787. A joint stiffness identification method based on finite element modeling and frequency response functions

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Abstract. Accurate finite element (FE) modeling of mechanical structures is extremely difficult with unknown joints or boundary conditions. An alternative joint stiffness identification method that involves a hybrid of FE model and frequency response functions (FRFs) is presented. Firstly, the joint stiffness is assumed by experience and the mechanical structure is modeled with the FE method. Secondly, the FRFs at the concerned nodes of the structure are simulated and measured, respectively. Then the norm of residual FRFs between the simulations and measurements is calculated. Finally, a sensitivity-based iterative algorithm is derived for minimizing the norm of residual FRFs and the least square method is used to solve over-determined iterative equation. The joints stiffness parameters are identified through the iteration process, while the FE model is updated simultaneously. The proposed joint stiffness identification method is applied on a clamped beam assembly. The first three natural frequencies calculated by the FE model are compared with the measured values. The largest relative error of the simulation deceases from 16.7 % to 2.5 % after the joint stiffness parameters are identified, which demonstrates the effectiveness of the presented method.

Keywords: joint parameters identification, model updating, finite element method, frequency response function, joint stiffness.

1. Introduction

Structural systems are composed of substructures and joints. The overall dynamic characteristics of the structural system, such as natural frequencies, mode shapes, and non-linear response characteristics to external excitations, are affected largely by the joints. Accurate mathematical models representing the dynamic characteristics of structure systems have been required in the design and analysis stage. Although finite element (FE) analysis is a well-established and accepted technique for the dynamic analysis of structural systems at the design stage, proper dynamic model still cannot be obtained if joint parameters are not accurately identified. For instance, the welded or less-than-rigid bolted joints in an assembly are often inappropriately modeled as rigid connections, which results in mismatch between FE analysis and experimental measurements.

Identification of joint parameters is an important task in predicting the dynamic characteristics of structural systems. The main joint parameters considered in structural dynamics are stiffness and damping properties. Considerable studies have been conducted to extract joint properties from measured data. Measured modal parameters were used in early studies to identify the joint parameters. For example, Inamura and Sata [1] proposed an approach to identify the stiffness and damping properties based on the use of the complete mode shapes and eigenvalues. Kim et al. [2] used a condensed FE model and incomplete mode shapes to identify properties for a taper joint. These methods require accurate modal parameters which are difficult to extract, especially for structures containing closely spaced modes or large modal damping [3], and thus the application is limited.

In order to circumvent the difficulties of extracting accurate modal parameters, some methods based on frequency response functions (FRFs) have been proposed to identify joint parameters. Tsai and Chou [4] proposed a method to obtain the properties of a single bolted joint directly from the measured responses. Wang and Liou [5] improved Tsai and Chou's method by simplifying the mathematics in the inverse operation. Ren and Beards [6] discussed the techniques for improving the accuracy of FRF-based joint identification method. Yang et al. [7] combined the substructure synthesis method with FRFs, and a coupled stiffness matrix was used to model a joint.

Recently, Celic and Boltezar [8, 9] considered not only the mass, stiffness and damping effects, but also the effects of rotational degrees of freedom (DOFs). An improved joint-identification method was presented with higher accuracy. The approach easily allowed an expansion to more DOFs or to a larger number of joints and therefore more FRFs were available. Furthermore, Wang et al. [10] developed a new unmeasured FRFs estimation method, which is capable of estimating all of the unmeasured FRFs. Since all of the FRFs are included, the identification accuracy and robustness are thus improved.

The basic idea of the most above joint identification methods are the same, i.e. the joint properties can be extracted from the measured FRFs of the assembled structure and its substructures. However, such experimental methods may become impractical for the cases when some joints are not measurable. Other attempts were made to employ the FE model updating methods for the joint identification problems [5, 11]. Because a mechanical joint may simply be treated as a lumped element in a FE model, any general model updating technique can be essentially used to identify the joint parameters [12]. Model-based techniques that involve a hybrid of experimental data and FE model results have been widely used [12, 13]. The FE model was updated by estimating the mechanical joint parameters.

