885. Self-synchronization theory of a dual mass vibrating system driven by two coupled exciters. Part 2: Numeric analysis

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Abstract. The coupling dynamic characteristics of the vibrating system with dual mass are analyzed quantitatively. Both the load torque and the coupling torque have three items. Two of them are concerned with the translation of the system, and the third item is related to the rotation of the system. Through numerical computation, the effects of translation and rotation in the system are considered in relation to the self-synchronization. The phase difference of two eccentric blocks is caused by the difference of the rated revolution of two motors. The stability of the synchronous operation is dependent on the structural parameters of the system, such as the mass ratio of two eccentric blocks and the distance between motor and centroid of the rigid frame. Simulation is carried out to verify that the system can be synchronized and the model can ensure the stability of synchronization if the parameters of the system meet the conditions of synchronous implementation and stability. Simulations are also performed for the case of self-synchronization of two motors with different rated revolutions.

Keywords: self-synchronization, stability, vibrating system.

Introduction

Blekhman I. I. [1-5] developed a formal, algorithmic guide to deriving the conditions for the existence and stability of self-synchronized motions of unbalanced rotors on mechanical systems, which laid the foundation for the research of self-synchronization theory. In the first paper, a vibration model has been put forward for studying the vibrating system with dual mass based on the self-synchronization theory. And the conditions of synchronous implementation and stability are derived by dynamical analysis. Recently, many researchers have already contributed to numerical analysis of synchronous characteristics for self-synchronization system. Zhang Nan [6] used an omnipotently vibrating machine as a prototype for the synchronous experiment to quantitatively discuss the variation rule of each parameter and synchronous characteristics about self-synchronization vibrating system at several representative states. Wang Degang and Zhao Chunyu [7] used the computer to simulate the process of self-synchronization, and the results indicated that the synchronization of vibrating system came true in either speed or phase to enable the system to be in a good self-synchronization state.

In this paper, the quantitative analysis of the coupling dynamic characteristics of the vibrating system with two masses is considered. During operation the system exerts an extra torque on each motor, which can be divided into load torque and coupling torque. As mentioned in the first part, the load torque reduces the speed of both motors. The coupling torque acts on the motor at a higher speed to reduce its angular velocity as the load torque, while it also acts on the other motor at a lower speed to speed it up as the driving torque. Finally, two motors reach the same speed. Even if the two motors have different revolutions, they can also implement self-synchronization with the coupling torque. Specifically, if the coupling torque is high enough, two motors reaching the synchronization can maintain the same speed by cutting off the power supply of one motor.

The paper is organized as follows. In Section 2, we analyze the load torque and coupling torque which rigid frame acts on two motors. Section 3 describes the simulation method used in this paper. In Section 4, the impact of structural parameters on the synchronization is discussed and the conclusions are given in Section 5.

Analysis of coupling characteristics of two exciters

When the two motors operate at the steady state, the motion of the system has an effect on their toques. The total torque of the motors can be represented as follows:

$$\begin{aligned}&(J_{m3}+m_{01}r^2)\omega_{m0}(\dot{\bar{\varepsilon}}_1+\dot{\bar{\varepsilon}}_2)+f_{d1}\omega_{m0}(\bar{\varepsilon}_1+\bar{\varepsilon}_2+1)=T_{e1}-T_{L1}\\&(J_{m4}+m_{02}r^2)\omega_{m0}(\dot{\bar{\varepsilon}}_1-\dot{\bar{\varepsilon}}_2)+f_{d2}\omega_{m0}(\bar{\varepsilon}_1-\bar{\varepsilon}_2+1)=T_{e2}-T_{L2}\end{aligned}\tag{1}$$

