817. Free vibration of transversely isotropic magneto-electro-elastic plates in contact with fluid

Tai-Ping Chang
National Kaohsiung First University of Science and Technology, Kaohsiung, Taiwan, R. O. C.
E-mail: *tpchang@ccms.nkfust.edu.tw*

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Abstract. In this study, one investigates the free vibration behaviors of transversely isotropic Magneto-electro-elastic (MEE) rectangular plates in contact with fluid. In particular, one derives the mathematical formulation on the determination of added virtual mass for MEE rectangular plates with uniform thickness, which is in contact with fluid. A fluid-structure interaction model is constructed and analyzed on the basis of the recently derived differential equation governing the dynamical responses of the MEE rectangular plates. The added virtual mass incremental (AVMI) factor of the system is computed by adopting the proposed method and the added virtual mass can then be estimated. The natural frequencies based on the proposed approach play an important role in the vibration analysis and design of the fluid-contacting MEE plate.

Keywords: fluid-structure interaction, magneto-electro-elastic (MEE) plates, vibration, added virtual mass.

Introduction

The added virtual mass incremental factor (AVMI factor) is often used to denote the ratio between the kinetic energy of the fluid and that of the structure, furthermore, the natural frequencies of the structure coupled with fluid can be determined by using the added virtual mass, which can be calculated following the determination of AVMI factor.

Chang and Liu [1] studied the free vibration behavior of a rectangular isotropic plate in contact with liquid by using the double Fourier Transform. Amabili [2] discussed the solution to the fully coupled problem of the vibrations of circular plates resting on sloshing liquid. Later on, Chang and Liu [3] performed the forced vibration analysis of a rectangular composite plate in contact with fluid as well as discussed the variation of AVMI factor for different plates with various widths and length. Besides, Liu and Chang [4] studied the vibration behavior of a varying-thickness circular plate in contact with fluid by adopting the Galerkin's method in conjunction with Hankel transform. More recently, Liu and Chang [5] have offered a compact form expression for the transverse vibration of a magneto-electro-elastic (MEE) thin plate, in which a new and simplified differential equation is presented.

The main theme of this study is to estimate the natural frequencies of a transversely isotropic magneto-electro-elastic (MEE) rectangular thin plate which is in contact with fluid.

Formulations

Consider the physical model of a transversely isotropic Magneto-Electro-Elastic (MEE) rectangular plate in contact with fluid as illustrated in Fig. 1, where $2a$ and $2b$ represent the width and length of the MEE rectangular plate and h is the thickness respectively.

F denotes the fluid domain, S_F denotes the surface between the fluid and an infinite rigid wall and S_B denotes the surface between the fluid and the plate, also S_R denotes the surface at infinity.

By neglecting the effects of rotatory inertia and transverse shear deformation, and utilizing the differential equation for MEE rectangular plates stated in Ref. [5], the governing equation of the undamped free vibration of a transversely isotropic Magneto-Electro-Elastic (MEE) rectangular plate in contact with fluid can be written as follows:

$$\left(D+E+M\right)\nabla^4 w+(\rho_P h+M_f)\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

where w is the transverse deflection of the plate, ρ_p is the mass density of the plate, h is the thickness of the plate, M_f denotes the fluid-added mass and $D\equiv\frac{c_{11}h^3}{12}$, $E\equiv\frac{e_{31}h^3}{12}\cdot\frac{\Delta_1}{\Delta}$, $M\equiv\frac{q_{31}h^3}{12}\cdot\frac{\Delta_2}{\Delta}$ represent the plate rigidity, effective rigidities due to the presences of electricity and magnetism, respectively. Here, $\Delta=\varepsilon_{33}\mu_{33}-d_{33}^2$, $\Delta_1=(e_{31}\mu_{33}-d_{33}q_{31})$, $\Delta_2=(\varepsilon_{33}q_{31}-d_{33}e_{31})$, c_{ij}, ε_{ij}, e_{ij}, q_{ij}, d_{ij} and μ_{ij} are the elastic, dielectric, piezoelectric, piezomagnetic, magnetoelectric, and magnetic constants, respectively. For free vibration analysis in the air, Eq. (1) yields:

$$\left(D+E+M\right)\nabla^4 w+\rho_P h\frac{\partial^2 w}{\partial t^2}=0 \tag{2}$$

