823. Finite element model updating of micromachined torsion structures using experimental eigendata

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Abstract. In this paper, the dynamic characteristics including natural frequencies and mode shapes of a cross-shaped torsion structure, fabricated by a micromachining process, are measured by using two full-field and non-contact experimental techniques: Electronic Speckle Pattern Interferometry and Stroboscopic Interferometry. In addition, the finite element method is also applied to analyze the microstructure. Since the mechanical properties of a microstructure are usually not accurately known and neither is its depth dimension, it is very likely that the measured and the predicted results show a significant inconsistency. This research performs a finite element model updating procedure to determine a set of much more reliable values for the mechanical properties and the thickness of the torsion structure. The result is a refined finite element model capable of accurately predicting the dynamic behaviors of the micromachined device and suitable for further design modification studies. Two updating cases both show significant improvements in frequency prediction. With the inclusion of the thickness parameter, the second case reduces the frequency differences from over 17 % to under 0.4 %.

Keywords: torsion structure, micromachined structure, finite element model updating, ESPI, SI.

1. Introduction

The functional performance of micromachined torsion structures, widely utilized in projection televisions and optical switches, is critically dictated by their dynamic characteristics including the natural frequencies and mode shapes. During the process of design and development of a Micro-Electro-Mechanical-System (MEMS) structure, there is always a concern that the actual behavior of a manufactured device is significantly different from the simulation result of the counterpart virtual model. If this deviation is substantial, the consequence might be catastrophic. The finite element method (FEM) has been commonly employed to simulate the static and dynamic behavior of microstructures, e.g. [1, 2]. Applying FEM to analyze a microstructure requires mechanical and geometrical properties, such as the Young's modulus and Poisson's ratio of the material used and the thickness of the structure, as input parameters. A small error in one input parameter can produce large deviations from the true structural responses. To obtain a more accurate and reliable finite element (FE) model, the FE model updating technique can be implemented to fine tune the model based on measured data [3-5]. For dynamic characteristics measurement of MEMS devices, Electronic Speckle Pattern Interferometry (ESPI) and Stroboscopic Interferometry (SI), which both are full-field and non-contact experimental techniques, play two important roles. ESPI and SI methods employ optical and image processing techniques and analysis software to extract the resonance frequencies of microstructures and their three dimensional mode shapes. Holographic interferometry [6], the underlining principle used by ESPI and SI, is a procedure for recording and analyzing the brightness and darkness, colors, and phases of two interfering light beams, one reflected from the test object and the other forming the reference. Holographic interferometry has been extensively applied to various engineering applications, including measurements of temperature field [7], three dimensional deformation [8], and structural dynamics measurement in nano-scale [9, 10].

In this paper, both ESPI and SI techniques are used to measure the frequencies and mode shapes of a micromachined, cross-shaped torsion structure. By slowly varying the frequency of an alternating voltage applied to a piezoelectric transducer, the microstructure mounted on the transducer can be excited and resonated. The input frequencies producing a resonant response are the resonant frequencies, and the mode shapes of the structure at these frequencies can also be determined. In addition, the finite element method is applied to analyze the microstructures as well. Since the mechanical properties of a microstructure are usually not accurately known and neither is its depth dimension, it is very likely that the measured and the predicted results will show a significant inconsistency. This research also utilizes the FE model updating concept to determine the mechanical properties and the thickness of the torsion structure. The result is a precise FE model capable of accurately predicting the dynamic behaviors of the micromachined torsion structure and suitable for further design modification studies.

2. Interferometry experiments

2. 1. The torsion structure

The cross-shaped torsion structure, whose SEM micrograph is shown in Fig. 1, was fabricated by a micromachining process. The micro device is a symmetrical structure and consists of a long beam (the horizontal beam shown in Fig. 1) and two short beams (the vertical beams). Table 1 gives the geometrical parameters of the microstructure, ***A*** and ***a*** denoting the half-length and beam width, respectively, of the long beam, and ***B*** and ***b*** the full beam length and width of the short beam. The torsion structure, made of single crystalline silicon (Si [100]) has a nominal thickness of 25 μm and its material properties are assumed to have: Young's modulus 165 GPa, Poisson's ratio 0.22, and density 2330 kg/m^3.

