

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References

[1] **G. A. Nikolaev, V. I. Loshtchilov** Ultrasonic Technology in Surgery. Moscow, 1980, 272 p., (in Russian).

[2] **M. G. Kiseliov et al.** Ultrasound in Medicine. Minsk, 2009, 428 p., (in Russian).

[3] **B. O'Daly et al.** High-power low-frequency ultrasound: a review of tissue dissection and ablation in medicine and surgery. Journal of Materials Processing Technology, Vol. 200, 2008, p. 38-58.

[4] **K. Hynynen, G. Clement** Clinical applications of focused ultrasound – the brain. International Journal of Hyperthermia, Vol. 23, 2007, p. 193-202.
[5] **J. E. Kennedy et al.** High intensity focused ultrasound: surgery of the future? British Journal of Radiology, Vol. 76, 2003, p. 590-599.
[6] **J. E. Kennedy** High-intensity focused ultrasound in the treatment of solid tumours. Nature Reviews Cancer, Vol. 5, 2005, p. 321-327.
[7] **T. Nagoya, M. K. Kurosawa** A micro ultrasonic scalpel with sensing function. Proc. of the IEEE Ultrasonics Symposium, 2003, p. 1070-1073.
[8] **S. J. Lee, K. H. Park** Ultrasonic energy in endoscopic surgery. Yonsei Medical Journal, Vol. 40, 1999, p. 545-549.
[9] United States Patent 3352303, Cl. 128-24. Method for Blood Clot Lysis / L. J. Delaney, Ser. No. 475337, Filed 28.07.65, Patented 14.11.67.
[10] **A. Sobbe et al.** Die ultraschall-auflösung von thrombin. Klinische Wochenschrift, B. 52, 1974, p. 1117-1121.
[11] **G. Trübestein et al.** Die emboliegefahr bei der ultraschall-thrombolyse. Langenbecks Archiv für Chirurgie, B. 340, 1976, p. 199-205.
[12] Offenlegungsschrift DT 24 38 648 A1, Int. Cl. A 61 B 17-22. Verfahren zum Entfernen von Thromben / R. Pohlman, U. Stumpff, Aktenzeichen P 24 38 648.9, Anmeldetag 12.08.74, Offenlegungstag 26.02.76.
[13] **U. Stumpff** Die Erzeugung und Übertragung von Ultraschalldehnwellen Hoher Energiedichte in Flexiblen Wellenleitern im 20 kHz-Bereich für Therapeutische Anwendungen. Dissertation, RWTH Aachen, 1978.
[14] **R. J. Siegel et al.** Ultrasonic plaque ablation. A new method for recanalization of partially or totally occluded arteries. Circulation, Vol. 78, 1988, p. 1443-1448.
[15] **U. Rosenschein et al.** Experimental ultrasonic angioplasty: Disruption of atherosclerotic plaques and thrombi in vitro and arterial recanalization in vivo. Journal of the American College of Cardiology, Vol. 1990, p. 711-717.
[16] **U. Rosenschein et al.** Ultrasonic angioplasty in totally occluded peripheral arteries. Circulation, Vol. 83, 1991, p. 1976-1986.
[17] **R. J. Siegel et al.** Use of therapeutic ultrasound in percutaneous coronary angioplasty. Experimental in vitro studies and initial clinical experience. Circulation, Vol. 89, 1994, p. 1587-1592.
[18] **M. J. Hallisey** Ultrasonic energy treatment of deep vein thrombosis. Endovascular Today, April 2006, p. 80-82.
[19] **J. H. Rogers, J. R. Laird** Overview of new technologies for lower extremity revascularization. Circulation, Vol. 116, 2007, p. 2072-2085.
[20] United States Patent Application 2010/0023037 A1, Int. Cl. A61B 17/22. Ultrasound Catheter and Methods for Making and Using Same, H. Nita et al., Appl. No. 12/538009, Filed 07.08.2009, Pub. date 28.01.2010.
[21] **T. Li et al.** Horn-type piezoelectric ultrasonic transducer: modeling and applications. Advances in Piezoelectric Transducers, Ed. by F. Ebrahimi, InTech, 2011, p. 3-26.
[22] International Application published under PCT WO 2010/027325 A1, IPC A61B 17/22, A61N 7/00, A61B 8/12. Micro-Emulsifier for Arterial Thrombus Removal, J. Ma et al., International Application Number PCT/SG2008/000323, International Filing Date 03.09.2008, International Publication date 11.03.2010.
[23] **M. Redtenbacher et al.** Endarterektomie mit ultraschall. Acta Chirurgica Austriaca, H. 5, 1974, p. 114-115.
[24] **M. Redtenbacher et al.** Über die einwirkung des ultraschalles auf die gefäßwand. Acta Chirurgica Austriaca, H. 6, 1975, p. 125.
[25] **A. La Rosa et al.** Ultrasonic endarterectomy: experimental and initial clinical results in carotid stenosis. Journal of Endovascular Surgery, Vol. 1, 1994, p. 25-30.
[26] **Tezel S. Mitragotri** Interactions of inertial cavitation bubbles with stratum corneum lipid bilayers during low-frequency sonophoresis. Biophysical Journal, Vol. 85, 2003, p. 3502-3512.
[27] **M. P. Wylie et al.** A linear finite element acoustic fluid-structure model of ultrasonic angioplasty in vivo. International Journal for Numerical Methods in Biomedical Engineering, Vol. 26, 2010, p. 828-842.