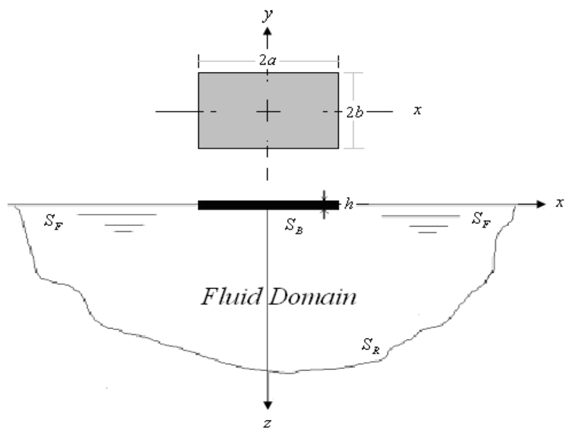

Fig. 1. The MEE rectangular plate in contact with fluid

By adopting the separation of variables, the solution of Eq. (2) can be expressed as follows:

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}W(x,y)T(t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}X(x)Y(y)T(t) \tag{3}$$

where:

$$T(t)=\sin\omega t\,. \tag{4}$$

and $X_m\left(x\right)$, $Y_n\left(y\right)$ are the orthogonal mode shape functions which satisfy the boundary conditions of Magneto-Electro-Elastic (MEE) rectangular plate in the x and y directions individually. In addition, the natural frequency ω can be computed from the boundary conditions of the MEE rectangular plate.

In the present study, the MEE rectangular plate is considered to be in contact with fluid on one side only, and the fluid is assumed to be incompressible and inviscid. Furthermore, the fluid flow is considered as irrotational under plate vibration only so that its velocity potential can be represented as:

$$U(x,y,z,t)=\phi(x,y,z)\dot{T}(t) \tag{5}$$

where ϕ is the spatial distribution of the velocity potential and "$\cdot$" denotes the derivative with respect to time. Based on the assumption of the fluid, ϕ has to satisfy the Laplace equation in F as follows:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{6}$$

where F denotes the fluid domain. As described in Fig. 1, S_F denotes the surface between the fluid and an infinite rigid wall and S_B denotes the surface between the fluid and the plate, besides S_R denotes the surface at infinity. The condition of the rigid wall on S_F, can be stated in the following:

$$\left.\frac{\partial\phi(x,y,z)}{\partial z}\right|_{z=0} = 0 \tag{7}$$

In addition, the interaction between the fluid and the plate can be expressed as follows:

$$\left.\frac{\partial\phi(x,y,z)}{\partial z}\right|_{z=0} = -\overline{W}(x,y) \tag{8}$$

where $\overline{W}$ denotes the "wet" mode shape of the MEE plate vibrating in contact with the fluid. Moreover, we must impose the conditions that the velocity potential ϕ and the velocities $\partial\phi/\partial x$, $\partial\phi/\partial y$ and $\partial\phi/\partial z$ approach zero on S_R, that is:

$$\phi, \frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z} \to 0 \quad \text{as } x,y,z \to \infty \text{ on } S_R \tag{9}$$

Based on the results verified by several researchers [6], it is assumed that the "wet" mode shape of the MEE plate in contact with the fluid is the same as the "dry" mode shape of the MEE plate when vibrating in the air. Hence, the approximation $\overline{W}(x,y) = W(x,y)$ will be used in the following derivations.

First of all, denote the double Fourier transform in the following:

$$\overline{\phi}(u,v,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x,y,z)\, e^{+i(ux+vy)} dxdy \tag{10}$$

Utilizing double Fourier transform on Eq. (6) and applying the boundary conditions stated in Eq. (9), then the velocity potential $\phi(x,y,z)$ can be represented as:

$$\phi(x,y,z) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi}(u,v,z)\, e^{-i(ux+vy)} dudv = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B(u,v)\, e^{-(u^2+v^2)^{\frac{1}{2}} z} e^{-i(ux+vy)} dudv \tag{11}$$

Applying the boundary conditions specified in Eqs. (7) and (8), $B(u,v)$ in Eq. (11) can be determined in the following:

$$B(u,v) = (u^2+v^2)^{-1/2} \int_{-b}^{b}\int_{-a}^{a} X_m(x) Y_n(y)\, e^{+i(ux+vy)} dxdy \tag{12}$$

Generally $B(u,v)$ is a complex function of both u and v.