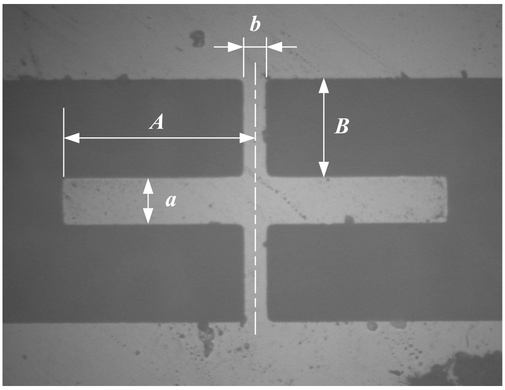

Fig. 1. The torsion structure under study

Table 1. Geometrical parameters of the torsion structure, all units in μm

Long beam half length, ***A***	Long beam width, ***a***	Short beam length, ***B***	Short beam width, ***b***
800	200	400	100

2. 2. Electronic speckle pattern interferometry

A laser-based, commercial ESPI system is used for measuring resonant frequencies and mode shapes of the micromachined structure. The system is capable of full-field and non-contact vibration measurements for frequencies up to 20 MHz. Two signal generators produce voltages with varying frequencies to a laser system and to an adjustable tri-axial stage equipped with a piezoelectric transducer. The transducer vibrates at various frequencies according to the supplied signal. By slowly changing the frequency of the alternating voltage entered to the piezoelectric actuator, the microstructure mounted on the transducer can be excited and resonated. The input frequencies producing a resonant response are the resonant frequencies, and also the mode shapes of the structure at these frequencies can be identified. Fig. 2 depicts schematically the ESPI measurement system that can display the mode shapes of a microstructure in a fringe form or a three dimensional format. Due to equipment limitation, only four modes of the torsion device are measured, and they are 16.60, 45.46, 117.38 and 232.32 kHz, respectively. Fig. 3 shows these four mode shapes in fringe format. The reason only four modes were obtained is that it required too high a voltage, exceeding the safety range recommended by the manufacturer, to excite the higher modes.

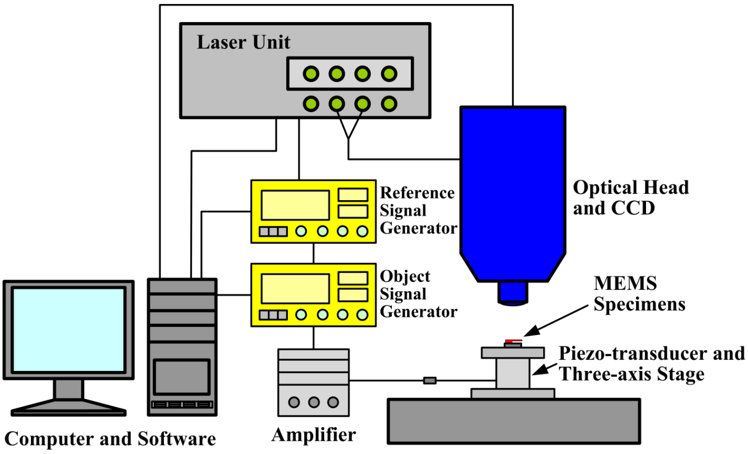

Fig. 2. Systematic diagram for the ESPI measurement system

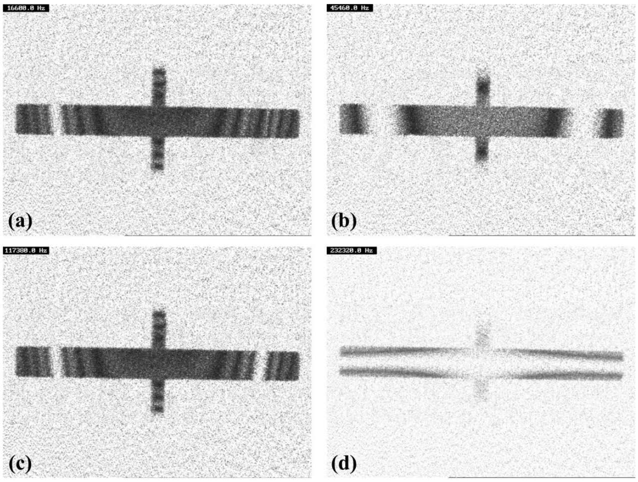

Fig. 3. Frequencies and mode shapes measured using ESPI: (a) first mode, 16.60 kHz; (b) second mode, 45.46 kHz; (c) third mode, 117.38 kHz and (d) fourth mode, 232.32 kHz