In this study an alternative model-based joint parameters identification method using FRFs is presented. The basic idea is that the analytical FE model of a mechanical structure can be modified with the changes of joint parameters, which lead to an updated FE model, whose predicted FRFs can match well with the measured FRFs. A sensitivity-based iterative algorithm is introduced for minimizing residuals between simulations and measurements. The dynamic characteristics of the joints are identified through the iteration process, while the FE model is updated simultaneously. A clamped beam assembly example is provided to describe the identification procedure and to validate the proposed method.

2. The joint parameter identification method

2. 1. The sensitivity-based iterative algorithm

The principle of sensitivity-based iterative algorithm can be described in the form of:

$$[S]\{\Delta u\} = \{\varepsilon\}, \tag{1}$$

where $[S]$ is the sensitivity matrix, $\{\Delta u\}$ is the vector of joint parameters or updated parameters, and $\{\varepsilon\}$ is the residual i.e., the difference between the analytical and measured dynamic FRFs. Such system of equations is often over-determined and solved by using least squares method. After solving Eq. (1), the vector of joint parameters $\{\Delta u\}$ is obtained and the FE model is updated. Following an eigen-solution of the updated FE model, a new residual is obtained. This process of solving and updating the system are repeated until the residual is zero or smaller than a defined threshold.

The existence of residuals and their theoretical treatment rely on the assumption that the analytical FE model (a) and the experimental test specimen (x) can both be represented by:

$$\left[Z^a(\omega)\right]\left\{X^a(\omega)\right\} = \left\{F^a(\omega)\right\} \quad \left[Z^x(\omega)\right]\left\{X^x(\omega)\right\} = \left\{F^x(\omega)\right\}, \tag{2}$$

where $[Z]$ is the dynamic stiffness, $\{X\}$ and $\{F\}$ are the displacement (or output) and force (or input) vectors. The dynamic stiffness matrices of the analytical FE model and experimental test specimen are given as:

$$\left[Z^a(\omega)\right]=[K]^a-\omega^2[M]^a+j\omega[C]^a \quad \left[Z^x(\omega)\right]=[K]^x-\omega^2[M]^x+j\omega[C]^x .$$

The model updating algorithm is derived from the input residual $\{\varepsilon_F(\omega)\}$ between the analytical and measured forces in the form of:

$$\{\varepsilon_F(\omega)\}=\{F^a(\omega)\}-\{F^x(\omega)\} . \tag{3}$$

Using Eq. (2) and assuming that the v^{th} DOF is excited by the force of unit magnitude, the force residual $\{\varepsilon_{F_j}\}_v$ at a selected frequency point ω_j may be equivalently defined as:

$$\{\varepsilon_{F_j}\}_v=\{1\}_v-\left[Z^a(\omega_j)\right]\{H^x(\omega_j)\}_v , \tag{4}$$

where $\{1\}_v$ is unity at the excitation point v and zero elsewhere, $\{H^x(\omega_j)\}_v$ is the experimental FRF vector at the selected frequency point ω_j.

Similar to the input residual equation Eq. (4), the residual FRF $\{\varepsilon_{H_j}\}_v$ at the selected frequency point ω_j between the analytical and experimental FRFs can be given as:

$$\{\varepsilon_{H_j}\}_v=\{H^a(\omega_j)\}_v-\{H^x(\omega_j)\}_v . \tag{5}$$

With the analytical FRF matrix $\left[H^a(\omega_j)\right]$, the residual FRF $\{\varepsilon_{H_j}\}_v$ can be expressed in another form as:

$$\{\varepsilon_{H_j}\}_v=\left[H^a(\omega_j)\right]\{\varepsilon_{F_j}\}_v . \tag{6}$$

Assuming that the force residual $\{\varepsilon_{F_j}\}_v$ is a function of the updated parameter vector $\{u\}=\{u_1,u_2,...,u_i,...,u_{N_u}\}^T$, then the linear approximation of $\{\varepsilon_{F_j}\}_v$ is obtained as:

$$\{\varepsilon_{F_j}\}_v=\{\varepsilon_{F_j}(\{u^0\})\}_v+\sum_{i=1}^{N_u}\frac{\partial\{\varepsilon_{F_j}\}_v}{\partial u_i}\Delta u_i , \tag{7}$$

where $\{u^0\}$ is the initial value of $\{u\}$, Δu_i is the change of the joint parameters u_i, N_u is the number of updated parameters.

