795. Characterization of mechanical properties by inverse technique for composite reinforced by knitted fabric.

Part 2. Experimental evaluation of mechanical properties by frequency eigenvalues method

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Abstract. This paper is the second part of the research work dedicated to evaluation of mechanical properties of polymer composites reinforced by knitted fabric. Three different approaches were applied for the task. Two of them: a) FEM analysis using Solid Works combined with structural modeling based on experimentally-determined mechanical and geometrical properties of the reinforcement and matrix, and b) direct measurement of mechanical properties (described in Part 1). Present investigation (Part 2) is based on application of vibrational analysis. Modal testing in combination with the mathematical optimization procedure were used for evaluation of elastic properties of a layered material. It is worth mentioning that the application of this approach for materials with high damping ability (laminated composites reinforced by knitted fabric) is still poorly investigated. The inverse technique exploited in this work is based on the direct orthotropic plate free vibration measurements and subsequent mathematical optimization procedure (the planning of experiments or response surface technique), which is based on minimization of error functional. Finally, elastic constants established by the inverse technique were discussed and compared with the results obtained in Part 1.

Keywords: textile composites, weft knitted fabric, modal analysis.

Introduction

Precise evaluation of the in-plane elastic properties for orthotropic composite plates is very important and is still a challenging problem, especially if we have material with high damping potential such as laminates reinforced by knit fabrics. As the possibility to overcome these obstacles, we can mention an attempt to estimate these elastic parameters using an inverse technique. Traditionally, it is performed using experimentally measured data sets that can also be obtained through the analytical or numerical modeling, and the associated inverse problem can be solved in a few ways. The determination of stiffness parameters for such materials as fiber-reinforced composites is not easy and sometimes has many discrepancies. Some inverse methods used for non-destructive evaluation of elastic properties can be found in literature [1-10]. The present study is focused on the identification of elastic properties for laminated plates using the vibration test data [2, 3, 9, 10]. The core idea of the method is that it is necessary to convert modal frequencies of composite plate free vibrations into elastic constants of plate material. A standard method for solving this problem is to use eigenvalue data in combination with numerical-experimental model and optimization techniques [5-6, 9-12]. The identification function represents the discrepancy between the numerical model response and the experimental one. This discrepancy must be minimized taking into account the constraints on the design variables (elastic constants). The minimization problem can be solved by using non-linear mathematical programming techniques and sensitivity analysis [6, 9-12]. The experimental design and the response surface approach was used in [8] instead of the direct function minimization, in such way reducing the amount of necessary computations. All the aforementioned approaches rely on experimental measurement of free vibration frequency eigenvalues. Modal analysis is an approach used to determine the natural frequencies (frequency eigenvalues) and mode shapes (eigenshapes) of structural members. The resonant frequency for construction member of simple shape (rectangular plate) made out of orthotropic material is dependent on material stiffness values and mass distribution, so modal analysis can used for identification of material properties. In this paper modal analysis approach was used for defining mechanical properties for polymer composite plate reinforced by knitted fabrics.

Numerical-experimental method

The numerical-experimental method [3] consists of several steps. In the beginning the physical experiments have been performed. Second step is to identify the domain of search and choose the criterion containing experimental data. Then finite element method is used in order to model the frequency response of the structure. FEM eigenvalue results are employed as numerical experimental data. Then experiment design points are determined. In the next step the numerical data are obtained by FEM in the reference points with the goal to determine simple functions using response surface method for calculation of the eigenfrequencies. After that, by using simple models and experimental data of the measured eigenfrequencies the identification of the material properties is performed minimizing corresponding functional (using method of non-linear programming).

Parameters will be identified. Elaborated mathematical procedure allows one to evaluate all orthotropic elastic constants of the plate. In our investigation we are reducing the parameters set to be identified. Only in plane elastic constants will be identified: Young's modulus E_1 and E_2, shear modulus: G_{12}, Poisson's ratio v_{12}. According to above mentioned, in the model was accepted: $E_2 = E_3$; $G_{12} = G_{13}$; $v_{12} = v_{23} = v_{13}$ and further, because for a composite plate, some elastic constants are less sensitive to frequencies, one independent elastic constant will be fixed. Shear modulus $G_{23} = 1.5\times10^5$ MPa.

