842. Identification of modal parameters from structural ambient responses using wavelet analysis

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Abstract. Runyang suspension bridge is the longest suspension bridge in China with a main span of 1490 meters. During the construction of the bridge, a structural health monitoring system was installed, which was designed by the Southeast University. Since the bridge was open to traffic, quantities of structural ambient vibration responses have been recorded by the monitoring system. It is important to extract dynamic characteristics from these responses for structural assessment and maintenance. This paper presents the study on extraction of modal parameters from the Runyang suspension bridge structural health monitoring system records using wavelet analysis. Time-frequency domain modal identification using wavelet analysis is studied with an emphasis on the efficient approach for determination of the dilatation parameter. Then the wavelet analysis based method was adopted to identify dynamic properties including modal frequencies, mode shapes and damping ratios from the ambient vibration responses recorded by the monitoring system. Identified results were compared with those from Enhanced Frequency Domain Decomposition method and Stochastic Subspace Identification method. The differences between results are analyzed. Suggestions on future study are also given.

Keywords: modal identification, structural ambient responses, wavelet analysis.

Introduction

More and more complex civil infrastructures are built in China with the development of its economy, e.g., long-span bridges and high-rise buildings. Among them, long-span cable-supported bridges attract much attention because of two reasons. The first is that these bridges have much more significant spanning ability than traditional bridges such as arched bridges and beam bridges. This makes cable-supported bridges such as suspension bridge and cable-stayed bridge priority choices for the engineer to build a bridge crossing the sea or river. The second is that the long-span cable-supported bridges are complex and sophisticated that nearly every bridge could be seen as the development landmark of the national structural engineering.

In 1997, Tsing Ma Bridge, which was built in Hong Kong Special Administrative Region of China, was opened to traffic. Since then, the span of suspension bridge in China has increased from 1377 m (Tsing Ma Bridge) to 1490 m (Runyang Suspension Bridge). Also, the span of cable-stayed bridge has increased rapidly to 1088 m (Sutong Yangtze River Bridge, or Sutong Bridge). These bridges make great contributions to the development of China economy. Meanwhile, these bridges are becoming sensitive to various loads such as wind and traffic because the flexibility increases with the span. Besides, the working environments of the bridge are really critical, for example, the fluctuating temperature and heavy humidity. Hence, the damages will possibly occur in the structures during the service life. The damages would lead to terrible consequences if they are not detected in time. Therefore, it is necessary to perform structural health monitoring (SHM) of the bridge [1].

The acceleration is indispensable for structural health monitoring because it is the most important type of response which implies dynamic characteristics such as the modal frequencies, modal damping ratios and mode shapes. These modal parameters could be adopted in health monitoring oriented finite element modeling, damage identification vibration control and response prediction [1, 2]. Due to its importance, modal parameters of most of the long-span bridges in the world have been identified either in the dynamic testing before opening to traffic or from the structural health monitoring system (SHMS) records [3-6]. And, there is a trend to develop automatic identification algorithms [4].

In the past decades, great progresses have been made in the field of modal identification and varieties of identification methods have been proposed. The available identification methods could be classified into three categories depends on the domain to perform the identification [7, 8]: time domain methods, frequency domain methods and time-frequency domain methods. Wavelet analysis is a time-frequency domain signal processing techniques. It is able to provide time-frequency description of signals. In recent year, modal identifications using wavelet analysis have been studied [9-11]. In the study of Lardies [9], a method based on wavelet analysis to determine the modal frequency, damping ratio and mode shapes has been proposed. A modified Morlet wavelet function is proposed to balance the time resolution and frequency resolution so that the close modes could be identified.

In this paper, modal identification method based on wavelet analysis is studied. Then, the method is used to identify modal parameters from SHMS records of a long-span Runyang suspension bridge which crosses the Yangtze River in Jiangsu Province, China. The bridge has a main span of 1490 meters as mentioned above. It is the longest suspension bridge in China and the third longest suspension bridge in the world when it was opened to traffic in 2005.

The paper is organized as follows: The formula of modal identification using wavelet analysis is given in section two. The efficient approach for determination of dilatation parameter is analyzed in section three. In section four, modal identification of Runyang suspension bridge is described in detail.

1. Modal identification using wavelet analysis

Wavelet transform of signal is the time-frequency decomposition of signal using the scaled and translated wavelet basis function. The transformation of time domain signal $x(t)$ could be described as follows:

$$w_x(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}x(t)\psi^*(\frac{t-b}{a})dt \tag{1}$$

in which $\psi^*\left(\frac{t-b}{a}\right)$ is the complex conjugate of wavelet function, a and b represent the dilatation parameter and translation parameter respectively.

