

# 786. Parameter identification of aircraft thin-walled structures using incomplete measurements

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**Abstract.** Early parametric identification is critical for the decision making of repair or replacement in order to guarantee structural safety. Nowadays, aircraft thin-walled structures are widely applied in aero-/astronautics areas and their health conditions receive considerable attention. Parameter identification in aircraft thin-walled structures is more challenging because of the structural complexity. In this research, a new time-domain analysis method, the sequential nonlinear least square estimation (SNLSE) method, along with model reduction technique is proposed to identify the parameters of aircraft thin-walled structures using vibration data, which is referred to as the reduced order model based SNLSE approach. Herein, model reduction technique is used to reduce the number of degrees of freedom for conducive to the placement of sensors and high-efficiency calculation by SNLSE method. Simulation and experimental studies have been conducted for the parameter identification of the aluminum thin-walled structure. As demonstrated by simulation and experimental results, the proposed approach using incomplete measurements is very effective in parameter identification of aircraft thin-walled structures.

**Keywords:** structural health monitoring, parameter identification, sequential nonlinear least square estimation, aircraft thin-walled structure, model reduction.

## 1. Introduction

The structural health monitoring (SHM) methodology offers the possibility to assess the integrity of a structure without using visual inspections. This is of great advantage especially in areas where the accessibility of structures is not provided, e.g. aero- and astronautics applications [1, 2]. One important problem in SHM is parameter identification leading to the detection of damages [3, 4]. This problem is more challenging for the aircraft thin-walled structures, which have been widely used in aero- and astronautics areas as key components, because of their complex non-linear mechanical properties [5]. Herein, aircraft thin-walled structures have been identified based on an incomplete measurement approach.

For the on-line or nearly on-line identification of structural parameters based on vibration data, various time-domain analysis approaches have been proposed in the literature [6, 7]. In particular, the methods of least square estimation (LSE) [8, 9] and the extended Kalman filter (EKF) [10, 11] can be used to identify constant system parameters, without the requirement for accurate modal parameters. However, for practical applications of LSE, acceleration responses are measured on-line, and velocity responses and displacement responses are usually obtained through a single numerical integration and a double numerical integration from the acceleration data respectively, which can cause a significant numerical drift that is also magnified seriously when a damage occurs, and it is difficult to remove the drift on-line. Furthermore, due to the linearization of the state equation, resulting in the fact that some identified parameters may easily lie on the imaginary axis, the EKF solution may become unstable. Besides, the EKF solution may not converge if the initial guesses of the parametric values are outside the region of convergence. In order to eliminate these drawbacks, a new approach, referred to as the sequential nonlinear least square estimation (SNLSE) approach, has been proposed recently [12]. In SNLSE approach, the unknown parameter vector and the unknown state vector are estimated

sequentially in two steps. Herein, SNLSE approach is used to identify the stiffness parameters of aircraft thin-walled structures.

Recently, aircraft thin-walled structural conditions have received considerable attention [13]. The need to identify the physical properties of aircraft thin-walled structures given its force-response relationship is driven primarily by the approximate solution models, such as finite element models. Further, the selected finite element models should be able to represent the real structures as accurately as possible [14, 15]. Generally, a finite element model involves a large number of degrees of freedom (DOFs) and requires a large number of sensor measurements, especially in some complex structures. However, due to practical limitations, it may not be possible to install enough sensors in SHM system to measure all the responses at all the DOFs. Furthermore, some locations are hard to install sensors, and some vibration responses are difficult to measure, such as the rotational acceleration at a nodal point. Consequently, it is highly desirable to develop an incomplete measurement technique to reduce the number of sensors required in SHM system and to avoid some difficult measures. Herein, the static condensation technique is used to reduce the dimension of finite element models leading to the parameter identification based on incomplete measurements [16, 17].

In this research, a new time-domain analysis method, the sequential nonlinear least square estimation method, along with model reduction technique is proposed to identify the stiffness parameters of aircraft thin-walled structures using vibration data, which is referred to as the reduced order model based SNLSE approach. Firstly, model reduction technique is used to reduce the number of DOFs for conducive to the placement of sensors. Then, parameters are identified based on SNLSE method in the reduced model for high-efficiency calculation. Simulation and experimental studies have been conducted for parameter identification of an aluminum thin-walled structure. Simulation and experimental results demonstrate that the identified stiffness parameters are consistent with the finite element method (FEM) values, which confirms that the proposed approach using incomplete measurements is very effective in parameter identification of aircraft thin-walled structures.