The plan of experiments for the composite plate was formulated for 4 design parameters and 101 experiments. The limits of the search region were:

$5.5 \leq E_1 \leq 6.5$ GPa;
$4.5 \leq E_2 = E_3 \leq 5.0$ GPa;
$1.5 \leq G_{12} = G_{13} \leq 2.5$ GPa;
$0.29 \leq v_{12} = v_{23} = v_{13} \leq 0.37$

The parameters to be identified are defined through non-dimensional quantities. An experimental frequencies eigenvalues was designated as $\overline{f}_1, \overline{f}_2, \overline{f}_3, \ldots \overline{f}_i$, where i is the number of measured frequencies eigenvalues. The value of i is typically taken between 7 and 15 [3]. The corresponding numerical eigenfrequencies f_i for our material parameters α_i are represented by $f_1, f_2, \ldots f_i$. The identification process is carried out through minimization of an error function that expresses the relative difference between the measured $\overline{f}_i$ and numerically calculated f_i frequencies:

$$\Phi(X) = \sum_{i=1}^{n} \frac{\left(\overline{f}_i - f_i\right)^2}{\left(\overline{f}_i\right)^2} \Rightarrow \min,$$

where: $\overline{f}_i$ – experimentally obtained frequency eigenvalue [Hz]; f_i – FEM calculated frequency eigenvalue [Hz].

Finite element solution. The eigenvalue problem for the harmonic vibrations can be represented by:

$$\mathbf{Ku} = f^2\mathbf{Mu}, \tag{1}$$

here **K** is the stiffness matrix of the plate, **M** is the mass matrix of the plate and **u** is the displacement vector. The eigenvalue relation (1) for mode $\mathbf{u}_1$, which corresponds to the first experimental eigenfrequency f_1 can be written in an equivalent form putting E_1 in evidence:

$$E_1\mathbf{K}*\mathbf{u}_1 = \bar{f}_i^2\mathbf{M}\mathbf{u}_1, \tag{2}$$

here $E_1\mathbf{K}^* = \mathbf{K}$ is the stiffness matrix. Taking into account the relation:

$$C = \frac{\bar{f}_1^2}{\tilde{f}_1^2}, \ \tilde{f}_1 = Cf_1, \tag{3}$$

this equation can be written as:

$$CE_1^0\mathbf{K} * \mathbf{u_1} = C\,\tilde{f}_1^2\,\mathbf{Mu_1}, \tag{4}$$

hence:

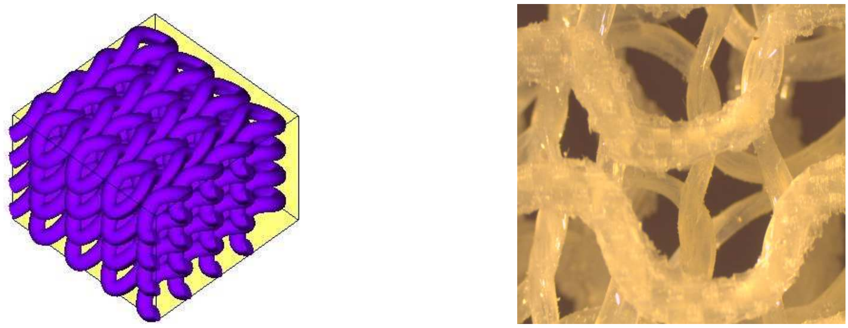
$$E_1 = CE_1^0, \tag{5}$$

where E_1^0 is the initial guess value given to the Young's modulus in the fiber direction of the layer and E_1 is the corresponding identified mechanical property. After the evaluation of the optimum values of $\boldsymbol{x}$ the remaining mechanical properties are then calculated through the inverse relations (1).

Calculated numerical eigenfrequencies were approximated according to material properties. Second order approximations were used. For example, for the sample SP1 first eigenfrequency approximation according to the properties of the material looks like:

$$\begin{aligned} f_1 = {} & 14.068 - 9.216\times10^{-10}\times E_1 - 1.284\times10^{-9}\times E_2 + 8.537\times10^{-9}\times G - 4.455\times n + \\ & + 3.126\times10^{-20}\times E_1^2 + 8.414\times10^{-21}\times E_1\times E_2 - 3.296\times10^{-20}\times E_1\times G + 1.830\times10^{-9}\times E_1\times n + \\ & + 1.479\times10^{-19}\times E_2^2 - 8.598\times10^{-20}\times E_2\times G + 1.169\times10^{-10}\times E_2\times n - 7.918\times10^{-19}\times G^2 + \\ & + 6.874\times10^{-10}\times G\times n - 11.779\times n^2. \end{aligned}$$

