832. Nonlinear vibration of rectangular plate under the parametric excitation

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Abstract. In this paper, the dynamic behavior of rectangular plate under the in-plane load is studied. The partial differential equation based on the mechanical model is established, which will be deduced into two ordinary differential equations by use of Galerkin method. The existence of 1/2 harmonic solutions of the dynamical system applying the harmonic balance method is analyzed. The amplitude-frequency relationship is found, and the stability of solutions is investigated. The stable zone of dynamical system is determined.

Keywords: rectangular plate, nonlinear vibration, dynamics behavior, parametric excitation.

Introduction

Chia [1], Sathyamoorthy [2] and Chia [3] gave a review of work on the nonlinear vibration of plate. Chu and Herrmann [4] made a fundamental study on the analysis of large-amplitude vibrations of rectangular plates. They studied simply supported rectangular plates and obtained the resonant curves. Many researchers [5–8] compared different results for the backbone curve with those of Chu and Herrmann [4]. Leung et al. [7] also investigated simply supported rectangular plates with movable edges. Hui [9] studied the effect of geometric imperfection. Ribeiro [10] investigated the forced response of simply supported plates with immovable edges. Kadiri et al. [11] developed a simplified analytical approach for the case studied by Chu and Herrmann.

In this paper, the dynamic behavior of rectangular plate under the loading of parameter cycle is studied. The partial differential equation based on the mechanical model is established. Applying the harmonic balance method, the existence of 1/2 harmonic solutions of dynamical system is analyzed. The amplitude-frequency relationship is determined, and the stability of solutions is confirmed.

Basic Equations

It is assumed that the deflection of plate is small enough to make the intersection angle of various parts to be far less than 1.

The transversal vibrating equation of plate is shown as:

$$D\Delta\Delta w = N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} - m\frac{\partial^2 w}{\partial t^2} - (\mu_1 + \mu_2 w^2)\frac{\partial w}{\partial t}. \tag{1}$$

Boundary conditions are shown as:

$$\begin{cases} w(o,y) = w(a,y) = 0 \\ w(x,o) = w(x,b) = 0 \\ w''(o,y) = w''(a,y) = 0 \\ w''(x,o) = w''(x,b) = 0 \\ u(o,y) = u(a,y) = 0 \\ v(x,o) = 0 \end{cases} \tag{2}$$

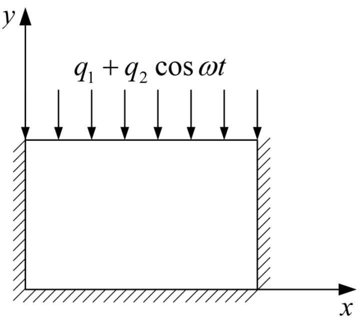

Fig. 1. Sketch of plate and coordinate system

When the plate is bending, the internal forces in the mid-surface are:

$$\begin{cases} N_x = \dfrac{Eh}{1-\nu}(\varepsilon_{xx} + \nu\varepsilon_{yy}) \\ N_y = \dfrac{Eh}{1-\nu}(\varepsilon_{yy} + \nu\varepsilon_{xx}) \,. \\ N_{xy} = \dfrac{Eh}{2(1+\nu)}\varepsilon_{xy} \end{cases} \tag{3}$$

The geometric equations are:

$$\begin{cases} \varepsilon_{xx} = \dfrac{\partial u}{\partial x} + \dfrac{1}{2}(\dfrac{\partial w}{\partial x})^2 \\ \varepsilon_{yy} = \dfrac{\partial v}{\partial y} + \dfrac{1}{2}(\dfrac{\partial w}{\partial y})^2 \\ \varepsilon_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{cases} \tag{4}$$

The motion equations along to *x* and *y* directions are:

$$\begin{cases} \dfrac{\partial N_x}{\partial x} + \dfrac{\partial N_{xy}}{\partial y} - m\dfrac{\partial^2 u}{\partial t^2} = 0 \\ \dfrac{\partial N_{yx}}{\partial x} + \dfrac{\partial N_y}{\partial y} - m\dfrac{\partial^2 v}{\partial t^2} = 0 \end{cases} \tag{5}$$