## 2. SNLSE approach based on model reduction

In this section, a theoretical derivation of SNLSE approach based on model reduction is presented. The equation of motion of a  $s$ -DOF non-linear structure can be expresses as

$$\underline{\mathbf{M}}\ddot{\underline{\mathbf{x}}}(t) + \underline{\mathbf{F}}_c[\dot{\underline{\mathbf{x}}}(t)] + \underline{\mathbf{F}}_s[\underline{\mathbf{x}}(t)] = \underline{\boldsymbol{\eta}}\mathbf{f}(t) \quad (1)$$

in which  $\underline{\mathbf{x}}(t) = [x_1(t), x_2(t), \dots, x_s(t)]^T = s$ -displacement vector;  $\underline{\mathbf{M}} = (s \times s)$  mass matrix;  $\underline{\mathbf{F}}_c[\dot{\underline{\mathbf{x}}}(t)] = s$ -damping force vector;  $\underline{\mathbf{F}}_s[\underline{\mathbf{x}}(t)] = s$ -stiffness force vector;  $\mathbf{f}(t) =$  excitation vector;  $\underline{\boldsymbol{\eta}} =$  excitation influence matrix. In what follows, the bold face letter represents either a vector or a matrix.

The acceleration vector  $\ddot{\underline{\mathbf{x}}}(t) = [\ddot{x}_1(t), \ddot{x}_2(t), \dots, \ddot{x}_s(t)]^T$  in Eq. (1) is divided into two vectors, denoted by  $\ddot{\underline{\mathbf{x}}}(t) = [\ddot{x}_1, \ddot{x}_2, \dots, \ddot{x}_m]^T$  and  $\ddot{\underline{\mathbf{x}}}^*(t) = [\ddot{x}_1^*, \ddot{x}_2^*, \dots, \ddot{x}_{s-m}^*]^T$ , in which  $\ddot{\underline{\mathbf{x}}}(t)$  ( $i = 1, 2, \dots, m$ ) and  $\ddot{\underline{\mathbf{x}}}^*(t)$  ( $i = 1, 2, \dots, m-s$ ) are known (measured) and unknown (unmeasured) acceleration responses, respectively. Herein,  $\ddot{\underline{\mathbf{x}}}(t)$  considered as main DOFs could be measured by fixed sensors, and  $\ddot{\underline{\mathbf{x}}}^*(t)$  regarded as secondary DOFs is reduced using the static condensation technique [16, 17]. Then, the reduced order equation of motion can be expresses as

$$\underline{\mathbf{M}}\ddot{\underline{\mathbf{x}}}(t) + \underline{\mathbf{F}}_c[\dot{\underline{\mathbf{x}}}(t)] + \underline{\mathbf{F}}_s[\underline{\mathbf{x}}(t)] = \underline{\boldsymbol{\eta}}\mathbf{f}(t) \quad (2)$$

in which  $M = (m \times m)$  mass matrix;  $F_c[\dot{x}(t)] = m$ -damping force vector;  $F_s[x(t)] = m$ -stiffness force vector. The acceleration responses  $\ddot{x}(t)$  and the excitation forces  $f(t)$  are measured, and the unknowns to be identified are the state vector  $X = [x^T, \dot{x}^T]^T$ , including displacement and velocity vectors, and the parametric vector  $\theta = [\theta_1, \theta_2, \dots, \theta_n]^T$ , involving  $n$  unknown parameters of the structure, such as stiffness, damping, and non-linear parameters.

The observation associated with the equation of motion, Eq. (2), can be written as

$$\varphi[X; t]\theta + e(t) = y(t) \tag{3}$$

in which  $y(t) = \eta f(t) - M\ddot{x}(t)$  is known and  $e(t)$  is the model noise. Eq. (3) can be discretized at  $t = t_k = k\Delta t$  as

$$\varphi_k(X_k)\theta_k + e_k = y_k \tag{4}$$

Instead of solving  $X_k$  and  $\theta_k$  simultaneously by forming an extended state vector as in the EKF approach, the SNLSE approach will solve  $X_k$  and  $\theta_k$  in two steps. The first step is to determine  $\theta_k$  by assuming that  $X_k$  is given using the LSE solution. The second step is to determine  $X_k$  through a non-linear LSE approach, referred to as the SNLSE, as follows.