If the system achieves self-synchronization, i.e. $\bar{\varepsilon}_1=\bar{\varepsilon}_2=0$, T_{L1} and T_{L2} can be rewritten as:

$$\begin{aligned}&T_{L1}=\chi_a+\chi_{f1}\\&T_{L2}=-\chi_a+\chi_{f1}\end{aligned}\tag{2}$$

where:

$$\begin{aligned}&\chi_a=\frac{1}{2}m_{01}r^2\omega_{m0}^2W_c\sin 2\alpha\\&\chi_{f1}=\frac{1}{2}m_{01}r^2\omega_{m0}^2W_{s0}+\frac{1}{2}m_{01}r^2\omega_{m0}^2W_s\sin 2\alpha\\&\chi_{f2}=\frac{1}{2}\eta^2m_{01}r^2\omega_{m0}^2W_{s0}+\frac{1}{2}m_{01}r\omega_{m0}^2W_s\cos 2\alpha\\&W_c=-\eta(\mu_x\cos\gamma_x-\mu_y\cos\gamma_y+(l_x^2+l_y^2)\mu_\psi\cos\gamma_\psi)\\&W_{s0}=\mu_x\sin\gamma_x+\mu_y\sin\gamma_y-(l_x^2+l_y^2)\mu_\psi\sin\gamma_\psi\\&W_s=-\eta(-\mu_x\sin\gamma_x+\mu_y\sin\gamma_y-(l_x^2+l_y^2)\mu_\psi\sin\gamma_\psi)\end{aligned}$$

χ_a, χ_{f1} and χ_{f2} are greater than zero. Hence, χ_{f1} and χ_{f2} as load torques act on the two motors to decrease their angular velocity. χ_a acts on the motor with higher speed as the load torque to decrease its angular velocity while it acts on the motor with lower speed as the driving torque to increase its angular velocity. The load torque relates to the sine effects of the phase angles γ_x, γ_y and γ_ψ. Since the damping constants of the system are very small, $\sin\gamma_x$, $\sin\gamma_y$ and $\sin\gamma_\psi$ can be considered to be zero. χ_{f1} and χ_{f2} are so small that their effects on motors can be ignored. The main effect of rigid frame on motors comes from the coupling torque χ_a. χ_a represents the effect of motion excited by one exciter on the other one due to the motions of the system.

The coupling torque χ_a consists of three items. Each item involves motor kinetic energy $m_{01}r^2\omega_{m0}^2/2$, and the sine of phase difference of two eccentric blocks. The three items of χ_a are related to the motions of the system in the directions of *x*, *y* and ψ, respectively. Define the sum of W_{ct} and W_{cr} as W_c:

$$W_{ct} = -\eta(\mu_x \cos\gamma_x - \mu_y \cos\gamma_y)$$
$$W_{cr} = -\eta(l_x^2 + l_y^2)\mu_\psi \cos\gamma_\psi \tag{3}$$

where W_{ct} is caused by the translation of the system and W_{cr} is relative to the rotation of the system. Fig. 1 shows the relation between W_c and $l_0 (l_0 = l_x^2 + l_y^2)$. It can be observed from Fig. 1 that compared with the translation, the rotation has a greater impact on W_c.

The process of implementing self-synchronization of the two motors is given as follows. At the beginning, two motors acquire the same rotating speed quickly due to the driving torque that mainly comes from their electromagnetic torques. At the stage of the stable operation, when two eccentric blocks excite the motions of the system in *x*-, *y*-, *z*- and ψ -directions, the motions have a reaction to the motors concurrently. The result of the interaction is the coupling torque χ_a. As the speed of one motor changes, the phase difference between the two motors varies as well. χ_a has a positive correlation with phase difference. Thus χ_a acts on the faster motor as load torque to decrease its angular velocity and acts on the slower motor as driving torque to increase its angular velocity. And ultimately two motors reach the same speed.

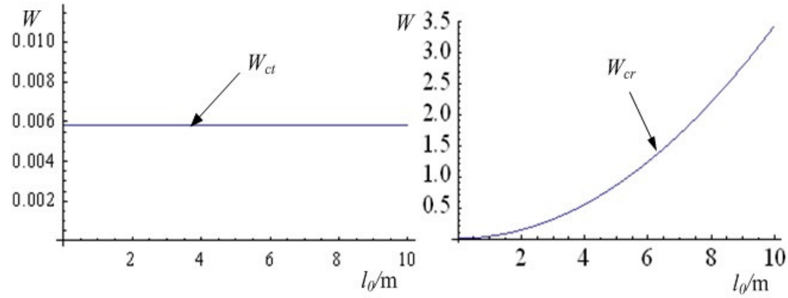

Fig. 1. Effects of translation and rotation on parameter W_c

Calculation of the angular velocity and phase difference

With the electromagnetic torques T_{e1} and T_{e2} shown in the part 1, angular acceleration in the different speed of two motors can be obtained at the beginning. After reaching the steady stage, the angular acceleration is determined by the electromagnetic torque, coupling torque and tiny disturbance. Selecting the appropriate time interval, we can get the speed simulation curve. The steps of this algorithm are summarized as follows:

Step 1: Input the structural parameters of the vibrating system, such as the mass of rigid frame, the mass of two eccentric blocks and the size of the system.

