829. An adaptive method for inertia force identification in cantilever under moving mass

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Abstract. The present study is concerned with the adaptive method based on wavelet transform to identify the inertia force between moving mass and cantilever. The basic model of cantilever is described and a classical identification method is introduced. Then the approximate equations about the model of cantilever can be obtained by the identification method. However, the order of modal adapted in the identification methods is usually constant which may make the identification results unsatisfied. As is known, the frequency of the highest order of modal is usually higher than the frequency of the input force in forward calculation methods. Therefore, wavelet transform is applied to decompose the data of deflection. The proportion of the low frequency component is chosen as the parameter of a binary function to decide the order of modal. The calculation results show that the adaptive method adapted in this paper is efficient to improve the accuracy of the inertia force between the moving mass and cantilever, and also the relationship between the proportion of low frequency component and the order of modal is indicated.

Keywords: adaptive, inertia force, cantilever, identification, wavelet transform.

1. Introduction

Time-varying parameter identification in cantilever under moving mass is an important inverse problem in the civil and structural engineering field. It is developed from time-varying parameter identification of structural dynamics problems. By accessing to the inertia force between moving mass and cantilever, more characteristics of cantilever will be obtained and it will provide some references for further engineering design. It is difficult to measure the inertia force directly between moving mass and cantilever because they are in motion and the inertia force itself is time-varying [1].

A serial of classical methods based on Euler-Bernoulli beam model for inertia force identification between moving mass and simply supported beam are presented by former researchers. T. H. T. Chan investigated the Interpretive Method (IM) [2], which is used to identify the inertia force according to modal analysis for bridge responses. S. S. Law developed the Time Domain Method (TDM) [3], which identifies the inertia force by using the modal superposition principle in time domain, and the Frequency–Time Domain Method (FTDM) [4], which calculates the inertia force spectrums by using the least-square method and then the inertia force can be obtained by the inverse Fourier transformation. Minzhuo Wang has improved the Interpretive Method [5], making it be successfully applied to identify the inertia force between moving mass and cantilever. However, the order of modal adapted in the identification methods is usually decided just by the experience of the researchers.

In this paper, an adaptive method based on wavelet transform is proposed. The data of deflection are decomposed by wavelet basis in order to get the proportion of the low frequency component by which the adaptive rule is decided.

2. Problem formulation

2. 1. Modeling for cantilever structure

In practical situation, the interaction between the cantilever and the moving mass is a complex process affected by different parameters. Simplified models are more effective to establish a clear connection between the parameters and the beam response than complex ones [6]. Therefore Euler-Bernoulli beam [7] model is chosen in this paper. The continuous beam model of cantilever is illustrated in Fig. 1.

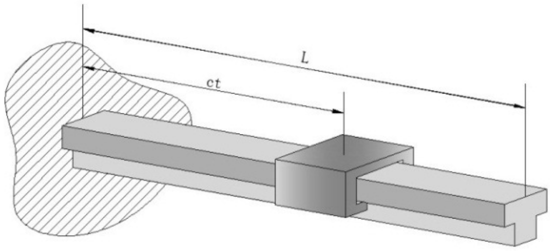

Fig. 1. Model of cantilever

Suppose an Euler-Bernoulli beam is defined with a span length L, constant flexural stiffness EI and constant mass per unit length ρ, then the equation of motion can be got as below [8]:

$$\rho\frac{\partial^2 v(x,t)}{\partial t^2}+EI\frac{\partial^4 v(x,t)}{\partial x^4}=\delta(x-ct)f(t) \tag{1}$$

where $v(x,t)$ is the beam deflection at point x and time t, δ is the Dirac delta function and damping is neglected.