Substituting (3) and (4) into (5), we obtain:

$$\begin{cases} \dfrac{\partial^2 u}{\partial x} + \dfrac{1-\nu}{2}\dfrac{\partial^2 u}{\partial^2 y} + \dfrac{1+\nu}{2}\dfrac{\partial^2 v}{\partial x \partial y} + \dfrac{\partial w}{\partial x}\left(\dfrac{\partial^2 w}{\partial x^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2 w}{\partial y^2}\right) \\ + \dfrac{1+\nu}{2}\dfrac{\partial^2 w}{\partial x \partial y}\dfrac{\partial w}{\partial y} - \dfrac{m\left(1-\nu^2\right)}{Eh}\dfrac{\partial^2 u}{\partial t^2} = 0 \\ \dfrac{\partial^2 v}{\partial y^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2 v}{\partial x^2} + \dfrac{1+\nu}{2}\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial w}{\partial y}\left(\dfrac{\partial^2 w}{\partial y^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2 w}{\partial x^2}\right) \\ + \dfrac{1+\nu}{2}\dfrac{\partial^2 w}{\partial x \partial y}\dfrac{\partial w}{\partial x} - \dfrac{m\left(1-\nu^2\right)}{Eh}\dfrac{\partial^2 v}{\partial t^2} = 0 \end{cases} \tag{6}$$

According to boundary conditions (2), vibrating modes of the plate can be assumed to be:

$$w(x,y,t) = \varphi(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{7}$$

Substitute (7) into (6) to get non-homogeneous equation:

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2 u}{\partial y^2} + \dfrac{1+\nu}{2}\dfrac{\partial^2 v}{\partial x \partial y} - \dfrac{m\left(1-\nu^2\right)}{Eh}\dfrac{\partial^2 u}{\partial t^2} + F_x\left(x,y,t\right) = 0 \\ \dfrac{\partial^2 v}{\partial y^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2 v}{\partial x^2} + \dfrac{1+\nu}{2}\dfrac{\partial^2 u}{\partial x \partial y} - \dfrac{m\left(1-\nu^2\right)}{2}\dfrac{\partial^2 v}{\partial t^2} + F_y\left(x,y,t\right) = 0 \end{cases} \tag{8}$$

where:

$$\begin{cases} F_x = -\dfrac{\pi^2\varphi^2}{4a}\sin\dfrac{2\pi x}{a}\left[\dfrac{1}{a^2} - \dfrac{\nu}{b^2} - \left(\dfrac{1}{a^2} + \dfrac{1}{b^2}\right)\cos\dfrac{2\pi y}{b}\right] \\ F_y = -\dfrac{\pi^2\varphi^2}{4b}\sin\dfrac{2\pi y}{b}\left[\dfrac{1}{b^2} - \dfrac{\nu}{a^2} - \left(\dfrac{1}{a^2} + \dfrac{1}{b^2}\right)\cos\dfrac{2\pi x}{a}\right] \end{cases} \tag{9}$$

The expression of the solution of equation (8) is shown as follows:

$$\begin{cases} u\left(x,y,t\right) = A_1\sin\dfrac{2\pi x}{a} + B_1\sin\dfrac{2\pi x}{a}\cos\dfrac{2\pi y}{b} + u_0\left(x,y,t\right) \\ v\left(x,y,t\right) = A_2\sin\dfrac{2\pi y}{b} + B_2\sin\dfrac{2\pi y}{b}\cos\dfrac{2\pi x}{a} + v_0\left(x,y,t\right) \end{cases} \tag{10}$$

where $u_0\left(x,y,t\right)$, $v_0\left(x,y,t\right)$ is the solution of homogeneous equations:

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2 u}{\partial y^2} + \dfrac{1+\nu}{2}\dfrac{\partial^2 v}{\partial x \partial y} = 0 \\ \dfrac{\partial^2 v}{\partial y^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2 v}{\partial x^2} + \dfrac{1+\nu}{2}\dfrac{\partial^2 u}{\partial x \partial y} = 0 \end{cases} \tag{11}$$

According to the displacement boundary condition (2), coefficients in equation (10) are determined as follows:

$$\begin{cases} A_1 = -\dfrac{\pi\varphi^2 a}{16}\left(\dfrac{1}{a^2} - \dfrac{\nu}{b^2}\right) \\ A_2 = -\dfrac{\pi\varphi^2 b}{16}\left(\dfrac{1}{b^2} - \dfrac{\nu}{a^2}\right) \\ B_1 = \dfrac{\pi\varphi^2}{16a} \\ B_2 = \dfrac{\pi\varphi^2}{16b} \end{cases} \tag{12}$$