Step I: Suppose the state vector  $X_k$  is known and the parametric vector  $\theta_k$  is constant, i.e.,  $\theta = \theta_1 = \theta_2 = \dots = \theta_k$ . Minimizing the objective sum-square errors

$$J(\theta) = \sum_{i=1}^{k+1} [y_i - \varphi_i(X_i)\theta_i]^T [y_i - \varphi_i(X_i)\theta_i] \tag{5}$$

The classical LSE recursive solution  $\hat{\theta}_{k+1}$  that is the estimate of  $\theta_{k+1}$  can be obtained as

$$\hat{\theta}_{k+1} = \hat{\theta}_k + K_{k+1}(X_{k+1})[y_{k+1} - \varphi_{k+1}(X_{k+1})\hat{\theta}_k] \tag{6}$$

$$K_{k+1}(X_{k+1}) = P_k \varphi_{k+1}^T(X_{k+1}) [I + \varphi_{k+1}(X_{k+1}) P_k \varphi_{k+1}^T(X_{k+1})]^{-1} \tag{7}$$

$$P_k = P_{k-1} - K_k(X_k) \varphi_k(X_k) P_{k-1} \tag{8}$$

in which  $K_{k+1}(X_{k+1})$  is the LSE gain matrix.

Step II: The recursive solution for  $\hat{X}_{k+1|k+1}$ , that is, the estimation of  $X_{k+1}$  can be obtained as

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + \bar{K}_{k+1} [y_{k+1} - \hat{y}_{k+1}(\hat{X}_{k+1|k})] \tag{9}$$

$$\hat{X}_{k+1|k} = \Phi_{k+1,k} \hat{X}_{k|k} + B_1 \ddot{x}_k + B_2 \ddot{x}_{k+1} \tag{10}$$

$$\bar{K}_{k+1} = \bar{P}_{k+1|k} \Psi_{k+1,k}^T [I + \Psi_{k+1,k+1} \bar{P}_{k+1|k} \Psi_{k+1,k+1}^T]^{-1} \tag{11}$$

$$\bar{P}_{k+1|k} = \Phi_{k+1,k} \bar{P}_{k|k} \Phi_{k+1,k}^T \tag{12}$$

$$\bar{P}_{k|k} = \bar{P}_{k|k-1} - \bar{K}_k \Psi_{k,k} \bar{P}_{k|k-1} \tag{13}$$

where

$$\hat{\mathbf{y}}_{k+1}[\hat{\mathbf{X}}_{k+1|k}] = \boldsymbol{\varphi}_{k+1}(\hat{\mathbf{X}}_{k+1|k})\hat{\boldsymbol{\theta}}_{k+1}(\hat{\mathbf{X}}_{k+1|k}) \quad (14)$$

$$\boldsymbol{\Phi}_{k+1,k} = \begin{bmatrix} \mathbf{I} & (\Delta t)\mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad \boldsymbol{\Psi}_{k+1,k+1} = \left. \frac{\partial \hat{\mathbf{y}}_{k+1}(\mathbf{X}_{k+1})}{\partial \mathbf{X}_{k+1}} \right|_{\mathbf{X}_{k+1} = \mathbf{X}_{k+1}(\hat{\mathbf{X}}_{k+1|k})} \quad (15)$$

$$\mathbf{B}_1 = \begin{bmatrix} (0.5 - \tilde{\beta})(\Delta t)^2 \mathbf{I} \\ (1 - \tilde{\gamma})(\Delta t) \mathbf{I} \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} \tilde{\beta}(\Delta t)^2 \mathbf{I} \\ \tilde{\gamma}(\Delta t) \mathbf{I} \end{bmatrix} \quad (16)$$

in which  $\tilde{\beta}, \tilde{\gamma}$  are parameters used in Newmark- $\beta$  method (usually  $\tilde{\beta} = 0.25$ ,  $\tilde{\gamma} = 0.5$  are used).  $\mathbf{I}$  is the unit matrix. Thus, the estimate  $\hat{\mathbf{X}}_{k+1|k+1}$  obtained from Eq. (9) to (13) will be used to replace  $\mathbf{X}_{k+1}$  in Eq. (6) to (8) for computing the unknown parametric vector  $\hat{\boldsymbol{\theta}}_{k+1}$ . The technique proposed above is referred to as the SNLSE approach based on model reduction.