Step 2: Select time interval δ and disturbance Δ, and calculate the angular acceleration of two motors according to T_{e1}, T_{e2}, δ, Δ and moments of inertia of two exciters.

Step 3: Calculate angular velocity based on the step 2 until the system reaches steady stage.

Step 4: Add the stochastic disturbance and assume the two motors have different speeds V_1 and V_2.

Step 5: Take into account the electromagnetic torques and update.

Step 6: If two motors are installed in the mechanism proposed in this paper, proceed to step 7. Otherwise, return to step 4.

Step 7: Calculate the phase difference 2α using the angular velocities of two motors.

Step 8: Calculate the coupling torque based on step 6.
Step 9: Update the angular velocities as a result of the coupling torque.
Step 10: Return to step 4.
Step 11: Draw the velocity curve.
Step 12: Draw the phase difference curve.

In this paper, the parameters of the vibrating system are as follows: $m_1 = 500$ kg, $m_2 = 100$ kg, $m_{01} = m_{02} = 10$ kg, $l_x = 1$ m, $l_y = 0.4$ m, $r = 0.15$ m, $k_x = k_y = k_f = 1560$ kN/m, $k_1 = 2222.5$ kN/rad, $J = 300$ kg/m^2, $f_1 = 2000$ N·s/m, $f_x = f_y = f_f = 700$ N·s/m, $\omega_{m0} = 980 \times 2 \times \pi / 60$ rad/s, $\gamma_y = -0.8214^\circ$, $\gamma_1 = -4.0628^\circ$, $\gamma_z = 4.7131^\circ$. Time-interval is 0.01s, 2000 points are used for the simulation; each step has a ±5-margin for error. $m_{01} = m_{02} = 10$ kg and $\eta = 1$. Fig. 2 provides the simulation of rotational velocities of two motors in their individual operation. Fig. 3 shows the speed difference of two motors after their angular velocities reach the steady state. During the initial phase, the speed of motor 2 is higher than that of the motor 1, as the electromagnetic torque of motor 2 is bigger than that of the motor 1. During the steady state, the two motors keep the speed difference of ±20 due to the disturbance. The rotational velocities of the two motors are illustrated in Fig. 4, while Fig. 5 illustrates their speed difference when they are installed in the mechanism. Fig. 6 shows the phase difference of two eccentric blocks when the two motors operate individually and in the coupling condition. Obviously, the rotational velocities of the two motors installed in the mechanism fluctuate with the disturbance, but the amplitude of the speed difference is nearly the same and the phase difference is much smaller in comparison with that in individual state. In other words, the two motors regain the same speed immediately through the reaction of the rigid frame, provided that they are installed in the mechanism proposed in this paper. When the velocities change, two motors can implement the synchronization very well.