Then, equation (10) is rewritten as:

$$\begin{cases} u(x,y,t) = \dfrac{\pi\varphi^2}{16a}\sin\dfrac{2\pi x}{a}\left(\cos\dfrac{2\pi y}{b} - 1 + \dfrac{\nu a^2}{b^2}\right) + u_0 \\ v(x,y,t) = \dfrac{\pi\varphi^2}{16b}\sin\dfrac{2\pi y}{b}\left(\cos\dfrac{2\pi x}{a} - 1 + \dfrac{\nu b^2}{a^2}\right) + v_0 \end{cases} \tag{13}$$

Therefore, according to the displacement boundary condition, the displacements in neutral plane are:

$$\begin{cases} u_0(x,y,t) = 0 \\ v_0(x,y,t) = -g(t)\, y\dfrac{1-\nu^2}{Eh} \end{cases} \tag{14}$$

The membrane force can be determined in accordance with the equilibrium condition on edge of the plate $y = b$:

$$\int_0^a N_y dx = -\left(q_1 + q_2\cos\omega t\right)\cdot a \tag{15}$$

Substitute (13) into (3) and obtain:

$$N_y = \frac{Eh}{1-\nu^2}\cdot\frac{\pi^2\varphi^2}{8b^2}\left[1 + \frac{\nu b^2}{a^2} - \left(1-\nu^2\right)\cos\frac{2\pi x}{a}\right] - g(t) \tag{16}$$

Combine (15) and (16) to work out $g(t)$:

$$g(t) = q_1 + q_2\cos\omega t + \frac{Eh}{1-\nu^2}\cdot\frac{\pi^2\varphi^2}{8h^2}\left(1 + \frac{\nu b^2}{a^2}\right) \tag{17}$$

The internal forces of the mid-surface of plate are obtained:

$$\begin{cases} N_x = \dfrac{\pi^2\varphi^2 Eh}{8a^2}\left(1-\cos\dfrac{2\pi y}{b}\right)-\nu\left(q_1+q_2\cos\omega t\right) \\ N_x = -\dfrac{\pi^2\varphi^2 Eh}{8b^2}\cos\dfrac{2\pi x}{a}-\left(q_1+q_2\cos\omega t\right) \\ N_{xy}=0 \end{cases} \tag{18}$$

Substituting (18) into (1) and applying the Galerkin variational method, we obtain ordinary differential equation:

$$\ddot{\varphi}+\alpha\varphi+\beta\varphi^3 = F\cos\omega t\varphi+\dot{\varphi}\left(\mu_1+\frac{9}{16}\mu_2\varphi^2\right), \tag{19}$$

where: $\alpha=\pi^2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)\sqrt{\dfrac{D}{m}}\left(1-\dfrac{q_1}{N^*}\right)$, $\beta=\dfrac{\pi^4 Eh}{16m}\left(\dfrac{3}{a^4}+\dfrac{1}{b^4}\right)$,

$F=\dfrac{\pi^4 D}{m}\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)^2\cdot\dfrac{q_2}{N^*}$ and $N^*=\dfrac{\pi^2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)^2 D}{\dfrac{\nu}{a^2}+\dfrac{1}{b^2}}$.

Dynamics Analysis

The linear approximate solution of equ. (19) can be supposed to be:

$$\psi_1 = A_0\cos\omega_0 t+B_0\sin\omega_0 t = a\cos\left(\omega_0 t+\theta_0\right). \tag{20}$$

In order to analyze primary resonance of the plate, we set:

$$\alpha=\omega_0^2\left(1+\alpha^*\right). \tag{21}$$

According to the harmonic balance method, we obtain:

$$\begin{cases} \left(\omega_0{}^2\alpha^*+\dfrac{3}{4}\beta a^2\right)A_0+\omega_0\left(\mu_1+\dfrac{9}{64}\mu_2 a^2\right)B_0+\dfrac{F}{2}\delta_\omega^{2\omega_0}A_0=0 \\ \left(\omega_0{}^2\alpha^*+\dfrac{3}{4}\beta a^2\right)B_0-\omega_0\left(\mu_1+\dfrac{9}{64}\mu_2 a^2\right)A_0-\dfrac{F}{2}\delta_\omega^{2\omega_0}B_0=0 \end{cases} \tag{22}$$

This shows that there exists sub-harmonic parametric resonances of $\omega=2\omega_0$, and the amplitude - frequency relationship can be obtained according to the above equation:

$$\omega_0^2\alpha^* = -\frac{3}{4}\beta a^2 \pm \sqrt{\frac{F^2}{4} - \omega_0^2\left(\mu_1 + \frac{9}{64}\mu_2 a^2\right)^2} \tag{23}$$