Supposing the response of a viscously damped single degree of freedom system is:

$$x(t)=Ae^{-\zeta\omega_n t}\cos(\omega_d t+\varphi_0) \tag{2}$$

where A is the amplitude, ω_n and ω_d are angular undamped and damped natural frequencies respectively, φ_0 is the initial phase and ζ is the damping ratio. If the damping ratio is smaller than 1, the system is an underdamped system. The signal can be considered asymptotic.

The wavelet transform of signal $x(t)$ is:

$$w_x(a,b)=\frac{\sqrt{a}}{2}Ae^{-\zeta\omega_n b}\psi^*(a\omega_d)e^{j(\omega_d b+\varphi_0)} \tag{3}$$

The modulus of the wavelet transform is:

$$\left|w_x(a,b)\right| = \frac{\sqrt{a}}{2} A e^{-\zeta\omega_n b} \left|\psi^*(a_0\omega_d)\right| \tag{4}$$

Its logarithm form is:

$$\ln\left|w_x(a,b)\right| = -\zeta\omega_n b + \ln\left(\frac{\sqrt{a}}{2} A \left|\psi^*(a_0\omega_d)\right|\right) \tag{5}$$

The slope of the line of the logarithm of the modulus is $-\zeta\omega_n$. Using s_1 to represent the absolute value of the slope:

$$s_1 = \zeta\omega_n \tag{6}$$

From equation (3), the phase of the wavelet transform is:

$$\text{phase}\left(w_x(a,b)\right) = \omega_d b + \varphi_0 \tag{7}$$

The slope of the phase line is ω_d. Using s_2 to represent the slope:

$$s_2 = \omega_d \tag{8}$$

Since:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{9}$$

equation (8) could be rewritten as:

$$s_2 = \omega_n\sqrt{1-\zeta^2} \tag{10}$$

From equation (6) and equation (10), modal frequency (undamped natural frequency) and damping ratio could be estimated as:

$$\zeta = \sqrt{\frac{(s_1)^2}{(s_1)^2 + (s_2)^2}} \tag{11}$$

$$\omega_n = \frac{s_1}{\zeta} \tag{12}$$

In the implementation, Morlet wavelet could be used as wavelet basis function:

$$\psi(t) = e^{j\omega_0 t} e^{-\frac{t^2}{2}} \tag{13}$$

For a given value of the dilatation parameter a, the spectrum of the Morlet wavelet has a fixed bandwidth. If the analyzed frequency is important, the dilatation parameter becomes small and the spectrum of the Morlet wavelet function is wide. There is then a bad spectral resolution. A balance parameter N is proposed by Lardies [9] to provide a better resolution for the identification of closely spaced modes:

$$\psi(t) = e^{j\omega_0 t} e^{-\frac{t^2}{N}} \tag{14}$$

2. Determination of dilatation parameters

The dilatation parameter a in equation (1) is the most important controlling parameters in the wavelet based modal identification. Hence, the parameter should be carefully determined. In the study of Lardies [9], the dilatation parameters should be chosen before identification as follows. First, wavelet transforms are performed using multiple dilatation parameters. Then, the wavelet coefficients are plotted as Fig. 1. The ordinate represents the dilatation parameters. The abscissa means the normalized time interval. The white line is a ridge line. Dilatation parameters for each mode are estimated according to those scales corresponds to the white ridge lines in the plot. When the parameter N is used, it is demonstrated that the ridge lines are more clearly separated than using the original Morlet wavelet function.