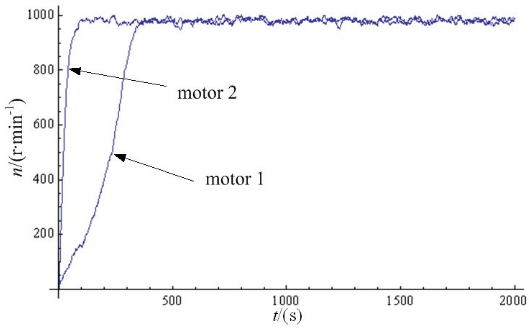

Fig. 2. Rotational velocities of the two motors operating individually

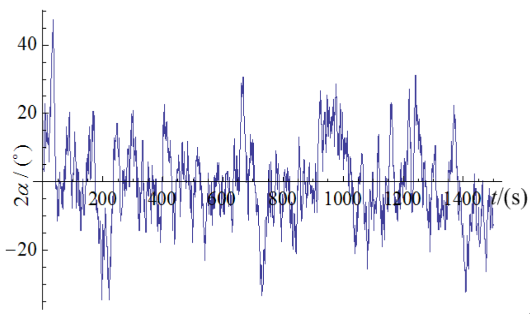

Fig. 3. Speed difference of two motors operating individually

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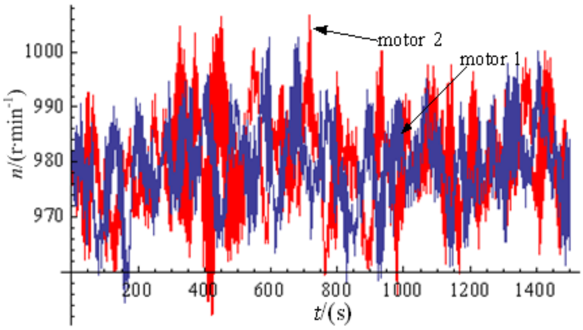

Fig. 4. Rotational velocities of two motors installed in the mechanism

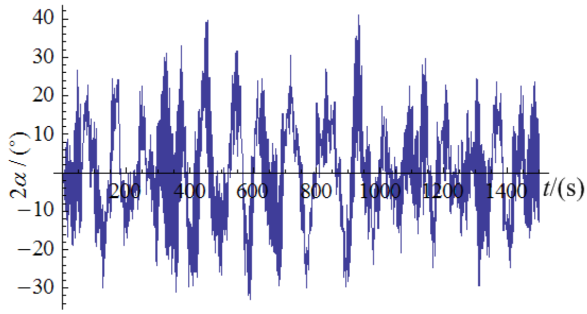

Fig. 5. Speed difference of two motors installed in the mechanism

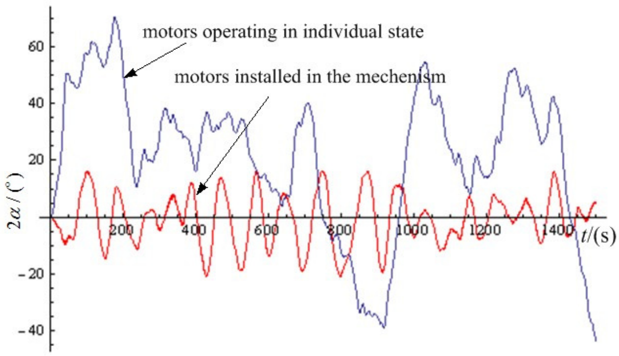

Fig. 6. Phase difference between the two eccentric blocks

Calculation of the self-synchronous stability

In the first part, the condition for the stability of the synchronous operation is obtained. The characteristic equation for eigenvalue λ is as follows:

$$\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0 \tag{4}$$

where $c_1 = 4\omega_{m0} h_1/h_2$, $c_2 = 2\omega_{m0} h_2/h_0$, $c_3 = 2\omega_{m0} h_3/h_0$.

The stability condition of the system can be expressed as follows:

$$h_0 > 0,\ h_1 > 0,\ h_3 > 0,\ 4 \cdot h_1 \cdot h_2 - h_0 \cdot h_3 > 0. \tag{5}$$

To investigate the stable domain of the system, $r_m (r_m = m_0 / M)$ and l_0 are defined as variable parameters, which range 0~0.1 and 0~20 m, respectively. Substitute r_m and l_0 into Eq. (6) to satisfy $h_0 = 0$, $h_1 = 0$, $4 \cdot h_1 \cdot h_2 - h_0 \cdot h_3 = 0$ and $h_3 = 0$. Fig. 7 indicates the stable domain of synchronization in $r_m - l_0$ plane and the curve of maximum value l_0 when the values of η are 0.2, 0.5 and 1, respectively.

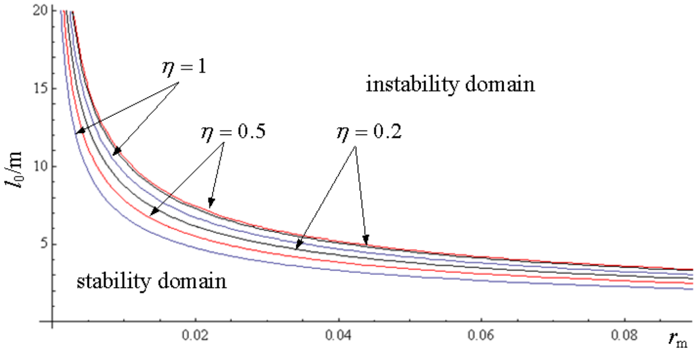

Fig. 7. Stability domain of synchronization in $r_m - l_0$ plane

It can be observed from Fig. 7 that with the increase of the mass ratio of the eccentric block and system, the stable region of the system shrinks. It also demonstrates that as the system size increases and the structural being more symmetrical, the stability becomes worse.