2. 3. Stroboscopic interferometry

The SI system used in the present study has an ultra-bright white-light LED as the flashing source, and its structure is basically an extension to an ordinary white-light interferometry microscope. The system includes two PZT micro-actuators, one for actuating the object lens module for phase-shifting scanning and the other mounted on an adjustable tri-axial stage. During an experiment, two signals with the same frequency, one pulse and one sine wave, are generated and fed synchronously into a pulsed light system and the actuating system, respectively. If the duration of the pulse is short enough, the interference phenomenon for measuring a vibrating microstructure can be explained by the common static interferometry. There are also four frequencies and mode shapes in interference fringe format obtained using the SI technique. The fringe files are loaded into a MATLAB program and then three dimensional images and animations of these mode shapes are created. Fig. 4 illustrates the four modes in a three dimensional form. There are apparent discrepancies between the two sets of frequencies obtained by the SI and ESPI methods. Since the mode shapes attained by the SI technique appear less ambiguous, the corresponding frequencies are adopted as the target frequencies to match in the finite element model updating procedure.

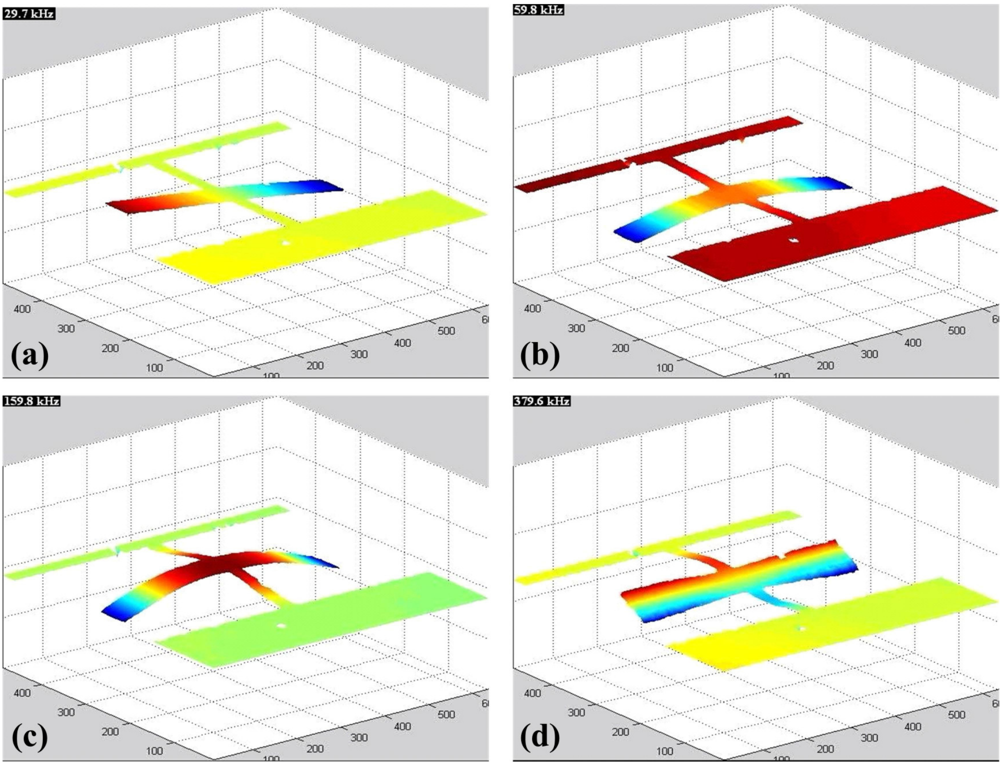

Fig. 4. Frequencies and mode shapes measured using SI: (a) first mode, 29.70 kHz; (b) second mode, 59.80 kHz; (c) third mode, 159.80 kHz and (d) fourth mode, 379.60 kHz

3. Finite element analysis of the torsion device

Two finite element software packages ANSYS and COMSOL Multiphysics are used to simulate the dynamic characteristics of the torsion structure. ANSYS is a general purpose FE analysis package, while COMSOL Multiphysics is a program mainly designed for analyzing MEMS. Both FE models comprise three dimensional solid elements with fixed boundary conditions applied on the areas at the outer ends of the short beams. Since only four modes of the torsion gadget were measured in the experiments, the same four FE modes are extracted using both FE software packages, and they are given in Table 2. By comparing the modal information obtained by using ANSYS and COMSOL, it is found that both results are in excellent agreement: the first mode is the short beams in torsion; the second a bending mode for the long beam; the third a bending mode for both types of beams and in phase; and the fourth the long beam in torsion. The FE analysis mode shapes extracted using COMSOL are illustrated in Fig. 5. Table 2 also compares all the frequency results obtained using the analytical and experimental means. The last column of the table illustrates the relative differences (errors) between the frequencies from the SI experiment and the ANSYS program. It is obvious that the errors are significant and that the FE results generally underestimated the SI measured data. The reasons to have caused such disagreements may include: a) the sizes, especially the depth dimension, of the microstructure deviating from its original design due to imprecision of the micromachining fabrication process; b) experimental error occurred during frequency measurement; and c) material properties of the FE models differing from the true values of the microstructure. The first and third causes of errors can be drastically improved by the finite element updating technique with the geometrical and material properties of the microstructure as the updating parameters. Although the measured frequencies are significantly different from the corresponding FEA frequencies, the results have indicated that the analytical and experimental mode shapes match very well.