2. 2. Theory of time-varying parameter identification

The boundary conditions for Eq. (1) are:

$$v(0,t)=0\,,\quad \left.\frac{\partial^2 v(x,t)}{\partial x^2}\right|_{x=0}=0\,,\quad v(x,0)=0\,,\quad \left.\frac{\partial(x,t)}{\partial t}\right|_{t=0}=0$$

Based on the modal superposition theory, the solution of Eq. (1) can be expressed as:

$$v=\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L}q_i(t) \tag{2}$$

where $q_i(t)$ are the modal displacements.

After substituting Eq. (2) into Eq. (1), both sides of the equations are multiplied by $\sin(i\pi x/L)$. Then integrate the equations with respect to x between 0 and L, use the properties of $\delta(t)$ and the boundary conditions, therefore the equations can be expressed as:

$$\ddot{q}_j(t) + \omega_{(j)}^2 q_j(t) = \frac{2F}{\rho L}\sin j\omega t,\ j = 1, 2, ... \tag{3}$$

where:

$$\omega_{(j)}^2 = \frac{j^4\pi^4}{L^4}\frac{EI}{\rho},\quad f_n(t) = F(t)\sin\left(\frac{n\pi\overline{x}}{L}\right)$$

are the n th modal frequency and modal force respectively.

In this paper, Interpretive Method (IM) which is previously based on simply supported beam is used to identify the inertia force in cantilever under moving mass. The equation of identification from Eq. (3) can be modified as:

$$\ddot{q}_j(t) + \omega_{(j)}^2 q_j(t) = \frac{F}{\rho L}\sin j\omega t,\ j = 1, 2, ... \tag{4}$$

Unfortunately, the ω in Eq. (4) can't be obtained as analytical solution. Therefore, an approximate solution is adapted in order to obtain the value of ω. By using the variables separation method and the boundary conditions of the cantilever, the frequency equation of the cantilever can be given as [9]:

$$\cos\beta L \cdot ch\beta L = -1 \tag{5}$$

The solution of Eq. (5) can be obtained by using the numerical method. Based on the results above, the modal function can be given as follows:

$$\varphi(n,x) = \mathrm{ch}\left(\frac{\beta_n \cdot x}{L}\right) - \cos\left(\frac{\beta_n \cdot x}{L}\right) - \frac{\mathrm{sh}(\beta_n) - \sin(\beta_n)}{\mathrm{ch}(\beta_n) + \cos(\beta_n)} \cdot \left(\mathrm{sh}\left(\frac{\beta_n \cdot x}{L}\right) - \sin\left(\frac{\beta_n \cdot x}{L}\right)\right) \tag{6}$$

where β_n is the nodes of the n th main vibration mode with the initial position of the steady cantilever, n is the order of modal, x is the x th one of all data of deflection.

Then, the equation of identification can also be expressed as:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_n \end{bmatrix} + \begin{bmatrix} \omega_1 q_1 \\ \omega_2 q_2 \\ \vdots \\ \omega_n q_n \end{bmatrix} = \frac{1}{\rho L}\begin{bmatrix} \varphi(1,1) & \varphi(1,2) & \dots & \varphi(1,m) \\ \varphi(2,1) & \varphi(2,2) & \dots & \varphi(2,m) \\ \vdots & \vdots & \vdots & \vdots \\ \varphi(n,1) & \varphi(n,2) & \dots & \varphi(n,m) \end{bmatrix}[P] \tag{7}$$

where P is the inertia force between moving mass and cantilever, n is the order of modal,

m is the number of the input data.

2. 3. An adaptive method based on wavelet transform

As it mentioned above, the order of modal which is adapted in the former methods is usually constant. But this constant order of the model adapted in the calculation only depends on the researchers' experience. In this paper, we propose an adaptive method that the order of the model become variable based on the results from wavelet transform. The input data is decomposed by using Daubechies 5 wavelet at a decomposition level of five. Then the proportion of the low frequency component is calculated. Decomposition model is shown in Fig. 2 as below [10].

