821. Study on analysis method for deepwater TTR coupled vibration of parameter vibration and vortex-induced vibration

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Abstract. Considering the vertical vibration, caused by floating platform, of top tensioned riser (TTR), an analysis method for the coupled vibration of parameter excited vibration and vortex-induced vibration is presented in this paper. With the section rotation and shear deformation due to the bending large displacement of TTR, a coupled vibration model of parameter excited vibration and vortex-induced vibration is put forward. And the vortex-induced vibration (VIV) of a TTR for 1500 m water depth is analyzed based on the model. The results show that the vertical vibration caused by floating platform increases transverse vibration displacement of TTR.

Keywords: deepwater riser, TTR, vortex-induced vibration (VIV), fluid-structure interaction, parameter excited vibration.

1. Introduction

Riser system is the main constituent of ocean infrastructure, and its safety becomes more and more important. However, with the depth increasing, the design analysis of riser system faces great challenges. Now long flexible deepwater riser vortex-induced models can be divided into: empirical models, mixed models and CFD models. A complete description is conducted [1]. Hartlen and Currie [2] originally established vortex-induced transverse vibrations model and wake oscillator model. After them, many scholars have it amended and improved. For example, Skop-Griffin model, Iwan-Blevins model, Landl model, Iwan wake oscillator model, Skop-Griffin wake oscillator model, Krent-Nielsen two oscillator model and so on.

The development of computer provides favorable conditions for the research and application of CFD, and makes the numerical method be widely used [3, 4]. The emphasis on vortex-induced vibration of deepwater riser with two degrees of freedom (in-line and cross flow) is increasing [5]. When transverse bending vibration happened under wave load, the vertical vibration caused by floating platform does not only affect the vertical parametrically excited vibrations, but also affect the transverse bending vibration, thus there is a coupled vibration of parameter excited vibration and vortex-induced vibration.

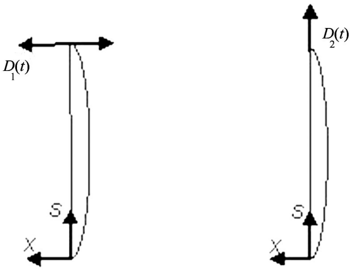
2. Mathematical model

Long flexible deepwater riser will have parameter excited vibration caused by floating platform heave [6], so when analysis the dynamical response of deepwater riser, not only consider the transverse excitation, but also consider axial excitation, because axial excitation caused by floating platform heave increases transverse vibration amplitude of deepwater riser. Fig. 1 displays the principle of parameter excited vibration and D_1, D_2 denote transverse load and axial load.

Existing deepwater riser bending vibration analysis methods generally doesn't consider transverse displacement caused by vertical displacement; only calculate the bending displacement caused by transverse loads. And the bending vibration equation of deepwater drilling riser is as follows:

$$EI\frac{\partial^4 y}{\partial x^4}-\frac{\partial}{\partial x}(T\frac{\partial y}{\partial x})+\overline{m}\frac{\partial^2 y}{\partial t^2}+c\frac{\partial y}{\partial t}=q(x,t) \tag{1}$$

where y is the transverse displacement, x - the riser axial coordination, t - the time, EI - the transverse flexural rigidity of TTR, T - the tension, it is the function of x and t, namely $T = T(x, t)$, $\overline{m}$ - the riser mass per unit length, c - the damping coefficient, $q(x, t)$ - the fluid force in the transverse direction.



a) b)

Fig. 1. a) Lateral vibration, b) Parametric vibration

Now, with the section rotation and shear deformation due to the bending large displacement of TTR, a coupled vibration model of parameter excited vibration and vortex-induced vibration is put forward. Its derivation process is as follows.

Select a micro-element of riser dx (Fig. 2). According to geometry relation, there is:

$$du-dx=\rho d\theta \tag{2}$$

where ρ is the curvature radius of selected infinitesimal section, θ the section corner, u the axial displacement of riser, the top of riser is influenced by platform motion, in this paper, we assume the platform on the top of the riser as a particle, and only consider its heave motion. du is the axial displacement of selected infinitesimal section.