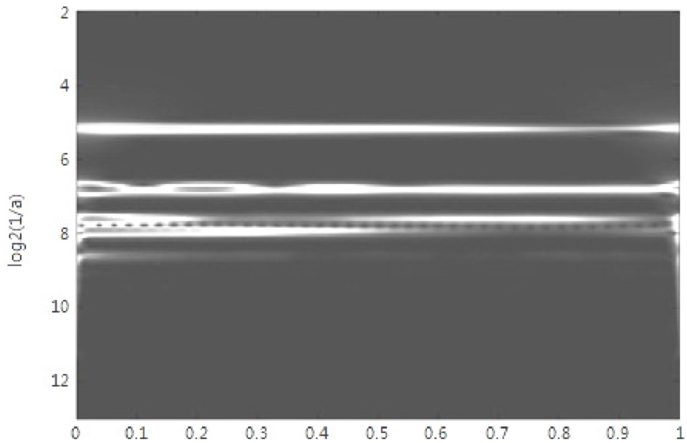

Fig. 1. Example of ridge line

The dilatation parameter determines the central frequency of the band-pass filter, which is actually the Fourier transform of the wavelet. The disadvantage of determination the dilatation parameter according to the plot is that wavelet transforms have to be computed on so many redundant scales. In fact, one structural mode corresponds to one scale. Hence, it is not necessary to perform wavelet transform on so many scales. It is really computationally intensive and time-consuming in the real application, especially in the SHMS case, where quantitative data have to be processed using this plot based method. Hence, using the plot and the ridge line to determine the dilatation parameter is not acceptable in practice. An efficient way to compute the dilatation parameter is described below.

The dilatation parameter a could be converted to frequency according to equation (15):

$$f = \frac{f_c}{a \times \Delta t} \tag{15}$$

where f_c represents the central frequency of the wavelet function and Δt represents the sampling interval. Supposing the damped natural frequency in Hz is f_d, which could be estimated using peak-picking from the PSD plot. Then, the dilatation parameter a could be estimated as:

$$a = \frac{f_c}{f_d \times \Delta t} \tag{16}$$

In this way, the number of dilatation parameter a is equal to that of structural modes. Hence, the time to compute wavelet transform is greatly reduced. Furthermore, the central frequency of band-pass filter will be consistent with modal frequency since the parameter a is estimated more objectively and accurately than those from ridge line. To ensure the estimation accuracy of peaks, it is also required to improve the frequency resolution. And, this could be easily implemented.

3. Modal identification of Runyang suspension bridge

3. 1. Runyang suspension bridge and its SHMS

Runyang cable-supported bridge is a newly built bridge that crosses the Yangtze River in Jiangsu Province, China. The bridge connects the two cities of Zhenjiang and Yangzhou. The north part of the bridge is a cable-stayed bridge with a main span of 406 meters and the south part of the bridge is a suspension bridge with a main span of 1490 meters. Fig. 2 shows the Runyang Suspension Bridge.

Fig. 2. Runyang suspension Bridge

During the construction of the bridge, a structural health monitoring system designed by the research group of Southeast University was installed [12]. More than ten kinds of sensors were installed to measure the loads and the responses, which always contain the static and dynamic features of the structure. To fully understand the dynamic properties of the bridge under the real working condition, the records of the SHMS should be analyzed to extract modal parameters. In this study, modal parameters of the vertical modes are concerned, therefore, those vibrations by accelerometers in the vertical direction and installed in sections along the girder are adopted.

For the suspension bridge, vibrations are measured by accelerometers installed in nine sections along the main girder. Two sections are assigned at the north end and the south end of the main span girder. The other seven sections are equally distributed between these two ends. Fig. 3 demonstrates the locations of the vibration monitoring sections in the main girder. The locations are marked using hollow circles. In each of the section, accelerometers are oriented to both the vertical direction and the lateral direction. Therefore, the vertical and lateral vibrations could be measured simultaneously. But, as mentioned before, only the vertical vibration responses are used in the study.

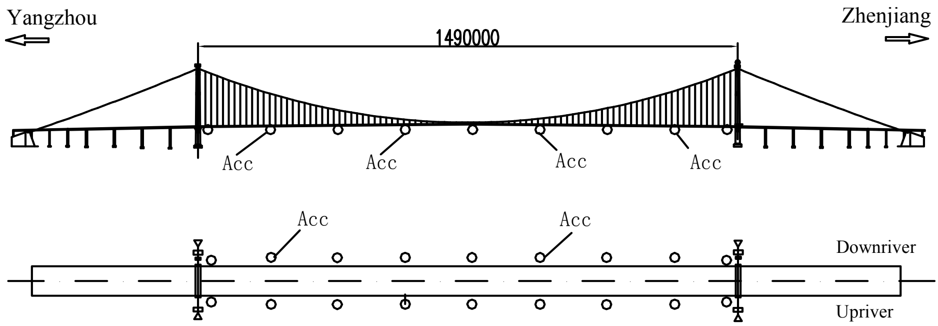

Fig. 3. Vibration monitoring sections in the main girder

Fig. 4 gives the number of accelerometers for the SHMS system of the bridge. The accelerometers in the upriver side are set to be 1, 3, 5, etc. The accelerometers in the downriver side are set to be 2, 4, 6, etc. In the figure, the serial numbers assigned in the SHMS are also shown. For example, the sensor installed in the middle of the upriver side is numbered 'ZD080104'. The counterpart in the downriver side is 'ZD080103'.