Materials. Glass fiber weft knitted fabrics were prepared on knitting machine. Precise preparation procedure is provided in Part 1. Fabrics were stacked and impregnated by polymer thermoset resin at room temperature. Epoxy resin was used. The laminate lay-up was $[0]_4$ (see Fig. 1). Glass fiber epoxy matrix composite plates were fabricated. Dimensions of the plates and matrix and fibers properties are listed in Table 1.



a) b)

Fig. 1. Layered Glass fiber / Epoxy matrix $[0]_4$ composite laminate. a) geometrical simulation; b) stacked fabric yarns impregnated by epoxy matrix (further all space between yarns will be fulfilled by epoxy resin)

Two plates were fabricated – SP1 and SP2 (see Fig. 2). The general experiment setup of the POLYTEC PSV-400-B Scanning Laser Vibrometer consists of a PSV-I-400 LR optical scanning head equipped with high sensitivity vibrometer sensor (OFV-505), an OFV-5000 controller, PSV-E-400 junction box, an amplifier Bruel&Kjaer type 2732, and a computer system with data acquisition board and PSV Software.

Table 1. Plates properties

	Laminated composite with knitted glass fabric reinforcement	
Layer number	4	
Plates dimensions, mm	280×380, thickness SP1 - 0.176, SP2 - 0.202	
Stitch density of the fabric	W = 1,053 loops/cm, C = 2 loops/cm	
Yarn diameter, mm	0,37	
Fiber-volume fraction W_f, %	11	
	matrix – epoxy resin	**reinforcement**
Density ρ, g/cm^3	1,36	2,54
Young modulus E, GPa	3,3	73,4
Poisson ratio v	0,22	0,35

The system requires defining the geometry of the object and set up scanning grid. Symmetrical points have been taken to cover a rectangular panel with regular grid. Free-free boundary conditions have been simulated by hanging up the panel with two thin threads bonded in two top corners of the plate. The test panel has been excited by a piezoelectric actuator (PZT), placed in the bottom of the composite plate. As a result of this excitation the plate starts to vibrate within the frequency band of the input signal.

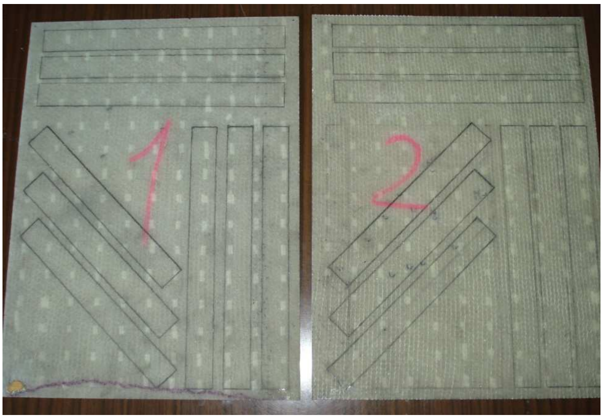


Fig. 2. Two fabricated plates: SP1 and SP2. On the plates it is possible to see the samples (are sketched and later were cut out of plates). Samples have diverse orientation to vertical each plate direction

After the measurement if performed in one point, the vibrometer automatically moves the laser beam to another point at the scan grid and measures the response using the Doppler principle and validates the measurement with the signal-to-noise ratio. The procedure is repeated until all scan points have been measured. The frequency spectrum of the panel is then obtained by taking the Fast Fourier Transform of the response signal. Fig. 3 and 4 shows experimental setup for modal testing of the laminated composite plate.

Results

Frequency response functions were obtained (see Figs. 5-6). Resonant frequencies for different vibrations modes, for both two plates were measured experimentally and calculated numerically and are given (for the first 18 modes) in Table 2. Corresponding vibrations modes are shown in Fig. 7.

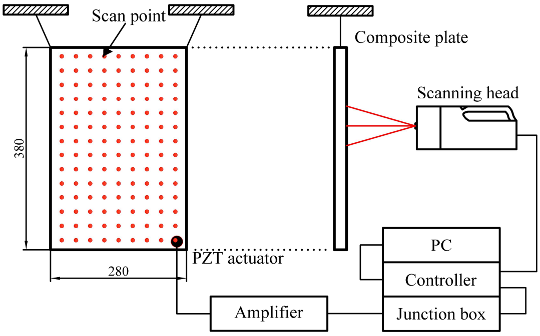


Fig. 3. Signal acquisition system for detailed plate vibration analysis

a) b)

Fig. 4. Plate modal analysis experimental system: a) scanning laser system; b) hanged plate with connected piezoactuator