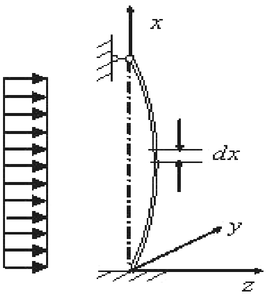


Fig. 2. Riser bending schematic diagram

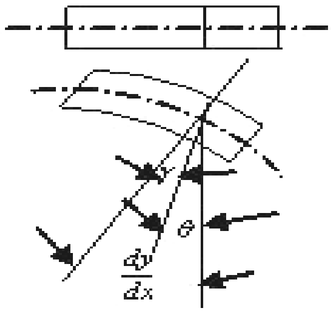


Fig. 3. Geometric description of beam deformation with the consideration of the effects of shearing

Take shear deformation (Fig. 3) into account:

$$\theta = \left(dy/dx\right) - \gamma \tag{3}$$

where γ is the shear strain and $\gamma = N/GA$.

Eq. (2) is expressed as:

$$(du/dx) - 1 = \rho(d^2y/dx^2 - d\gamma/dx) \tag{4}$$

We know: $1/\rho = -d^2y/dx^2$, thus Eq. (4) is expressed as:

$$1 - du/dx = 1 - \rho d\gamma/dx \tag{5}$$

$$du/\rho dx = dN/GAdx \tag{6}$$

$N = EId^3y/dx^3$, and we assume $\kappa = 1/\rho$, we can get Eq. (7):

$$GA\kappa\frac{du}{dx} = EI\frac{d^4y}{dx^4} \tag{7}$$

As known the beam bending equation under transverse loads is as follows:

$$EI\frac{d^4y}{dx^4} = q(x,t) \tag{8}$$

GAκdu/dx is equivalent to transverse loads, we can add it to the right of Eq. (1) directly, and we can get a coupled vibration model of parameter excited vibration and vortex-induced vibration:

$$EI\frac{\partial^4 y}{\partial x^4} - \frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\right) + \bar{m}\frac{\partial^2 y}{\partial t^2} + c\frac{\partial y}{\partial t} = q(x,t) + GA\kappa\frac{du}{dx} \tag{9}$$

GA is the riser's shear rigidity, other parameters are same with Eq. (1).

With the section rotation and shear deformation due to the bending large displacement of TTR, the new coupled vibration model is put forward. In the study, riser vibration analysis takes the effects of fluid within the riser and riser inner tension effects into account.

3. Fluid-structure interaction vortex-induced forces

The riser under the fluid loads will vibrate, and the vibration of the riser will affect the flow field in turn, vice versa, namely fluid-structure interaction. When considering the effects of fluid-structure interaction, the structure will have nonlinear damping force and inertia force in transverse direction, as the Morison equation [7] expresses:

$$f_y' = C_D\rho D\dot{v}\left|\dot{v}\right|/2 + C_m\rho\pi D^2\ddot{v}/4 \tag{10}$$

Cross-flow force expression as follows:

$$f_y = C_L\rho D(u_0 - \dot{w})^2\cos\omega_s t/2 + C_D\rho D\dot{v}\left|\dot{v}\right|/2 + C_m\rho\pi D^2\ddot{v}/4 \tag{11}$$

In-line force expression as follows:

$$f_z = C_L'\rho D(u_0 - \dot{w})^2\cos\omega_s' t/2 + C_D\rho D(u_0 - \dot{w})\left|u_0 - \dot{w}\right|/2 + C_m\rho\pi D^2\ddot{w}/4 \tag{12}$$

where C_L is the cross-flow lift coefficient, C_L' - the in-line lift coefficient, D - the riser diameter, ω_s - the vortex shedding frequency, C_D - the drag coefficient, C_m - the added mass coefficient, $\dot{v}$ - the vibration speed of riser in cross-flow direction, $\ddot{v}$ - the vibration acceleration of riser in cross-flow direction, $\dot{w}$ - the vibration speed of riser in in-line direction, $\ddot{w}$ - the vibration acceleration of riser in cross-flow direction.

4. Numerical analysis

Put formula (11) and formula (12) into Eq. (9), and use Newmark – β method incremental form to analysis of dynamic response of riser. Newmark – β method is a direct integration method, it's unconditionally stable.