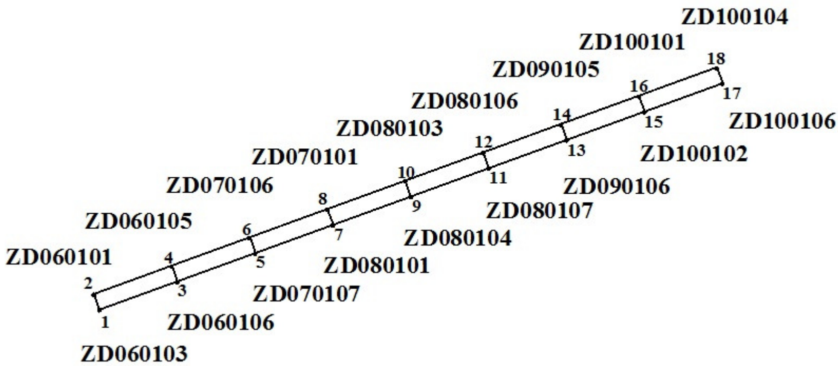

Fig. 4. Serial numbers of accelerometers in the suspension bridge

3. 2. Signal processing

For some engineers, modal identification seems to be a kind of standard-procedure operation. Or it is some kind of a black box, to which one just inputs the measured data and gets the output of the modal parameters. Actually, this is quite wrong in engineering application because the measurements could be quite un-idealistic. The identification could extract modes from responses only if there were information about modes implied in the responses. If the responses are poorly measured, none of the identification method works. Therefore, the data records must be pre-processed before being adopted in the identification.

In this study, the data record of the first hour in the early dawn of May 2nd, 2006 was employed. The sampling frequency of accelerometers in the main girder was 20 Hz. Therefore, the data from single accelerometer has 72000 points. It is clear from Fig. 4 that there are 18 sensors. That's to say, the data record consists of 18 channels. In the pre-processing, signal should be checked channel by channel. Fig. 5 to Fig. 7 show the response output by the vertical accelerometers at the upriver side of the main girder and corresponding power spectral density (PSD).

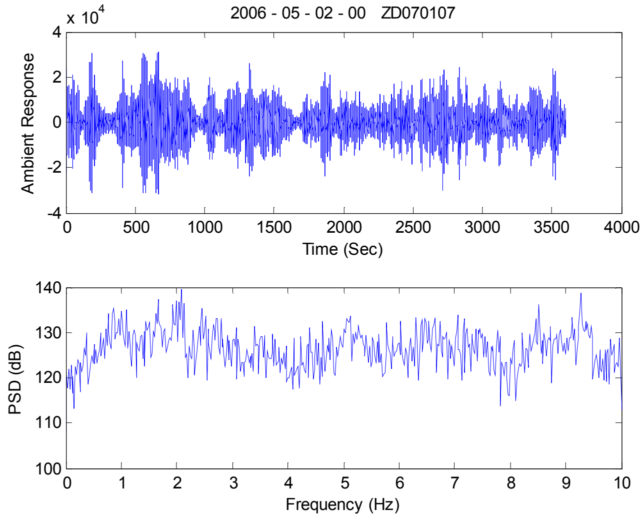

Fig. 5. Upriver side, quarter span

The reason why these accelerometers are paid much attention is that these points are key points in the expression of mode shapes. For example, the nodes at the middle span are either node with the maximum mode shape amplitude or with the minimum mode shape amplitude in both the symmetric modes or asymmetric modes. From the figures, it could be concluded as follows:

(1) The peaks of the PSD plots indicate the damped natural frequencies of the bridge. This means that the accelerometers at the quarter span, the middle span and the three-quarter span effectively measured the vibrations of the main girder.

(2) The peaks in the PSD plots of middle span responses are clearer than those in the PSD plots of quarter span or three-quarter span.