Based on the previous assumption that the wet mode shapes are almost the same as the dry mode shapes, the natural frequency of the MEE plate in contact with fluid ω_f can be evaluated as follows [7]:

$$\omega_{fmn} = \frac{\omega_{amn}}{\sqrt{1+\gamma_{mn}}} \tag{13}$$

where ω_{amn} is the natural frequency of the MEE plate in the air. Utilizing Eq. (2) in conjunction with the general boundary conditions for the MEE plate, it is quite feasible to determine ω_{amn} in the following form:

$$\omega_{amn} = \frac{(D+E+M)}{\rho_p h}(\alpha_m{}^4 + 2\alpha_m{}^2\beta_n{}^2 + \beta_n{}^4) \tag{14}$$

in which α_m and β_n are the corresponding eigenvalues according to the MEE plate's boundary conditions along the x and y directions, respectively. Also γ_{mn} in Eq. (13) is the AVMI factor that denotes the ratio between the reference kinetic energy of fluid induced by the plate vibration and that of the plate which can be expressed as:

$$\gamma_{mn} = \frac{T_F}{T_P} \tag{15}$$

The reference kinetic energy of the MEE plate can be calculated as:

$$T_p = \frac{1}{2}\rho_p h \int_{-b}^{b}\int_{-a}^{a} X_m{}^2(x)Y_n^2(y)\, dxdy \tag{16}$$

Employing the assumption on the irrotational movement of the fluid flow, the reference kinetic energy of the fluid can be determined as follows from its boundary condition:

$$T_F = -\frac{1}{2}\rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\phi(x,y,0)}{\partial z}\phi(x,y,0)dxdy \tag{17}$$

Substituting Eq. (7), (8) and (11) into Eq. (17) yields:

$$T_F = \frac{1}{2}(\frac{1}{2\pi})^2 \rho_F \int_{-b}^{b}\int_{-a}^{a}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_m(x)Y_n(y)B(u,v)e^{-i(ux+vy)}dudvdxdy\cdot \tag{18}$$

To calculate the above multiple integral, reverse the order of integration, thus Eq. (18) can be simplified as follows:

$$\begin{aligned} T_F &= \frac{1}{2}(\frac{1}{2\pi})^2 \rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{-b}^{b}\int_{-a}^{a} X_m(x)Y_n(y)e^{-i(ux+vy)}dxdy\right]B(u,v)dudv \\ &= \frac{1}{2}(\frac{1}{2\pi})^2 \rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[B^*(u,v)(u^2+v^2)^{\frac{1}{2}}\right]\left[B(u,v)\right]dudv \\ &= \frac{1}{2}(\frac{1}{2\pi})^2 \rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|B(u,v)\right|^2(u^2+v^2)^{\frac{1}{2}}dudv \end{aligned} \tag{19}$$

where * is the complex conjugate and it is noted that $B(u,v)$ can be evaluated from Eq. (12) as long as the dry mode shapes of the MEE plate are available. Hence, once T_F and T_P are calculated, γ_{mn} (AVMI factor) can be determined from Eq. (15) and finally the natural frequency ω_{fmn} of the MEE plate in contact with fluid is readily obtained from Eq. (13).

Numerical examples and discussion

In the numerical computations, we perform the free vibration analysis of the MEE plate in contact with fluid by considering a bi-layered $BaTiO_3 - CoFe_2O_4$ composite with variable volume fraction (v.f.) of $BaTiO_3$. The dimensions of the plate are assumed as follows: length $a = 2$ m, width $b = 2$ m and height $h = 0.05$ m. The density of the bi-layer plate is assumed to be proportional to the volume-faction of these two materials, i. e. $\rho_P = \rho_{BaTiO_3}$*v.f.+ $\rho_{CoFe_2O_4}$*(1-v.f.). Besides, the density of the fluid is assumed as $\rho_f = 1000$ kg/m^3. The numerical computations are carried out by selecting six different plates with volume fractions in steps of 20 %, i. e. 0 %, 20 %, 40 %, 60 %, 80 %, 100 %. The magneto-electro-elastic material properties with different volume fraction are given by Annigeri et al. [8]. The boundary conditions for the MEE plate are selected to be simply-supported on four sides (SSSS), nevertheless, any other kinds of boundary conditions can be considered without any difficulties.