The full variable form of direct integration method can be expressed as follows:

$$[M]\{\ddot{v}\}^e_{t_{i+\Delta t}} + [C]\{\dot{v}\}^e_{t_i+\Delta t} + [K]\{v\}^e_{t_i+\Delta t} = \{F_y\}_{t_i+\Delta t} \tag{13}$$

$$[M]\{\ddot{w}\}^e_{t_{i+\Delta t}} + [C]\{\dot{w}\}^e_{t_i+\Delta t} + [K]\{w\}^e_{t_i+\Delta t} = \{F_z\}_{t_i+\Delta t} \tag{14}$$

Incremental equations of motion of the system can be expressed as:

$$[M]_{t_i}\{\Delta\ddot{v}\}_{t_i} + [C]_{t_i}\{\Delta\dot{v}\}_{t_i} + [K]_{t_i}\{\Delta v\}_{t_i} = \{\Delta F_y\}_{t_i} \tag{15}$$

$$[M]_{t_i}\{\Delta\ddot{w}\}_{t_i} + [C]_{t_i}\{\Delta\dot{w}\}_{t_i} + [K]_{t_i}\{\Delta w\}_{t_i} = \{\Delta F_z\}_{t_i} \tag{16}$$

where [M] is the mass matrix of system, [C] - the damping matrix of system, we usually use Rayleigh damping $[C_u] = \alpha[M_u] + \beta[K_u]$, in which α and β are Rayleigh damping coefficients, their unit are s^{-1} and s, and their value according to:

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \frac{2\xi}{\varpi_m + \varpi_n}\begin{bmatrix}\varpi_m\varpi_n\\ 1\end{bmatrix}$$

and [16] $\xi = 0.05$, ϖ_m is fundamental frequency, ϖ_n is the third order natural frequency. [K] - the stiffness matrix of system, $\{v\}^e$, $\{w\}^e$ - the node displacement of element, $\{\dot{v}\}^e$, $\{\dot{w}\}^e$ - the node speed of element, $\{\ddot{v}\}^e$, $\{\ddot{w}\}^e$ - the node acceleration of element.

Due to introducing large deformation, the mass matrix, damping matrix and stiffness matrix change with time. So it can be used to solve geometrical and physical non-linear problem. In order to improve calculation accuracy, iteration is carried out in every step. Time step is $dt = 0.01$ s.

5. Results and discussion

Based on the new above program, the features of a top tension riser considering large deformation are further investigated. The selected TTR is a double casing production riser, its external diameter of external pipe is 324 mm, diameter of inner pipe is 222 mm, diameter of internal pipe is 114 mm. Based on criterions of bending stiffness equivalent this double-layered pipe is equivalent to single-layered pipe. The equivalent inner diameter is 292 mm. Top tension coefficient is 1.4. Boundary conditions are one fixed end and one articulated end.

Parameters of model riser are given in Table 1.

Table 1. Parameters of model riser

Riser length	1500 m
Outer diameter	0.3239 m
Inner diameter	0.285 m
Elastic modulus	207 GPa
Shear modulus	79000 Pa
Material density	7850 kg/m^3

Table 2. VIV response parameters

V (m/s)	C_D	C_m	C_L	C_L'
0.18 / 0.4	1.0	1.0	0.9	0.05

According to Fig. 4, the relation curves of reduced amplitude and reduced speed, TTR cross-flow vibration lock-in range is: $4 < U_r < 6.3$, thus in this paper, velocity 0.18 m/s, which is in lock-in range, and velocity 0.4 m/s which is outside lock-in range are selected as simulation speeds.

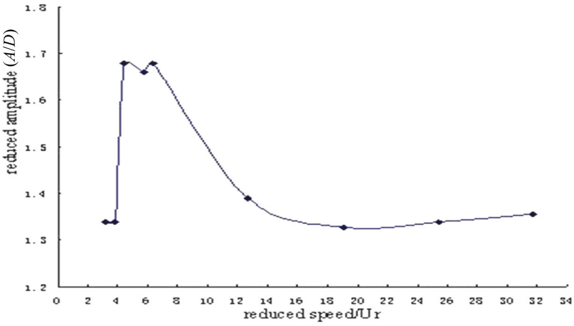


Fig. 4. Cross-flow reduced amplitude and reduced velocity

As that different velocity lead to different modes being excited by vortex-induced vibration of riser, this paper selects the points where maximum vibration displacement happened at 885 m water depth (velocity 0.18 m/s) and 1065 m water depth (velocity 0.4 m/s) as analysis points. In-line direction and cross-flow direction displacement time curve of analysis points are given in Fig. 5 - Fig. 6 and Fig. 7 - Fig. 8 under the two vortex-induced force models (considering parameter excited vibration and not considering parameter excited vibration model). By running it was determined that the program of the TTR is stable, so in this paper 0–50 s time history is selected.

Response results of vortex-induced vibration are given in Table 3 and Table 4.