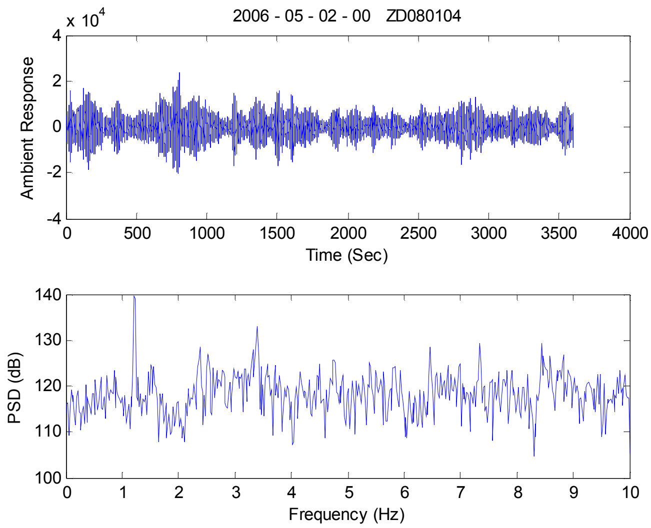

Fig. 6. Upriver side, middle span

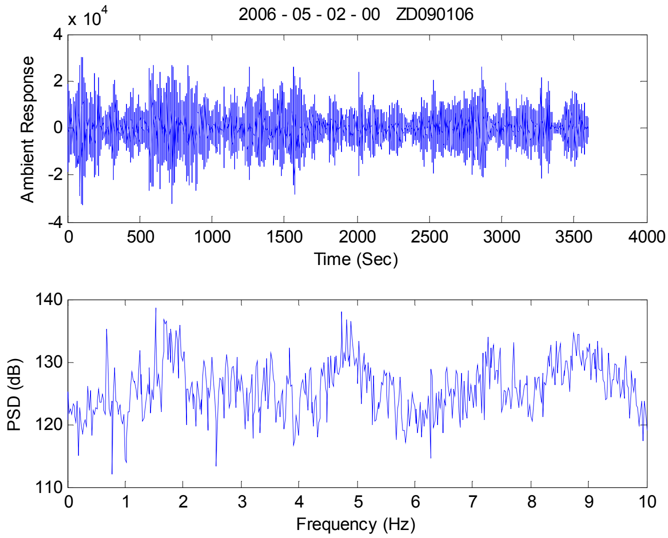

Fig. 7. Upriver side, three-quarter span

Responses of other accelerometers are also checked. Fig. 8 is the response measured by the accelerometer installed at the three-eighth span on the upriver side. It could be seen that the signal-to-noise ratio is not high comparing with the responses at the middle span.

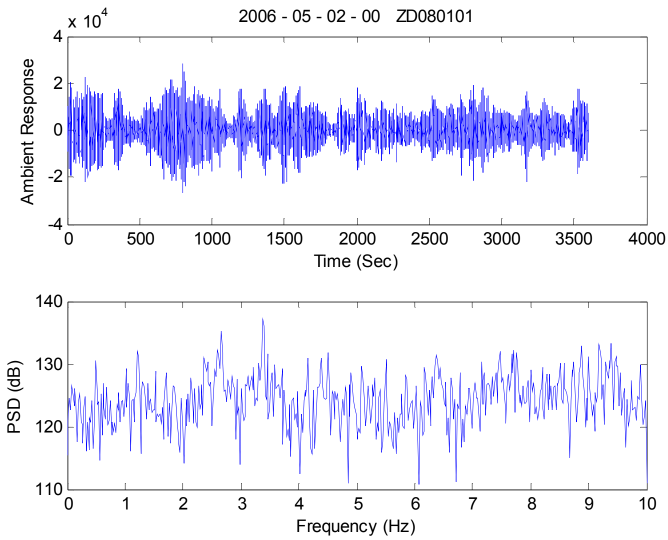

Fig. 8. Upriver side, three-eighth span

After signal processing, measurements of one accelerometer is abandoned, otherwise it will lead to a bad estimation of modal parameters. And, to display the mode shape, the motion of this node is set to be computed according to the motions of the neighboring nodes.

3. 3. Modal identification using wavelet analysis

The sampling frequency of SHMS accelerometers is 20 Hz, this is far more than it is required. According to the results of preliminary finite element analysis, modal frequencies of low order modes of vertical direction are less than 1 Hz (see the first column in Table 1). Hence, the data were down-sampled by 10. The autocorrelations for records of every accelerometer were computed since the proposed method is only able to extract modal parameters from free decay type of signal. And, the autocorrelation of ambient response has similar characteristics to free decay signal.

It is really difficult to identify the modal parameters of Runyang suspension bridge since it has high density modes or close modes. To circumvent these problems, the son wavelet is carefully selected. The modes to be identified are specified by peak-picking in the PSD plot. Then the dilatation parameters corresponding to these modes are estimated not by the ridge line but by the equation (16).