Substituting Eq. (7) into Eq. (6) leads to:

$$\{\varepsilon_{H_j}\}_v=\{\varepsilon_{H_j}(\{u^0\})\}_v+\sum_{i=1}^{N_u}\left[H^a(\omega_j)\right]\frac{\partial\{\varepsilon_{F_j}\}_v}{\partial u_i}\Delta u_i \tag{8}$$

The next step is to solve the equation $\{\varepsilon_{H_j}\}_v=0$, i.e.:

$$\{\varepsilon_{H_j}(\{u^0\})\}_v+\sum_{i=1}^{N_u}\left[H^a(\omega_j)\right]\frac{\partial\{\varepsilon_{F_j}\}_v}{\partial u_i}\Delta u_i=0 . \tag{9}$$

Eq. (9) can be reduced to the form of $[S]\{\Delta u\}=\{\varepsilon\}$, where the sensitivity matrix $[S]$, the vector of joint parameters or updated parameters $\{\Delta u\}$ and the residual FRF $\{\varepsilon\}$ are:

$$[S]=\left[H^a\left(\omega_j\right)\right]\left[\frac{\partial\left\{\varepsilon_{F_j}\right\}_v}{\partial u_1},\frac{\partial\left\{\varepsilon_{F_j}\right\}_v}{\partial u_2},\cdots,\frac{\partial\left\{\varepsilon_{F_j}\right\}_v}{\partial u_{N_u}}\right]$$

$$\{\Delta u\}=\begin{Bmatrix}\Delta u_1\\ \Delta u_2\\ \vdots\\ \Delta u_{N_u}\end{Bmatrix}$$

$$\{\varepsilon\}=-\left\{\varepsilon_{H_j}\left(\left\{u^0\right\}\right)\right\}_v.$$

2. 2. The proposed model-based joint parameters identification scheme

The basic idea of the proposed model-based joint parameters identification is that the analytical FE model of a mechanical structure can be modified with changes of joint parameters, and these changes lead to an updated FE model, whose predicted FRFs can match well with the measured FRFs.

On the basis of the sensitivity-based iterative algorithm, the proposed joint parameters identification scheme is given in Fig. 1.

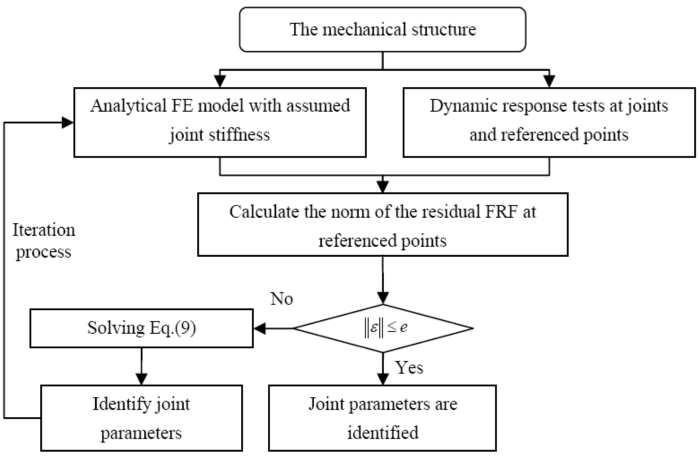


Fig. 1. The flow chart of the proposed scheme

The joint stiffness identification scheme starts from analytical modeling of the mechanical structure. For the complex structural systems, the joint dynamics between each substructure are usually unknown and therefore initial joint parameters are assumed by experience. To identify the joint parameters successfully, the measured FRFs with high accuracy are essential. Dynamic response tests at joints and referenced points are needed. Due to the constraints of experiment conditions, only the FRFs of translational DOFs can be measured directly while FRFs of rotational DOFs are usually estimated as described in [14].