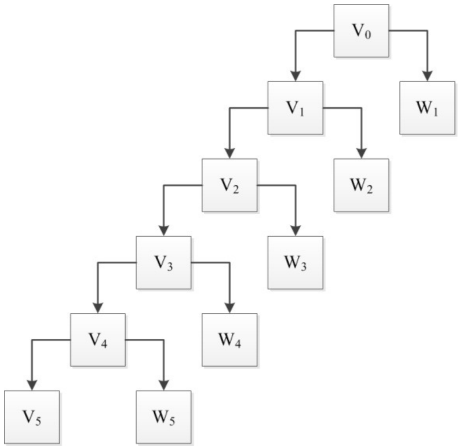

Fig. 2. Multiresolution analysis model

Mathematics decomposition results are given as follows:

$$\sum_k x_k^i \phi_{ik}(t) = \sum_k x_k^{i+1} \phi_{(i+1)k}(t) + \sum_k d_k^{i+1} \psi_{(i+1)k}(t)$$

and $P_i x(t) = P_{i+1}x(t) + D_{i+1}x(t)$ or $D_{i+1}x(t) = P_i x(t) - P_{i+1}x(t)$, where $\phi_{ik}(t) = \frac{1}{2^{i/2}}\phi(2^{-i}t - k)$, x_k^i is the scale function coefficient at layer i. $\psi_{ik}(t) = \frac{1}{2^{i/2}}\psi(2^{-i}t - k)$, d_k^i is the wavelet transform coefficient at layer i. $D_{i+1}x(t)$ is the projection of $x(t)$ in W_{i+1}, $P_i x(t)$ reflect the general view of signal $x(t)$ under the resolution I.

For each sampling point, we adapt a binary function which depends on proportion of its low frequency component. If the proportion of the low frequency component is higher than the threshold, the value of the binary function is 1. Or, if the proportion of the low frequency component is lower than the threshold, the value of the binary function is 0. Then, the order of modal is decided by the sum of the value of the binary functions of all the sampling points obtained at the same time.

In sum, the whole process of the calculation can be illustrated as it shown in the Fig. 3 below.

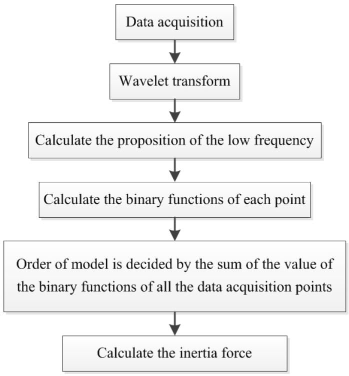

Fig. 3. The whole process of the calculation

3. Numerical calculation and results

The source of the data adapted in the identification is from the deflection of ten sampling points which are acquired by numerical methods. A relative percentage error (RPE) [11] is defined to compare the results of the identification which is expressed as:

$$\text{RPE}=\frac{\sum\left|f_{ture}-f_{ident}\right|}{\sum\left|f_{ture}\right|}\times 100\%$$

The parameters of the model in numerical calculation are defined as below: the span length L is 1.5 m, constant flexural stiffness EI is $3.42\cdot 10^5$ N·m^2 and constant mass per unit length ρ is $7.8\cdot 10^3$ kg/m^3. Sampling frequency is 10 kHz constantly. Three numerical examples with different velocity of the moving mass on cantilever are shown in this paper (v_1 = 5 m/s, v_2 = 10 m/s, v_3 = 20 m/s).

As mentioned above, the threshold of the binary function depends on the proportion of low frequency, which is applied to set the order of modal. According to the prior knowledge, the farther distance between the sampling point and supporting point on cantilever, the larger threshold must be chosen to calculate, and the higher velocity of moving mass, the threshold must be also larger. Thus, the threshold of the different three numerical examples is listed in Table 1.