### 3. Simulation studies

To demonstrate the accuracy and effectiveness of the proposed approach based on incomplete measurements in aircraft thin-walled structural parameter identification, a simulation research is conducted. Firstly, a thin-walled structural model will be built for simulation studies and acquire the finite element results as reference values. Then, structural parameters will be identified in the numerical model.

#### 3.1 Aircraft thin-walled structural model

A thin-walled structural model in Fig. 1 is built. This finite element model has 12 nodes with 21 members, including 16 beam elements and 5 plate elements. In this model, 2 nodes and 1 beam element are under clamped boundary condition. Consequently, there are 10 nodes needing to be considered in the finite element system where each node has 6 DOFs (3 translations and 3 rotations) leading to a total of 60 DOFs. Further, there are 15 equivalent stiffness parameters of beam elements needing to be identified. For acquiring the identified parametric FEM reference values of the real aircraft thin-walled structural model in laboratory, herein, a modal experiment is conducted and finite element updating using modal parameters is achieved to obtain the equivalent stiffness reference values of beam elements [18-20].

As shown in Fig. 1, 60-DOF original finite element model has lots of rotational DOFs and some locations are not suitable for sensor installation in practical applications. Therefore, a reduced order finite element model is generated based on model reduction to avoid the above measurements. If a  $Z$  axis excitation is input, as known through the mechanical properties of aircraft thin-walled structures and a traditional finite element analysis, force and acceleration responses in  $X$ ,  $Y$  and  $RZ$  axis are small enough to be neglected and a 30-DOF finite element model is equivalent. To avoid the need for measuring the rotational accelerations at nodal points, the  $RX$  and  $RY$  axis DOFs could be reduced leading to a 10-DOF finite element model. Finally, the equivalent stiffness of beam elements in aircraft thin-walled structures will be estimated based on the reduced order system.

#### 3.2 Simulation results

In this simulation example, consider that the thin-walled structural model in Fig. 1 is subject to a  $Z$  axis sinusoidal excitation applied horizontally at node 0 at frequency of 30 Hz. The measured responses include dynamic excitation and the horizontal accelerations at nodes 1, 2, ..., 5 & 7, 8, ..., 11. All the measured quantities are simulated by superimposing the theoretically

computed quantities with the responding stationary white noise with a 2 % noise-to-signal ratio. In this case, the root mean square (RMS) of a particular response signal is computed from the temporal average over 4 s. Then, a stationary white-noise process is generated using 2 % RMS. This noise process is superimposed to the corresponding acceleration response that is computed theoretically. The sampling frequency is 1000 Hz for all measured data. Then, parameter identification is conducted in the reduced order model proposed above using incomplete measurements.