After data is prepared, the analytical FRFs are compared with the measured FRFs at the referenced points. The norm of the residual ($\|\varepsilon\|$) between the analytical and measured FRFs is calculated. As the initial FE model is usually not accurate, the norm of the residual FRFs is larger than the threshold *e*. Then the iterative procedure starts from solving Eq. (9) with the least squares method. After the solution, the vector of joint parameters is obtained. With the solved joint parameters, the FE model of the mechanical structure is updated. Following an eigen-solution of the updated FE model, new analytical FRFs are obtained and the norm of the residual FRFs is refreshed. Then the norm of the residual FRFs is compared with the threshold again. This process of solving and updating the system are repeated until the norm of the residual is smaller than the defined threshold *e*. Then the iterative process stops, and the unknown dynamic characteristics of the joints are identified successfully.

3. Application case

A clamped beam assembly is used to validate the proposed joint parameters identification method. The clamped beam assembly consists of one beam with circular cross section and a vise, as shown in Fig. 2. The FRF tests of the beam were conducted by using a PCB hammer and several accelerometers. The FRFs were measured by applying an impact force on the surface and recording the vibration response at the opposite part. The measurements were made by CutPro-MalTF® with the sampling frequency of 8000 Hz. The material properties of the beam are listed in Table 1.

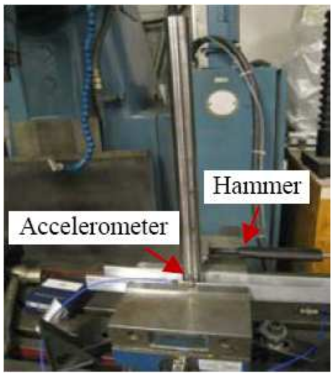


Fig. 2. The clamped beam assembly

Table 1. Material properties of the beam

Length (mm)	Diameter (mm)	Young's Modulus (N/m^2)	Poisson's ratio	Density (kg/m^3)
383	25	2.1×10^{11}	0.3	7800

As the mass of the vise is much larger than the beam, the modeling of the vise is avoided. Timoshenko beam element is used and thus accounted for the effects of shear on the vibration of the beams. The motion of each node is described by three translational ($\delta_x, \delta_y, \delta_z$) and two rotational ($\gamma_y, \gamma_z$) DOFs, as shown in Fig. 3.

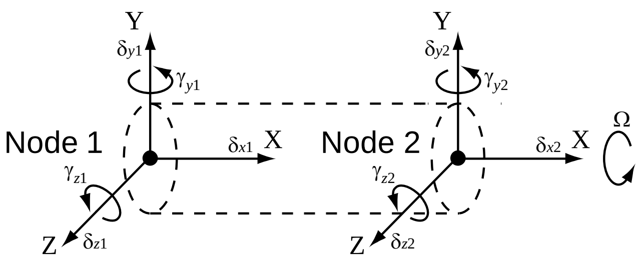


Fig. 3. The Timoshenko beam element

Firstly, as the beam is clamped firmly, the joint between the beam and the vise is considered to be rigid connection in a traditional way. Fig. 4 is the FE model of the clamped beam assembly with rigid connection. The beam is divided into 10 elements in total.

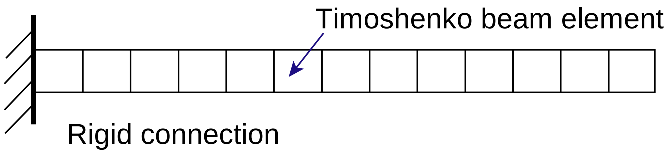


Fig. 4. FE modeling of the clamped beam assembly (rigid connection)

The comparison between the simulated and the experimental FRFs in the free-end are shown in Fig. 5. It can be seen that the simulation deviates from the experiment apparently. The comparison of natural frequencies between the simulation and the experiment is listed in Table 2. The largest relative error of the simulated natural frequencies is 16.7 %, which is unacceptable. From Fig. 5 and Table 2 in the manuscript, it can be seen that all the simulated natural frequencies are larger than the measured values for the first three modes. This means that the actual connection between the beam and the vise is less-than-rigid. Therefore, it is not reasonable to simplify the joint as rigid connection during the FE modeling process.