Based on Eq. (15), we first determine γ_{mn} (AVMI factor) of a MEE rectangular plate in contact with fluid. In particular, a bi-layered $BaTiO_3 - CoFe_2O_4$ MEE plate with 40 % $BaTiO_3$ is considered. Table 1a and Table 1b show the values of added virtual mass incremental factor (AVMI factor) for such a MEE plate. It can be detected from these tables that the values of γ_{11} is much larger than those of the other modes. This implies that γ_{11} plays a dominant role as far as AVMI factor is concerned; besides it can be found that γ_{mn} decreases with the mode order, meaning the effect of fluid also decreases with the mode order. Generally speaking, AVMI factor γ_{mn} of the lower mode number is larger than that of the higher mode number since the fluid movement stroke of the lower mode number is larger than that of the higher one. Hence, the fluid movement stroke will be reduced as the mode number increases, which will end up with the reduction of the added mass and AVMI factor. We now calculate the natural frequencies of a MEE plate in contact with fluid by using Eqs. (13) and (14). Tables 2a-2d and Tables 3a-3d present the natural frequencies of the bi-layered MEE thin plate in contact with fluid subjected to different volume fraction of the PZ material $BaTiO_3$, which are ranging from 0 % to 100 % with 20 % offset. The natural frequencies presented in Tables 2a-2d are for odd modes; while those presented in Tables 3a-3d are for even modes. It is noted that the natural frequencies listed in Tables 2a-3d are very useful for those engineers or researchers who are engaged in the vibration analysis and design of the MEE plate in contact with fluid.

Table 1a. The values of γ_{mn} for odd mode of a bilayered square plate (40 % of $BaTiO_3$)

γ_{mn}	$m = 1$	$m = 3$	$m = 5$
$n = 1$	3.084426	0.999956	0.578724
$n = 3$	0.999956	0.618340	0.436316
$n = 5$	0.578724	0.436316	0.349497

Table 1b. The values of γ_{mn} for even mode of a bilayered square plate (40 % of $BaTiO_3$)

γ_{mn}	$m = 2$	$m = 4$	$m = 6$
$n = 2$	0.962513	0.581607	0.403931
$n = 4$	0.581607	0.440930	0.341378
$n = 6$	0.403931	0.341378	0.286101

Table 2a. Natural frequencies for odd mode of a bilayered square plate (20 % of $BaTiO_3$)

Omega (rad/s)	$m = 1$	$m = 3$	$m = 5$
$n = 1$	293.27	2100.50	6154.17
$n = 3$	2100.50	4207.55	8442.12
$n = 5$	6154.17	8442.12	12813.44

Table 2b. Natural frequencies for odd mode of a bilayered square plate (40 % of $BaTiO_3$)

Omega (rad/s)	$m = 1$	$m = 3$	$m = 5$
$n = 1$	267.96	1914.69	5602.99
$n = 3$	1914.69	3831.34	7681.71
$n = 5$	5602.99	7681.71	11654.26

Table 2c. Natural frequencies for odd mode of a bilayered square plate (60 % of $BaTiO_3$)

Omega (rad/s)	$m = 1$	$m = 3$	$m = 5$
$n = 1$	248.95	1774.77	5187.43
$n = 3$	1774.77	3547.50	7107.96
$n = 5$	5187.43	7107.96	10779.44

Table 2d. Natural frequencies for odd mode of a bilayered square plate (80 % of $BaTiO_3$)

Omega (rad/s)	$m = 1$	$m = 3$	$m = 5$
$n = 1$	209.91	1493.07	4358.90
$n = 3$	1493.07	2981.40	5969.29
$n = 5$	4358.90	5969.29	9049.30

Table 3a. Natural frequencies for even mode of a bilayered square plate (20 % of $BaTiO_3$)