The amplitude - frequency relationship in the real space should satisfy the following essential condition:

$$\frac{F}{2} > \omega_0\left(\mu_1 + \frac{9}{64}\mu_2 a^2\right) \tag{24}$$

Backbone curve equation is shown as follows:

$$\omega_0^2\alpha^* = -\frac{3\beta}{4}a^2 \tag{25}$$

The maximum amplitude is shown as follows:

$$a_{\max} = \frac{4}{3}\sqrt{\frac{2F - 4\omega_0\mu_1}{\omega_0\mu_2}}. \tag{26}$$

The above equation means that the maximum amplitude of amplitude - frequency curve is only related with the excitation amplitude and damping, while the backbone curve only has the cubic non-linear relationship.

Qualitative Analysis

Apply a small perturbation ψ_0 on ψ_1, and then:

$$\varphi = \psi_1 + \psi_0 \tag{27}$$

Substitute it into (19), omit the high-order trace and get:

$$\ddot{\psi}_0 + \alpha\psi_0 + \left(\mu_1 + \frac{9}{8}\mu_2\psi_1\right)\dot{\psi}_1\psi_0 + \left(\mu_1 + \frac{9}{16}\mu_2\psi_1^2\right)\dot{\psi}_0 + 3\beta\psi_1^2\psi_0 - F\psi_0 = 0. \tag{28}$$

The perturbation ψ_0 should satisfy the following condition:

$$\ddot{\psi}_0 + \chi\psi_0 = P(t)\psi_0 + Q(t)\dot{\psi}_0, \tag{29}$$

where: $\alpha = \chi\left(1+\alpha^*\right)$, $Q(t) = \left(\mu_1 + \frac{9}{16}\mu_2\psi_1^2\right)$,

$$P(t) = -\left[\chi\alpha^* + \left(\mu_1 + \frac{9}{8}\mu_2\psi_1\right)\dot{\psi}_1 + 3\beta\psi_1^2 - F\cos\omega t\right].$$

To develop $P(t)$ and $Q(t)$ into the form of Fourier series:

$$P(t) = P_0 + \sum_{j=1}^{\infty}\left(P_j^1 \cos jt + P_j^2 \sin jt\right) \tag{30}$$

$$Q(t) = Q_0 + 2\sum_{j=1}^{\infty}\left(Q_j^1 \cos jt + Q_j^2 \sin jt\right) \tag{31}$$

where: $P_0 = -\left(\omega_0^2\alpha^* + \frac{3}{2}\beta a^2\right)$, $P_{2n}{}^1 = -\left[\frac{F}{2} + \frac{9}{16}\omega_0 A_0 B_0 \mu_2 + \frac{27}{64}\left(A_0^2 - B_0^2\right)\mu_2\right]$,
$P_{2n}{}^2 = -\left[\frac{9}{32}\omega_0\left(B_0^2 - A_0^2\right)\mu_2 + \frac{27}{32}A_0 B_0 \mu_2\right]$,
$Q_0 = -\left[\mu_1 + \frac{9}{32}\mu_2 a^2\right]$, $Q_{2n}{}^1 = -\frac{9}{64}\left(A_0^2 - B_0^2\right)\mu_2$, $Q_{2n}{}^2 = -\frac{9}{32}A_0 B_0 \mu_2$.

ψ_0 can be written in exponential form:

$$\psi_0 = e^{\rho t}\eta(t). \tag{32}$$

Then $\eta(t)$ should meet the following conditions:

$$\ddot{\eta} + \lambda\eta = \left[P(t) + \rho Q(t) - \rho^2\right]\eta + \left[Q(t) - 2\eta\right]\dot{\eta}. \tag{33}$$

First, the resonant condition of $\chi = \omega_0^2$ is discussed, and the linear approximate solution of equation (33) is assumed to be:

$$\eta(t) = p\cos\omega_0 t + q\sin\omega_0 t. \tag{34}$$

Substituting equation (34) into (33) and applying the harmonic balance method to obtain a group of homogeneous linear algebraic equations which use p and q as the variables:

$$\begin{gathered}\left[\left(P_0 + Q_0\rho - \rho^2\right) + \left(P_{2n}{}^1 + Q_{2n}{}^1\rho - \omega_0 Q_{2n}{}^2\right)\right]p + \left[\omega_0\left(Q_0 - 2\rho\right) - \left(P_{2n}{}^2 + Q_{2n}{}^2\rho - \omega_0 Q_{2n}{}^1\right)\right]q = 0\\ \left[\omega_0\left(Q_0 - 2\rho\right) - \left(P_{2n}{}^2 + Q_{2n}{}^2\rho - \omega_0 Q_{2n}{}^1\right)\right]p - \left[\left(P_0 + Q_0\rho - \rho^2\right) - \left(P_{2n}{}^1 + Q_{2n}{}^1\rho - \omega_0 Q_{2n}{}^2\right)\right]q = 0.\end{gathered} \tag{35}$$