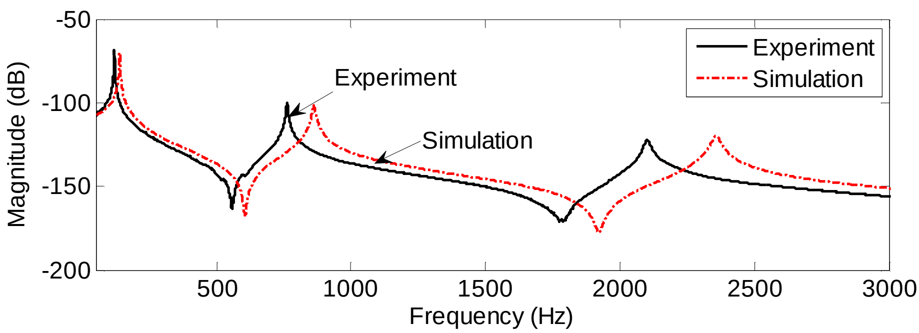


Fig. 5. The comparison of FRFs between the simulation and the experiment (rigid connection)

Table 2. The comparison of natural frequencies (rigid connection)

The order of modes	Experiment (Hz)	Simulation (Hz)	Relative error (%)
1	120	140	16.7
2	762	862	13.1
3	2101	2357	12.2

Then, the joint properties between the beam and the vise are included, as shown in Fig. 6. Joint stiffness is modeled by using two equivalent springs in translational and rotational directions, respectively, and expressed in a matrix form as

$$\mathbf{K}^J = \begin{bmatrix} k_{11}^J & k_{12}^J \\ k_{21}^J & k_{22}^J \end{bmatrix},$$

where the translational stiffness k_{11}^J, the rotational stiffness k_{22}^J and their coupling k_{12}^J or k_{21}^J are included.

The joint damping can be added in the same manner as joint stiffness. However, due to the nonlinear complexity of the damping, the damping matrix is ignored when modeling the system. The modal damping ratios are obtained by experimental modal analysis.

The next step is to identify the joint stiffness of equivalent springs. The proposed joint parameters identification method is applied here. The global stiffness matrix of the clamped beam assembly can be written as

$$\mathbf{K}^x = \mathbf{K}^a + \mathbf{K}^J,$$

where $\mathbf{K}^a$ is the analytical stiffness matrix of the beam.

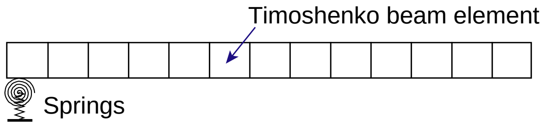


Fig. 6. FE modeling of the clamped beam assembly (flexible connection)

When only DOFs in XY or XZ plane are considered, the analytical stiffness matrix of the beam with N nodes is given as

$$\mathbf{K}^x = \begin{bmatrix} k_{1,1}+k_{11}^J & & S & & \\ k_{2,1}+k_{21}^J & k_{2,2}+k_{22}^J & & Y & \\ \vdots & \vdots & \ddots & & M \\ & & & k_{N-1,N-1} & \\ k_{N,1} & k_{N,2} & \cdots & k_{N,N-1} & k_{N,N} \end{bmatrix}.$$

where the first node of the beam is connected with the vise and the unknown variables are

$$\{\Delta u\} = \begin{Bmatrix} k_{11}^J \\ k_{21}^J \\ k_{22}^J \end{Bmatrix}.$$

The measured FRF $H_{21,21}$ at the free end (i.e., the 10$^{\text{th}}$ node) is chosen as the object FRF of the joint parameter identification algorithm, that is, the residual FRF is

$$\varepsilon_{H_{21,21}}(\omega) = H_{21,21}^a(\omega) - H_{21,21}^x(\omega).$$

Since the stiffness of the rotational DOF (k_{22}^J) is included in the joint stiffness matrix, the measured FRFs of the rotational DOFs are needed. Due to the high cost to obtain the rotational FRFs, we use the experimental translational FRFs to derive the rotational FRFs through a mathematical manipulation as described in [14].