Omega (rad/s)	$m = 2$	$m = 4$	$m = 6$
$n = 2$	1060.28	4020.15	10047.32
$n = 4$	4020.15	7932.69	13366.69
$n = 6$	10047.32	13366.69	18906.85

Table 3b. Natural frequencies for even mode of a bilayered square plate (40 % of $BaTiO_3$)

Omega (rad/s)	$m = 2$	$m = 4$	$m = 6$
$n = 2$	966.42	3660.29	9141.04
$n = 4$	3660.29	7218.29	12157.28
$n = 6$	9141.04	12157.28	17191.09

Table 3c. Natural frequencies for even mode of a bilayered square plate (60 % of $BaTiO_3$)

Omega (rad/s)	$m = 2$	$m = 4$	$m = 6$
$n = 2$	895.73	3388.76	8456.81
$n = 4$	3388.76	6679.25	11243.92
$n = 6$	8456.81	11243.92	15894.99

Table 3d. Natural frequencies for even mode of a bilayered square plate (80 % of $BaTiO_3$)

Omega (rad/s)	$m = 2$	$m = 4$	$m = 6$
$n = 2$	753.50	2847.58	7101.23
$n = 4$	2847.58	5609.44	9438.84
$n = 6$	7101.23	9438.84	13339.51

Conclusions

In this paper, the fluid-structure interaction problem is investigated. In particular, the vibration characteristics of transversely isotropic Magneto-electro-elastic (MEE) rectangular plates in contact with fluid are investigated.

It is well known that the natural frequencies of the uniform plate in contact with fluid can be estimated by using the added virtual mass incremental (AVMI) factor, which represents the kinetic energy due to the fluid. In the present study, the mathematical formulation on the determination of added virtual mass for water-contacting MEE rectangular plates with uniform thickness is performed. A recently proposed differential equation governing the dynamical responses of the MEE rectangular plates is adopted, and the attempt to extend the system into a fluid-interaction model in order to account for the water influence is accomplished. On the fluid-structure interface, some techniques are used to deal with the relationships between the velocity potential and the mode shapes, and then by imposing the Fourier transform, one can derive the formulations of the reference kinetic energy for both the fluid and the plate itself.

After these two energies have been computed, the added virtual mass incremental (AVMI) factor can be obtained rapidly and the added virtual mass can thus be acquired. From the engineering point of view, the proposed approach and some important findings can be adopted as a guide for those engineers or researchers who are engaged in the vibration analysis and design of the MEE plate in contact with fluid.

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References

[1] **Chang T. P., Liu M. F.** On the natural frequency of a rectangular isotropic plate in contact with fluid. Journal of Sound and Vibration, Vol. 236, No. 3, 2000, p. 547-553.
[2] **Amabili M.** Vibrations of circular plates resting on sloshing liquid: solution of the fully coupled problem. Journal of Sound and Vibration, Vol. 245, No. 2, 2001, p. 261-283.
[3] **Chang T. P., Liu M. F.** Vibration analysis of rectangular composite plates in contact with fluid. Mech. Struct. & Mach., Vol. 29, No. 1, 2001, p. 101-120.
[4] **Liu M. F., Chang T. P.** Axisymmetric vibration of a varying-thickness circular plate in contact with fluid. Mech. Based Design Struct. & Mach., Vol. 32, No. 1, 2004, p. 39-56.
[5] **Liu M. F., Chang T. P.** Closed form expression for the vibration problem of a transversely isotropic magneto-electro-elastic plate. ASME, Journal of Applied Mechanics, Vol. 77, No. 2, 2010, p. 024502-1-024502-8.
[6] **Espinosa F. M., Gallego-Juarez J. A.** On the resonance frequencies of water-loaded circular plates. Journal of Sound and Vibration, Vol. 94, 1984, p. 217-222.
[7] **Kwak M. K., Kim K. C.** Vibration of circular plates in contact with water. ASME, Journal of Applied Mechanics, Vol. 58, 1991, p. 481-483.
[8] **Annigeri A. R., Ganesan N., Swarnamani S.** Free vibration behavior of multiphase and layered magneto-electro-elastic beam. Journal of Sound and Vibration, Vol. 299, 2007, p. 44-63.