Modal parameters of vertical modes are identified. For each mode, there are nine groups of modal frequency and damping ratio since the records from nine accelerometers are used separately. The average of the nine groups of modal frequencies and damping ratios are taken as the final result. Averaged modal frequencies and damping ratios of the first four modes are listed in Table 1. From the modal frequencies shown in the table, it could be seen that modal frequencies of the second and the third modes are so close that differences between them is less than 0.02 Hz.

Table 1. Identified modal parameters

Mode No.	FEA frequency (Hz)	Modal frequency (Hz)	Damping ratio (%)	Mode shape
1	0.117	0.124	0.443	Symmetric bending
2	0.162	0.166	0.488	Symmetric bending
3	0.192	0.181	0.343	Anti-symmetric bending
4	0.287	0.285	0.266	Anti-symmetric bending

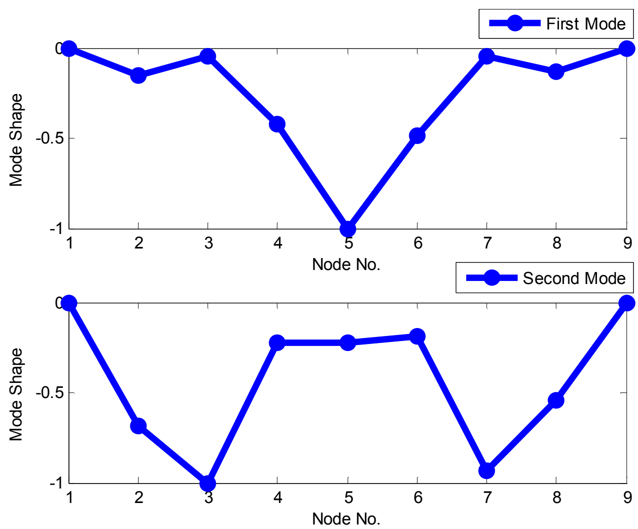

Fig. 9. Identified mode shapes

Fig. 9 gives the identified mode shapes of the two modes. The mode shapes are not smooth due to the limitation of the sensors. But still it could be seen that these two modes have symmetric mode shapes and able to describe the characteristics of the symmetric bending.

3. 4. Comparison with the result from EFDD and SSI

As comparison, modal identification results from Enhanced Frequency Domain Decomposition [13] method and Stochastic Subspace Identification [14] method are listed in Table 2. From the table it could be observed that the damping ratios from wavelet analysis method (WA) are similar to those from EFDD method but different from those identified using SSI method. This is because WA and EFDD actually identify damping ratios using the same technique: the logarithm decrement method. And, it is a problem worthy of studying in the future to improve the ability of these two methods to identify damping ratio from low signal-to-noise ratio ambient responses.

Table 2. Comparison of modal parameters of three methods

Mode No.	FEA frequency (Hz)	WA		EFDD		SSI	
		Frequency (Hz)	Damping ratios (%)	Frequency (Hz)	Damping ratios (%)	Frequency (Hz)	Damping ratios (%)
1	0.117	0.124	0.443	0.122	0.475	0.122	0.215
2	0.162	0.166	0.488	0.167	0.408	0.167	0.545
3	0.192	0.181	0.343	0.186	0.376	0.187	0.621
4	0.287	0.285	0.266	0.286	0.258	0.287	0.530

4. Conclusion

Wavelet analysis based modal identification method is applied to identify modal parameters from structural health monitoring system records of Runyang suspension bridge. The records are ambient vibration responses measured by accelerometers installed on nine sections in the main girder of the bridge. Determination of the dilatation parameter, data pre-processing and modal parameter identification are the three major issues discussed. An approach to determine the dilatation parameter which is much more efficient than plot-based method was discussed. In the data pre-processing, it was found that the SHMS records should be carefully processed. This will not only be helpful to improve the accuracy of modal identification but also be helpful for system maintenance to detect the malfunctioned sensors. In the modal identification, modal parameters, including modal frequencies, mode shapes and damping ratios, were identified using the wavelet analysis. And, the obtained results were compared with those from EFDD and SSI techniques. For modal frequencies, results from three methods are almost identical. But for damping ratios, results from WA and EFDD are similar but different from those identified using SSI. This is reasonable because WA and EFDD actually both adopt the logarithm decrement method to extract damping ratios. It is an important problem worthy of studying in the future to improve the ability of these two methods to identify damping ratio from low signal-to-noise ratio ambient responses.

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