Fig. 7 shows the convergence history of the norm of the residual FRF ($\|\varepsilon\|$). After 30 iteration steps, the trend of the residual FRF becomes stable and acceptable.

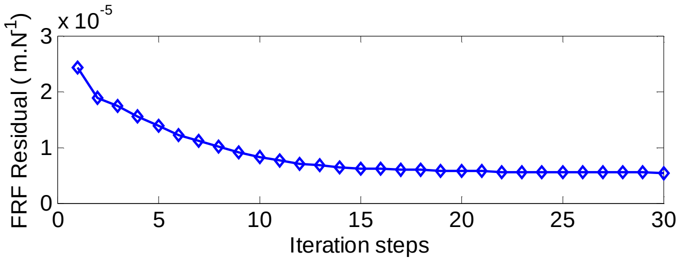
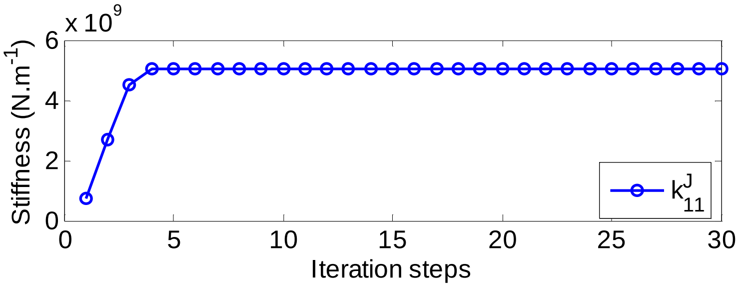
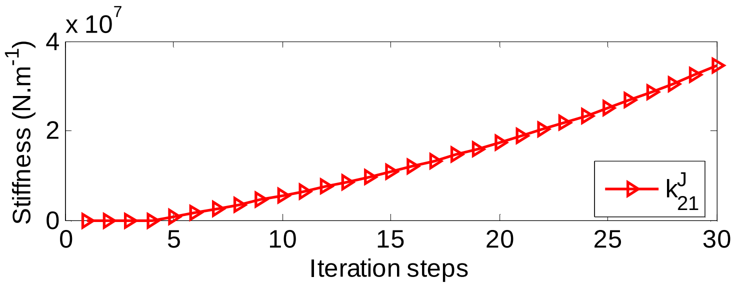


Fig. 7. The convergence history of the FRF residual (flexible connection)

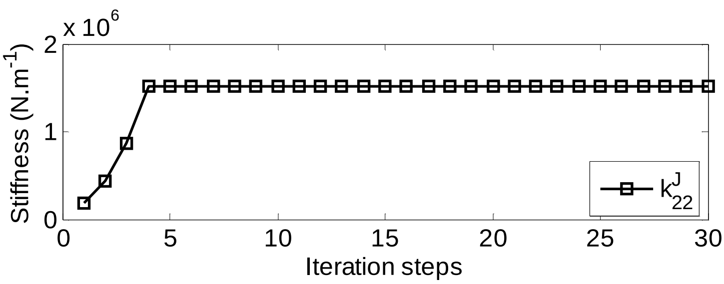
The identifying process of the joint stiffness is displayed in Fig. 8. In five iteration steps, the translational stiffness k_{11}^{J} (Fig. 8a) and the rotational stiffness k_{22}^{J} (Fig. 8c) converges to 5.05×10^{9} N/m and 1.52×10^{6} N/m, respectively. However, the coupling stiffness k_{12}^{J} (Fig. 8b) doesn't converge and the updated stiffness becomes 3.44×10^{7} N/m in 30 iteration steps. The changes of joint stiffness are listed in Table 3.