Table 1. The threshold of the different three numerical examples

Sampling point	v_1 = 5 m/s	v_2 = 10 m/s	v_3 = 20 m/s
1	95.0 %	97.0 %	98.0 %
2	95.2 %	97.2 %	98.2 %
3	95.4 %	97.4 %	98.4 %
4	95.6 %	97.6 %	98.6 %
5	95.8 %	97.8 %	98.8 %
6	96.0 %	98.0 %	99.0 %
7	96.2 %	98.2 %	99.2 %
8	96.4 %	98.4 %	99.4 %
9	96.6 %	98.6 %	99.6 %
10	96.8 %	98.8 %	99.8 %

The results of identification with different velocity of the moving mass are illustrated below in Figures 4-6 and the relative percentage errors of identification are listed in Tables 2-4.

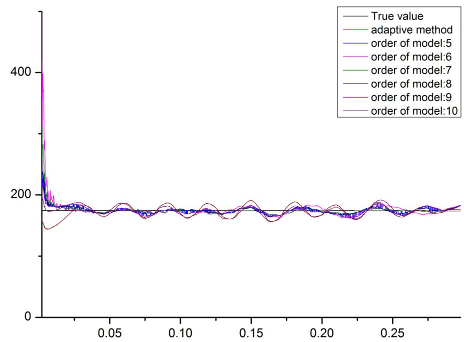

Fig. 4. Identification results (velocity of moving mass is 5 m/s)

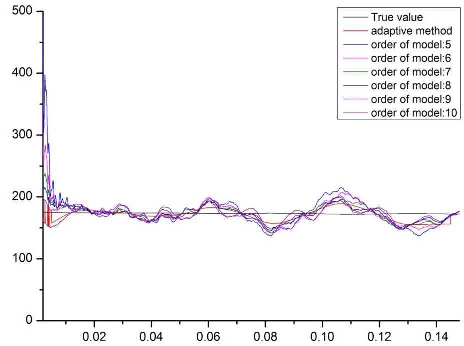

Fig. 5. Identification results (velocity of moving mass is 10 m/s)

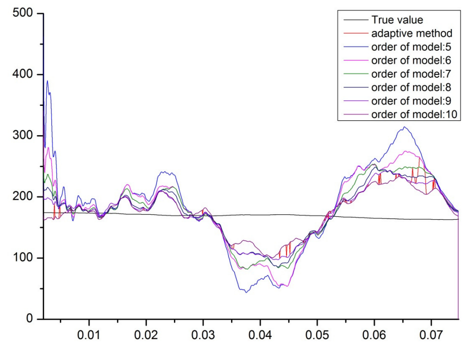

Fig. 6. Identification results (velocity of moving mass is 10 m/s)

4. Conclusions

In this paper, an adaptive method based on wavelet transform is applied to identify the inertia force between moving mass and cantilever. The input data is decomposed by using wavelet transform. Then the proportion of the low frequency component is calculated in order to set the threshold which is used to determine the order of modal.

Three different results are obtained by changing the speed of moving mass. Some recommendations based on these results are:

(1) 20 percent or more improvement can be obtained by using the adaptive method based on

wavelet transform.

(2) The relationship between the proportion of low frequency component and the order of modal has been proved.

(3) The improvement of the results is better as the velocity of the moving mass is higher among three numerical examples.

Table 2. Identification results (velocity of moving mass is 5 m/s)

Order of modal	RPE
5	8.51
6	4.53
7	3.74
8	3.70
9	4.34
10	4.91
adaptive	2.67

Table 3. Identification results (velocity of moving mass is 10 m/s)

Order of modal	RPE
5	20.55
6	12.92
7	9.27
8	7.37
9	6.66
10	6.32
adaptive	5.01

Table 4. Identification results (velocity of moving mass is 20 m/s)

Order of modal	RPE
5	57.70
6	39.86
7	30.95
8	25.19
9	22.51
10	18.32
adaptive	13.43

Future work of this study would do more research on the relationship between the proportion of low frequency component and the order of modal and apply more advanced adaptive methods.

Acknowledgments

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