[28] **T. Tun** Efficiency of Recanalization of Arteries Affected by Atherosclerosis Using Various Modifications of Ultrasonic Waveguides in Vitro. Ph. D. Dissertation, Minsk, 2006, 110 p., (in Russian).

[29] **F. Siddiqi et al.** Binding of tissue-plasminogen activator to fibrin: effect of ultrasound. Blood, Vol. 91, 1998, p. 2019-2025.

[30] **I. E. Adzerikho et al.** Ultrasound fibrin clot destruction in vitro in the presence of fibrinolytic agent. Ultrasonics Sonochemistry, Vol. 8, 2001, p. 315-318.

[31] **Goncharuk M. M. et al.** Efficiency of ultrasonic angioplasty in treatment of chronic arterial occlusions. Medical Panorama, No. 15, 2007, p. 16-20, (in Russian).

[32] **M. J. Dapino** On magnetostrictive materials and their use in adaptive structures. Structural Engineering and Mechanics, Vol. 17, 2004, p. 303-329.

[33] **J. Twiefel, T. Morita** Utilizing multilayer lithium niobate elements for ultrasonic actuators. Sensors and Actuators A, Vol. 166, 2011, p. 78-82.

[34] **J. Tschepe et al.** Transmission of laser radiation and acoustical waves via optical fibers for surgical therapy. Proc. SPIE, Vol. 2131, 1994, p. 155-166.

[35] **T. Gudra, S. Muc** Some problems of ultrasonic and laser cutting of biological structures. The European Physical Journal. Special Topics, Vol. 154, 2008, p. 85-88.

[36] http://epo.metolit.by/ru/dir/index.php/1537. Accessed 2012.

[37] **A. Bubulis et al.** Experimental investigation of ultrasound vibrations of a flexible waveguide. Journal of Vibroengineering, Vol. 10, 2008, p. 74-78.

[38] **S. E. Kvashnin** Ultrasonic Electro-Acoustic Transducers and Waveguide Tools for Medicine. Moscow, 1995, 43 p., (in Russian).

[39] **D. Stepanenko et al.** Computer modelling and experimental studies of wire ultrasonic waveguides for minimally-invasive surgery. Lékař a Technika, Vol. 40, No. 2, 2010, p. 1-5.

[40] **D. A. Stepanenko, V. T. Minchenya** Optimization of flexible waveguides for ultrasonic thrombectomy. Proceedings of the National Academy of Sciences of Belarus. Series of Physical and Technical Sciences, No. 3, 2011, p. 92-98, (in Russian).

[41] Ultrasonics: Data, Equations and their Practical Uses. Ed. by D. Ensminger and F. B. Stulen, Boca Raton, L., N. Y.: CRC Press, Taylor & Francis Group, 2009, 496 p.

[42] **J. A. Lappo-Danilevskij** Application of Matrix Functions to Theory of Linear Systems of Ordinary Differential Equations. Moscow, 1957, 456 p., (in Russian).

[43] **P. Lankaster** Theory of Matrices. N. Y., L.: Academic Press, 1969, 316 p.

[44] **D. A. Stepanenko, V. T. Minchenya** Modeling of flexible waveguides for ultrasonic vibrations transmission: longitudinal and flexural vibrations of non-deformed waveguide. Ultrasonics, Vol. 50, 2010, p. 424-430.