If p and q have non-zero solutions, the following formula should be met:

$$\left[\left(P_0 + Q_0\rho - \rho^2\right)^2 - \left(P_{2n}{}^1 + Q_{2n}{}^1\rho - \omega_0 Q_{2n}{}^2\right)^2\right] + \left[\omega_0^2\left(Q_0 - 2\rho\right)^2 - \left(P_{2n}{}^2 + Q_{2n}{}^2\rho - \omega_0 Q_{2n}{}^1\right)^2\right] = 0 \tag{36}$$

Assuming that P_0, Q_0, P_{2n} and Q_{2n} are enough small, then the following formula can be deduced from formula (36):

$$\rho^4 + 4\omega_0^2\rho^2 = 0 \tag{37}$$

The above equation has only one solution in real space, that is $\rho = 0$.

It is obvious that ρ, P_0 and Q_0 are the same-order trace. We can omit the high-order trace in equation (36) and get:

$$2\omega_0 \rho = \omega_0 Q_0 \pm \left[\left(P_{2n}{}^1 - \omega_0 Q_{2n}{}^2 \right)^2 + \left(P_{2n}{}^2 - \omega_0 Q_{2n}{}^1 \right)^2 - P_0^2 \right]^{\frac{1}{2}} \tag{38}$$

Similarly, the stability condition of ψ_0 is deduced as:

$$\mu_1 + \frac{9}{32}\mu_2 a^2 > 0 \tag{39}$$

and

$$\left[\frac{9}{32}\omega_0 A_0 B_0 \mu_2 + \frac{27}{64}\left(A_0{}^2 - B_0{}^2 \right)\mu_2 + \frac{F}{2} \right]^2 + \left[\frac{27}{64}\omega_0 \left(A_0{}^2 - B_0{}^2 \right)\mu_2 - \frac{9}{32} A_0 B_0 \mu_2 \right]^2$$
$$- \left[\omega_0{}^2 \alpha^* + \frac{27}{32}\mu_2 a^2 \right]^2 < \omega_0{}^2 \left[\mu_1 + \frac{9}{32}\mu_2 a^2 \right]^2 . \tag{40}$$

The amplitude - frequency relation is:

$$\omega_0{}^2 \alpha^* = -\frac{27}{64}\mu_2 a^2 \pm \sqrt{\frac{F^2}{4} - \omega_0{}^2 \left(\mu_1 + \frac{9}{32}\mu_2 a^2 \right)^2} \tag{41}$$

The trivial solution is: $a = 0$.
Then, there are:

$$\frac{1}{2} F\left(A_0{}^2 - B_0{}^2 \right) = -\left(\omega_0{}^2 \alpha^* + \frac{27}{64}\mu_2 a^2 \right) \tag{42}$$

The stability condition of the trivial solution is:

$$\left| \alpha^* \right| > \frac{1}{\omega_0{}^2} \sqrt{\frac{F^2}{4} - \omega_0{}^2 \mu_1{}^2} . \tag{43}$$

For non-trivial solutions, the stability condition (40) is changed into:

$$\pm \frac{27\mu_2}{16} \sqrt{\frac{F^2}{4} - \omega_0{}^2 \left(\mu_1 + \frac{9\mu_2}{64} a^2 \right)^2} + \frac{9\omega_0{}^2}{16}\mu_2 \left(\mu_1 + \frac{9\mu_2}{64} a^2 \right) > 0 . \tag{44}$$

The instability of the trivial solution is an important factor that leads to the occurrence of chaotic motion in the dynamic system.

Conclusions

This work analyzed the nonlinear dynamic behavior of rectangular plate under external periodic in-plane load. The harmonic balance method is applied to investigate the amplitude-frequency relationship and the stability of solutions. The following conclusions could be made:

1) There exist even order subharmonic oscillation in this dynamical system.
2) The nonlinear damping has strong effect on the largest vibrating amplitude.
3) The nonlinear damping plays an important role on the amplitude-frequency relationship.
4) The trival solutions beyond the resonant zone are stable.

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