Calculation of the response of the system

In the first part, when the system reaches the steady stage, the response of the system can be given by:

$$\begin{aligned}
x &= r\mu_x[\sin(\varphi + \alpha + \gamma_x) + \eta \sin(\varphi - \alpha + \gamma_x)] \\
y &= r\mu_y[\cos(\varphi + \alpha + \gamma_y) - \eta \cos(\varphi - \alpha + \gamma_y)] \\
z &= r\mu_z[\sin(\varphi + \alpha + \gamma_z) + \eta \sin(\varphi - \alpha + \gamma_z)] \\
\psi &= r\mu_\psi[l_x \cos(\varphi + \alpha + \gamma_x) + l_y \sin(\varphi + \alpha + \gamma_x) - l_x \eta \cos(\varphi - \alpha + \gamma_x) - l_y \eta \sin(\varphi - \alpha + \gamma_x)]
\end{aligned} \tag{6}$$

Substituting the system parameters into Eq. (6), we obtain the response of the vibrating system. Fig. 8 shows the displacement in x-direction of the body 1, the displacement in y-direction of the body 1, the displacement in x-direction of the body 2 and the displacement in ψ-direction of the body 1. The red line indicates the operational state of the two motors installed in the mechanism proposed in this paper, while the blue line indicates the operational state of two motors operating individually.

As indicated in Fig. 8, in the directions of y and ψ, the system fluctuates less comparing with the motors running separately, when the vibrating parameters of the system meet the synchronous conditions. The simulation results also verify the correctness of the theoretical analysis.

Impact of system parameters

The condition of implementing the synchronization is:

$$m_0 r^2 \omega_{m0}^2 W_c > Abs[(T_{e01} - T_{e02}) - (f_{d1}\omega_{m0} - f_{d2}\omega_{m0}) - \frac{1}{2}(1-\eta^2) m_0 r^2 \omega_{m0}^2 W_{s0}] = T_{diff} \tag{7}$$