Table 2. Frequency results obtained using FEM and experiments

Mode No.	ANSYS (kHz)	COMSOL (kHz)	ESPI (kHz)	SI (kHz)	Difference[#] (%)
1st mode	25.73	25.04	16.60	29.70	-15.4
2nd mode	50.90	50.81	45.46	59.80	-17.5
3rd mode	143.62	143.84	117.38	159.80	-11.3
4th mode	344.87	344.20	232.32	379.60	-10.1

[#] Relative difference between the ANSYS and SI frequencies.

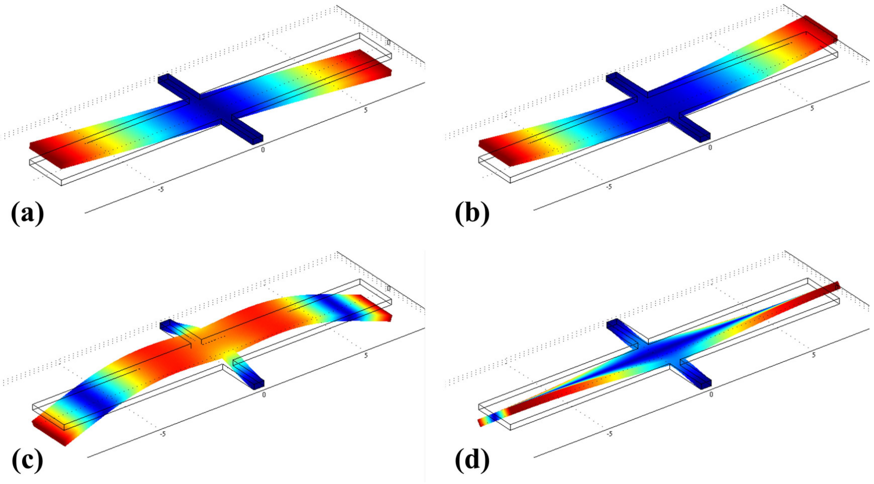

Fig. 5. FEA mode shapes obtained using COMSOL: (a) first mode, 25.04 kHz; (b) second mode, 50.81 kHz; (c) third mode, 143.84 kHz and (d) fourth mode, 344.20 kHz

4. Finite element model updating

4. 1. The updating scheme

The updating procedure, which combines two sets of eigenmodes of the same microstructure, one from a vibration experiment and the other from a finite element analysis, starts by measuring the natural frequencies and mode shapes of the microstructure under study. Also, the finite element model of the structure is created and then analyzed. Since the material properties of the structure are often not known precisely and the geometrical properties are sometimes also in doubt (especially in the depth direction), it is quite possibly that the measured and the predicted results will show a significant disagreement. By carefully comparing the experimental and FEA mode shapes, matching modes are correctly paired, which is important since the order of the FEA modes can be different from that of the measured data. By defining the error vector as a vector containing the relative differences between the experimental and FEA natural frequencies, an optimization problem can be formulated to minimize the length of the error vector as follows:

$$\text{Minimize} \quad f(\boldsymbol{x}) = \left[\sum_{i=1}^{m} \left(\frac{f_i^a - f_i^e}{f_i^e} \right)^2 \right]^{\frac{1}{2}} \tag{1}$$

$$\text{Subject to} \quad F_i^l \le f_i^a \le F_i^u \qquad i = 1, \ldots, m \tag{2}$$

$$X_j^l \le x_j \le X_j^u \qquad j = 1, \ldots, n \tag{3}$$

where f_i is the natural frequency for the matched mode i; the superscripts a and e represent FEA and experimental results, respectively; the superscripts l and u denote the lower and upper limits; x_j is the j^{th} FE input parameter; m is the number of modes included in the optimization process and n the number of FE input parameters to be updated (updating parameters). The number of updating parameters should be kept small and only the uncertain ones should be selected as the updating parameters [5]. In general it is required that $n \le m$ to ensure a physically meaningful result.