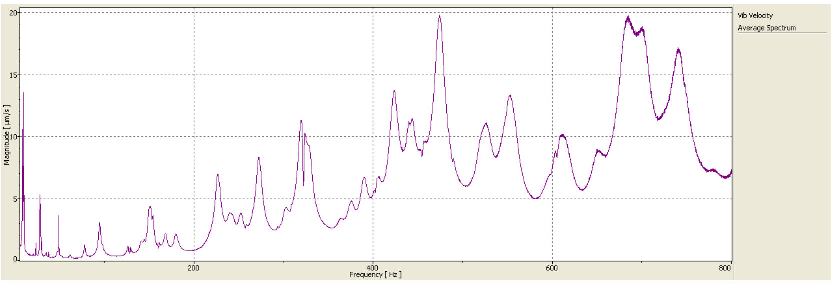


Fig. 5. Frequency response function of the laminated composite plate SP1

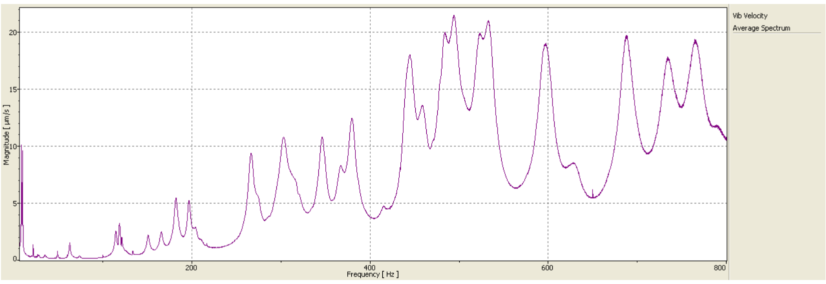
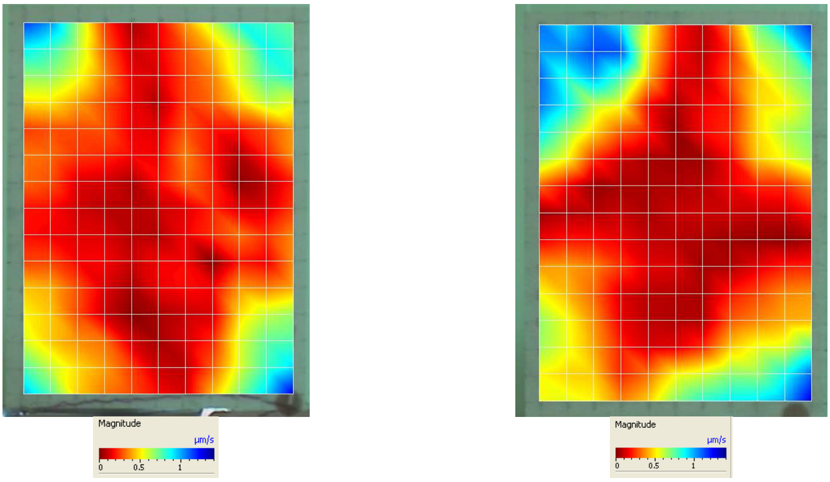
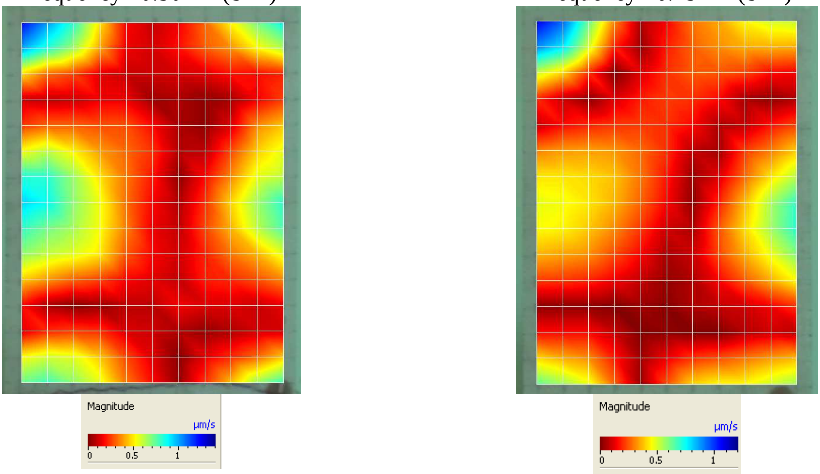