(a) The translational stiffness k_{11}^{J}



(b) The coupling stiffness k_{12}^{J}



(c) The rotational stiffness k_{22}^{J}

Fig. 8. The identifying process of joint stiffness (flexible connection)

Table 3. The change of joint stiffness

Joint stiffness	Initial stiffness (N·m^{-1})	Identified stiffness (N·m^{-1})
k_{11}^{J}	1.0×10^{8}	5.05×10^{9}
k_{22}^{J}	0	1.52×10^{6}
k_{12}^{J}	0	3.44×10^{7}

The comparison of the FRFs in the free end of the clamped beam assembly is shown in Fig. 9. The initial simulated FRF is obtained with the initial stiffness, which is quite different from the measured FRF. With the identified joint stiffness, the FE model of the clamped beam assembly is updated, and the updated FRF matches with the experimental FRF very well.

The comparison of natural frequencies between the simulation and the experiment is listed in Table 4. The largest relative error of the simulation is 2.5 %, which demonstrates that the identified joint stiffness parameters are reasonable. Therefore, the updated FE model can describe the dynamic properties of the clamped beam assembly reliably.

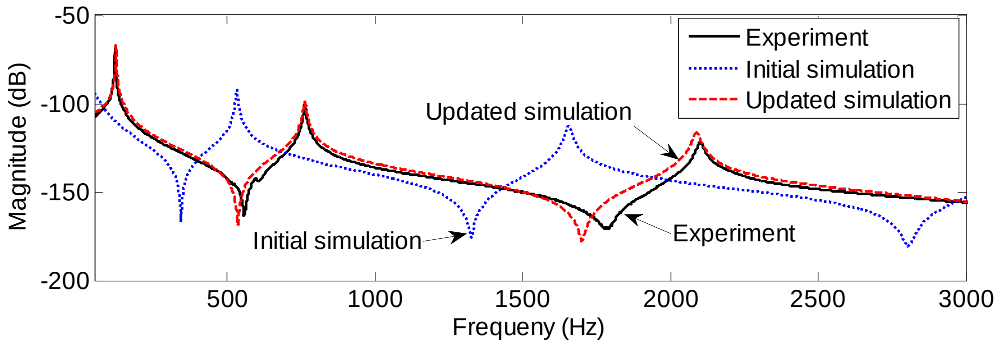


Fig. 9. The comparison of FRFs at the free end of the clamped beam assembly

Table 4. The comparison of natural frequencies (flexible connection)

The order of modes	Experiment (Hz)	Simulation (Hz)	Relative error (%)
1	120	123	2.5
2	762	761	-0.1
3	2101	2088	-0.6

Conclusions

A model-based joint parameters identification method using FRFs has been described. The method takes advantages of both the FE model and the experimental FRFs. Sensitivity-based iterative algorithm is the kernel of the proposed method. The least square method is used to solve the over-determined iteration equation. The dynamic characteristics of the joints are identified through the iteration process, while the FE model is updated simultaneously.

Since the FRF data is used, the difficulties of extracting accurate modal parameters are avoided, and it becomes much easier to form an over-determined equation for the iteration. Therefore, the ill-conditioned problem of the iteration equation is avoided.

The proposed method has been applied to identify the elastic constraints imposed on a beam. The first three natural frequencies calculated by the FE model are compared with the measured values, and the largest relative error of the simulation deceases from 16.7 % to 2.5 % after the joint dynamics are identified reasonably. Meanwhile, the simulated FRFs of the updated FE model match with the experimental FRF very well.

Finally, it should be pointed out that even though the given example is focused on the joint stiffness identification of a simple beam assembly, the proposed method is applicable to other mechanical structures as well. Damping parameters of the joint have not been studied in this paper, which will be addressed in the future work.

Acknowledgements

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