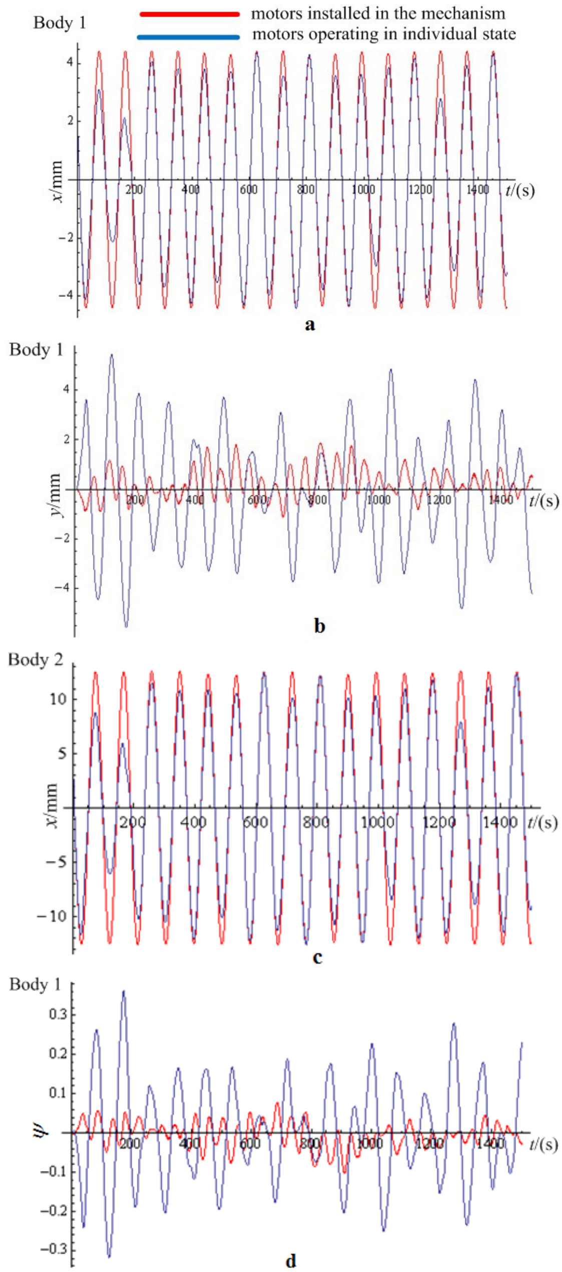

Fig. 8. Results of the simulation: (a) displacement in x-direction of body 1; (b) displacement in y-direction of body 1; (c) displacement in x-direction of body 2; (d) displacement in ψ -direction of body 1

It can be seen from the Eq. (7) that the two motors can implement the self-synchronization even though their rated angular velocities are distinct. If the angular velocities of the two motors are assumed to be ω_1 and ω_2, respectively, the angular velocity of the synchronous operation ω_{m0} must satisfy: $\min(\omega_1, \omega_2) < \omega_{m0} < \max(\omega_1, \omega_2)$.

In this paper, the allowed maximum of T_{diff} is $101.86\ \mathrm{N}\cdot\mathrm{m}$. Fig. 10 provides synchronizing characteristics when $\omega_1 = 980$ r/min and $\omega_2 = 940$ r/min. In this case $T_{diff} \approx 50.33\mathrm{N}\cdot\mathrm{m} < 101.86\ \mathrm{N}\cdot\mathrm{m}$ and the system can implement the self-synchronization.

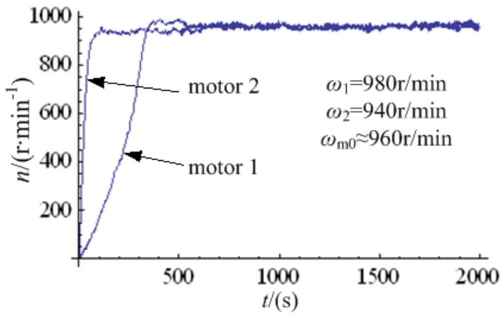

Fig. 9. Synchronizing characteristics in the different rated rotational velocities

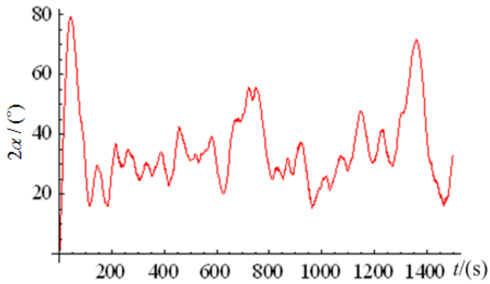

Fig. 10. Phase difference of two motors in different rated rotational velocities

Before the time t reaches 500 s, the two motors operate individually and the difference of rotational velocity is about 40 r/min. When t is in the interval of (500, 1000), the coupling torque is added, and the coupling speed is approximately 960 r/min. Fig. 10 illustrates the phase difference of the two eccentric blocks after the system achieves self-synchronization, $2\alpha \approx 30^\circ$, and $m_0 r^2 \omega_{m0}^2 W_c \sin 2\alpha \approx 46.86$. Compared with the theoretical results, its error is lower than 10 %. Fig. 11 presents synchronizing characteristics when $\omega_1 = 980$ r/min and $\omega_2 = 880$ r/min. Here $T_{diff} \approx 119.62\mathrm{N}\cdot\mathrm{m} > 101.86\mathrm{N}\cdot\mathrm{m}$, Eq. (7) cannot meet, and the system cannot implement the self-synchronization.