The above optimization problem is solved to yield a set of updated parameters, and then the results are checked for convergence. If converged, the process can be stopped; otherwise, FEA is once again performed using the updated parameters to produce a new set of modal data leading to the next iteration, and the procedure is continued in an iterative way. To perform optimization, eigensensitivities with respect to changes in the updating parameters are calculated to direct the optimization path. Solution of the optimization problem defined in Eqs. (1)-(3) requires an integration of an FEA program and an optimization routine. ANSYS equipped with such capability is employed in this study. The first order optimization method of the design optimization module in ANSYS, which uses a forward difference approximation to calculate the eigensensitivities and the required gradient vectors, is selected to perform the optimization.

In this paper, two FE model updating cases are presented: Case 1 using only the material properties as the updating parameters and Case 2 employing both geometrical and material parameters. Both cases treat the measured frequencies acquired from the SI experiments as the targets for the FE analysis results to match while tuning the updating parameters.

4. 2. Results of finite element model updating

The first case utilizes the material properties Young's modulus (E) and Poisson's ratio (ν) as the updating parameters. An ANSYS script file is coded to perform the optimization task defined in Eqs. (1) through (3) with $m = 4$ and $n = 2$. The updating process quickly converges in just a few iterations, and the final updated parameters are $E^* = 205$ GPa, compared to the original value of 165 GPa, and $\nu^* = 0.211$, compared to the original value of 0.220, in which the superscript asterisk denotes the final updated value. With these updated parameters, the prediction of natural frequencies by the ANSYS program is greatly improved, as shown in Table 3. However, the updated parameter E^* seems uncharacteristically high. One of the possible causes of this circumstance could be due to an inappropriate selection of the updating parameters.

In fact, since the depth dimension of a microstructure is probably one of the most difficult parameters to control during a micromachining procedure, the thickness of the torsion device should bear a great uncertainty and it should be included in the updating process.

The second case adds the thickness as the third updating parameter. Consequently, the parameters to be updated in this case are the material properties: Young's modulus and Poisson's ratio, and the geometrical parameter: the thickness of the torsion structure (t). Again, the optimization sequence converges rather fast. The final updated parameters are $E^* = 181.06$ GPa, $\nu^* = 0.221$, and $t^* = 28.09$ μm, compared to the original value of 25 μm. With these values as inputs, the natural frequencies predicted by the ANSYS program are very close to the experimental modal data, and they are also given in Table 3. Obviously, the outcome of Case 2 is much better than that of Case 1. All frequency differences are under 0.4 % in the second case and, furthermore, the inclusion of the thickness parameter makes the final updated values for all three parameters much more physically meaningful.

Table 3. Comparison of the updated and experimental results for Cases 1 and 2

Mode No.	SI (kHz)	Case 1		Case 2	
		ANSYS (kHz)	Difference (%)	ANSYS (kHz)	Difference (%)
1st mode	29.7	28.76	-3.15	29.74	0.14
2nd mode	59.8	56.77	-5.06	59.73	-0.12
3rd mode	159.8	163.40	2.25	159.19	-0.38
4th mode	379.6	390.97	2.99	380.87	0.33

5. Conclusions

A cross-shaped torsion device, fabricated by a micromachining process, was analyzed by finite element analysis with ANSYS and COMSOL software packages. The microstructure's resonance frequencies and mode shapes were also measured by using the Electronic Speckle Pattern Interferometry and Stroboscopic Interferometry techniques, which both are full-field and non-contact experimental procedures. Four sets of modal data, two from FE analysis and two from experiments, were compared and shown having a significant discrepancy in their frequency values, although their mode shapes were quite consistent. Inconsistency in the frequency results due to erroneous inputs of geometrical and material parameters to the FE analysis could be salvaged by applying the finite element model updating procedure. When administrating the updating technique, the number of updating parameters should be kept small and only the uncertain ones should be selected as the updating parameters. Such a parameter that was mostly doubtful as the depth dimension of a microstructure should definitely be included in the optimization process. Two updating cases showed that the optimization sequence converged rather quickly for both cases and significant improvements in frequency prediction were achieved. With the inclusion of the thickness parameter, the second case yielded frequency differences under 0.4 % in all four modes, and all three updating parameters attained more reliable updated values. The resulted FE model is capable of accurately predicting the dynamic behaviors of the micromachined torsion structure and suitable for further design modification studies.

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