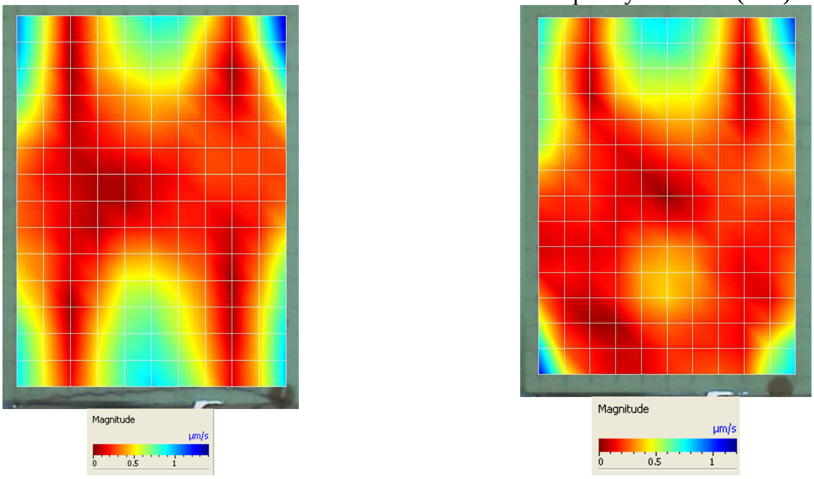
Fig. 6. Frequency response function of the laminated composite plate SP2



frequency 20.50 Hz (SP1) frequency 26.75 Hz (SP2)



frequency 47.50 Hz (SP1) frequency 62.75 Hz (SP2)



frequency 61.50 Hz (SP1) frequency 73.50 Hz (SP2)

Fig. 7. Mode shapes corresponding to different knitted fabric reinforced composite plate vibration frequencies

Table 2. Natural frequencies of composite plates

SP1				SP2		
Mode no.	FEM	Experimental	Δ, %	FEM	Experimental	Δ, %
1	20.30	20.50*	0.97	26.40	26.75*	1.34
2	26.68	-		31.62	-	
3	46.42	47.50	2.33	54.31	-	
4	49.31	-		62.45	62.75*	0.48
5	60.46	61.50*	1.71	74.02	73.50*	0.71
6	76.53	77.75*	1.59	91.50	-	
7	96.68	94.50*	2.25	119.40	118.50*	0.76
8	99.21	-		125.10	121.00*	3.28
9	126.85	126.25*	0.47	148.21	150.75*	1.71
10	141.41	141.25*	0.11	168.94	165.75*	1.89
11	148.53	150.75	1.49	177.16	182.00*	2.73
12	153.93	-		193.76	196.00*	1.16
13	167.25	168.00*	0.45	204.03	204.25	0.11
14	182.92	179.75*	1.73	224.86	-	
15	227.49	226.75*	0.32	282.25	-	
16	239.95	240.00*	0.02			
17	244.35	-				
18	250.51	252.00*	0.59			
	Mean difference, %		1.08			1.42

Data comparison shows good agreement between the calculated and measured results. Elastic constants for both plates are shown in Table 3. Both plates were cut into pieces and after that were tested by direct method (described in Part 1). Results in Table 4 indicate predictions from the three methods for longitudinal and transverse modulus of glass fiber knitted fabric reinforced composite plate. The results reveal that inverse method provides higher elastic properties in comparison with the direct experiment and numerical modeling. At the same time, this demonstrates the effectiveness of modeling approach and inverse method application to materials with high damping facilities such as textile reinforced polymer composite.

Table 3. Obtained elastic constants for plates SP1 and SP2

Sample designation	SP1	SP2
E_1 [GPa]	5,93	6,23
E_2 [GPa]	4,92	4,96
G_{12} [GPa]	1,80	2,30
ν_{12}	0,29*	0,32

Table 4. Comparison of elastic properties obtained by three different methods for glass fiber knitted fabric reinforced composites

Method	E_L, GPa	E_T, GPa
Solid Works model	5.825	4.555
Modal analysis	5.93/6.23	4.92/4.96
Experimental tensile tests	5.46	3.95

Conclusions

Three different approaches were successfully implemented: a) numerical structural modeling (FEM using Solid Works) based on application of experimentally measured mechanical and geometrical properties of reinforcement and matrix; b) direct measurements of mechanical properties; c) inverse method approach with the goal to predict mechanical properties of weft knitted fabric reinforced multilayered composite. Inverse method predicted higher elastic properties in comparison with direct experiment and FEM results. Thereby, the effectiveness of the modeling approach was demonstrated. Applicability of the inverse method to materials with high damping facilities such as textile reinforced polymer composite was confirmed.

Acknowledgement

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