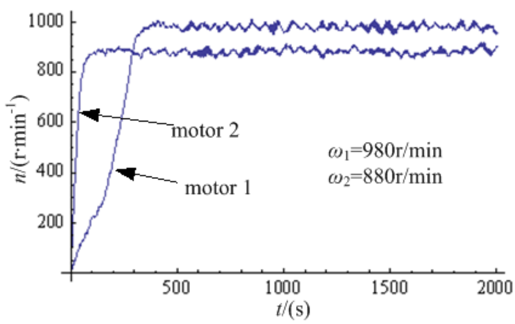

Fig. 11. Non-synchronizing characteristics in different rated rotational velocities

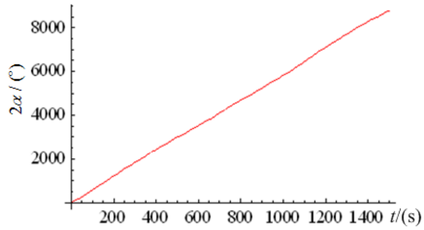

Fig. 12. Phase difference of two motors of non-synchronization

After the two motors reach self-synchronous state, an interesting phenomenon demonstrates that even if the power supply of one motor is cut off, the synchronous motion of the two motors can continue due to the coupling torque. This phenomenon can be explained by using Eq. (7). With one motor stops the power, the electromagnetic torque of this one is zero. In this paper $T_{diff} \approx 51.3\ \mathrm{N \cdot m} < 101.86\ \mathrm{N \cdot m}$ and the coupling torque is big enough to overcome the load torque. This specific synchronous rotation using only one power supply is called vibratory synchronization transmission [8]. Fig. 13 illustrates this case.

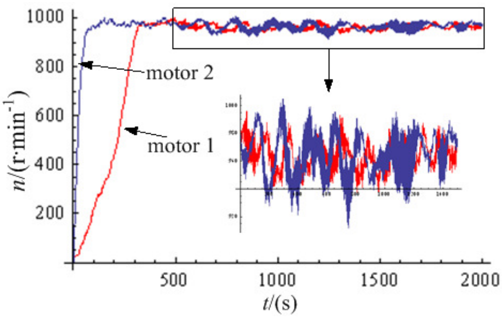

Fig. 13. Synchronizing characteristics of vibratory synchronization transmission

At t = 500 s the power supply of one of the two motors is cut off. The rigid frame transmits the driving torque from the motor (with the power supply) to the other, so they can still synchronize and the coupling speed is about 960 r/min. Fig. 14 provides the phase difference of vibratory synchronization transmission $2\alpha \approx 20^{\circ}$.

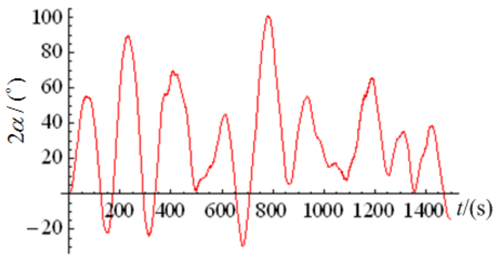

Fig. 14. Phase difference of vibratory synchronization transmission

Conclusions

A mechanism is proposed to analyze the coupling dynamic characteristics of the vibrating system quantitatively. The motions of the system operating at the stable stage are excited by the two eccentric blocks, while simultaneously these motions in the x-, y-, z- and ψ - directions react to the motors. As a result, this interaction produces the coupling torque including three items, among which two items are concerned with the translation and another item is related to the rotation. As the speed of one motor changes, the phase difference of the two motors changes as well. The coupling torque has a positive correlation with the phase difference. Thus the coupling torque acts on the motor with higher speed as load torque to decrease its angular velocity while it acts on the motor with lower speed as driving torque to increase its angular velocity. Ultimately the two motors reach the same speed. The phase difference of two eccentric blocks is caused by the difference of the rated revolution of two motors. When the phase difference reaches a certain value, the speeds of two motors are the same and the system implements self-synchronization. If the two motors and eccentric blocks are completely symmetrical, the motions in y- and ψ -directions of the system can be neglected. The system can only implement the horizontal movement.

When the phase difference is 90°, if the coupling torque is smaller than T_{diff}, the self-synchronization cannot be achieved. But at this time, the two motors are still influenced by the motions of the rigid frame and their angular velocities have a large cyclical variation. With the calculation for the stable domain of synchronization, it indicates that the stability depends on the structural parameters of the system, i.e. the mass ratio of the two eccentric blocks and the distance between the motor and the centroid of the rigid frame. The smaller the mass ratio and the distance are, the stronger is the ability of maintaining synchronous stability. The phenomenon, dubbed the vibratory synchronization transmission, is simulated. It demonstrates that even if the power supply of one motor is cut off, the synchronization of the two motors will not be influenced.

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