From Fig. 5 - 8 displacement time histories, we could see that: the riser displacement increase whatever in cross-flow direction or in in-line direction if we consider the parameter excited vibration caused by floating platform. When velocity is 0.18 m/s, at 1065 m water depth, the maximum vibration displacement of TTR in in-line direction increases 0.0154 m, the maximum vibration displacement of TTR in cross-flow direction increases 0.0348 m. When velocity is 0.4 m/s, at 885 m water depth, the maximum vibration displacement of TTR in in-line direction increases 0.091 m, the maximum vibration displacement of TTR in cross-flow direction increases 0.1464 m. According to above analysis, the parameter excited vibration caused by floating platform increases transverse vibration displacement of TTR.

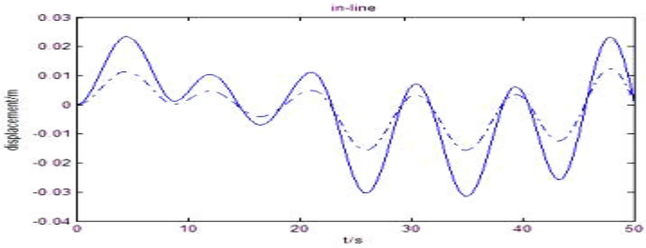


Fig. 5. Displacement time histories of in-line direction riser vibration at 1065 m water depth. Dotted line is on behalf of not considering parameter excited vibration, the solid line is on behalf of considering parameter excited vibration

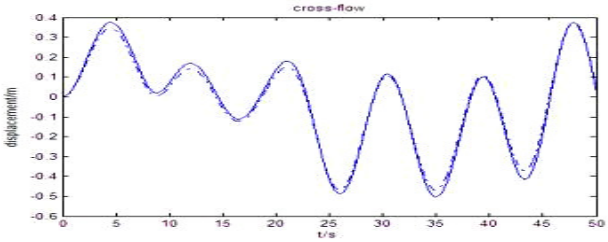


Fig. 6. Displacement time histories of riser vibration at 1065 m water depth in cross-flow direction. Dotted line is on behalf of not considering parameter excited vibration, the solid line is on behalf of considering parameter excited vibration

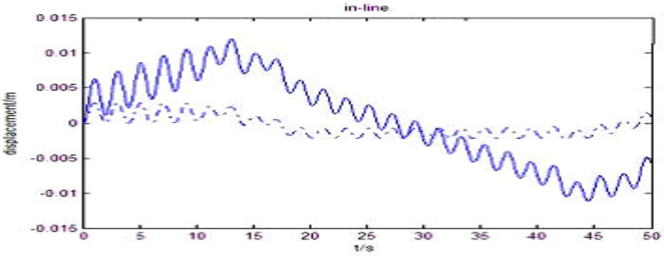


Fig. 7. Displacement time histories of in-line direction riser vibration at 885 m water depth. Dotted line is on behalf of not considering parameter excited vibration, the solid line is on behalf of considering parameter excited vibration

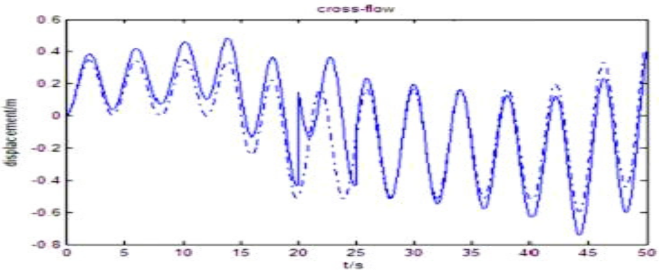


Fig. 8. Displacement time histories of cross-flow direction riser vibration at 885 m water depth. Dotted line is on behalf of not considering parameter excited vibration, the solid line is on behalf of considering parameter excited vibration

Table 3. Response results of vortex-induced vibration at 1065 m water depth

Direction	Maximum displacement		Increasing rate
	No coupled vibration model	The coupled vibration model	
In-line	0.0156 m	0.0310 m	98.7 %
Cross-flow	0.4684 m	0.5032 m	7.4 %

Table 4. Response results of vortex-induced vibration at 885 m water depth

Direction	Maximum displacement		Increasing rate
	No coupled vibration model	The coupled vibration model	
In-line	0.0029 m	0.0120 m	313 %
Cross-flow	0.5961 m	0.7425 m	24.5 %

6. Conclusions

The vertical vibration caused by floating platform increases transverse vibration displacement of TTR. So, when forecasting dynamic response of deepwater riser, the parameter excited vibration caused by floating platform shouldn’t be ignored, instead, it should be taken into account. The coupled vibration model of parameter excited vibration and vortex-induced vibration of TTR can provide reference for riser design and analysis.

Acknowledgments

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