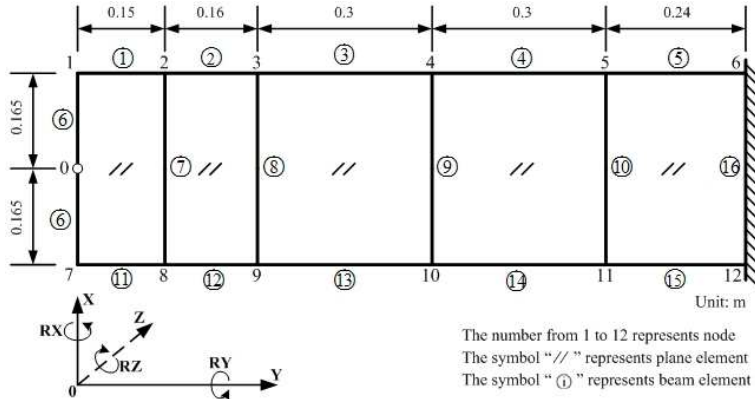


Fig. 1. Aircraft thin-walled structural model

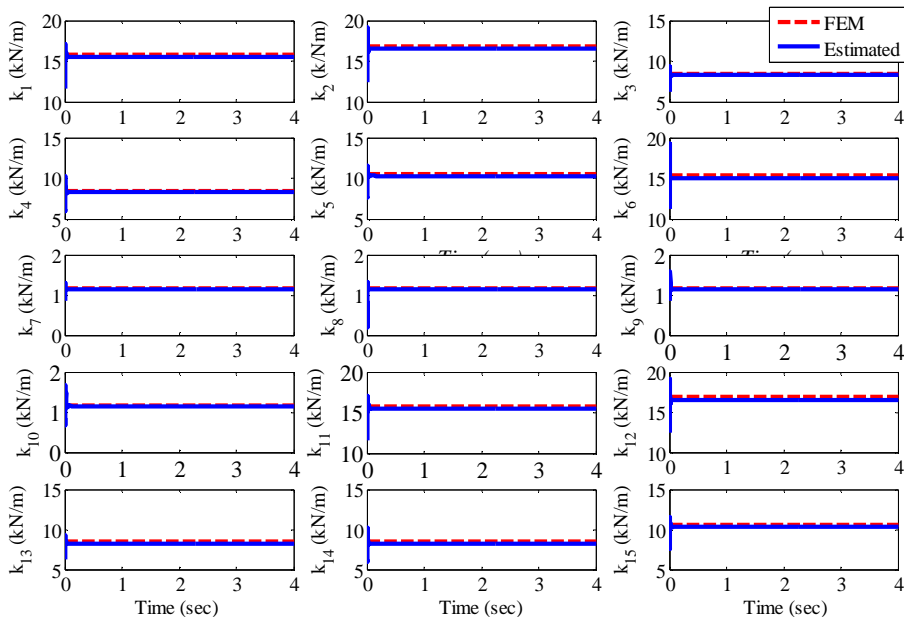


Fig. 2. Identified stiffness parameters in simulation

For the SNLSE approach based on model reduction described previously, the following initial values were assumed: (i) the identified parametric initial values  $k_1 = k_2 = \dots = k_{20} = 15\text{ kN/m}$ ; (ii) the initial values for the displacements and velocities are zero, i.e.,  $\mathbf{x} = 0, \dot{\mathbf{x}} = 0$ ; (iii) the initial gain matrix  $\mathbf{P}_0$  for the estimation of the parametric vector

and the initial gain matrix  $\bar{P}_{00}$  for the estimation of the state vector, are set to be  $P_0 = 10^6 I$  and  $\bar{P}_{00} = I$ , respectively. The simulation results are presented in Fig. 2 and Table 1.

**Table 1.** Identified stiffness parameters in simulation

Stiffness	$k_1$	$k_2$	$k_3$	$k_4$	$k_5$	$k_6$	$k_7$	$k_8$	
Ref. values	15.82	16.87	8.44	8.43	10.54	15.34	1.16	1.15	
Predicted	15.48	16.52	8.28	8.27	10.30	14.96	1.14	1.13	
Difference (%)	2.15	2.07	1.90	1.90	2.28	2.48	1.72	1.74	
Stiffness	$k_9$	$k_{10}$	$k_{11}$	$k_{12}$	$k_{13}$	$k_{14}$	$k_{15}$		Unit: kN/m
Ref. values	1.22	1.16	15.82	16.87	8.43	8.44	10.54		
Predicted	1.24	1.14	15.44	16.42	8.26	8.26	10.31		
Difference (%)	1.64	1.72	2.40	2.67	2.02	2.13	2.18		

Based on the parameter identification approach proposed in this paper and the incomplete measured data, the identified unknown parameters for the equivalent stiffness of beam elements in the aircraft thin-walled structural model above are presented in Fig. 2 as solid curves. Also, as shown in Fig. 2, the dashed curves for comparison are the results based on FEM. As shown in the Fig. 2 and Table 1 clearly, the difference between the estimated and FEM values is very small (less than 2.5 % under white noise), leading to the fact that the proposed approach is quite effective for identifying thin-walled structural parameters in theory.

#### 4. Experiment studies

Experimental studies are conducted in order to prove the effectiveness of the approach proposed for parameter identification in aircraft thin-walled structures. A dynamic experimental system is established, and some tests are carried out. Next, structural parameters will be identified on-line using the real-time test data.

##### 4.1 Experimental set-up

An aircraft thin-walled structural model, consisting of thin plates with strengthening stiffeners, as shown in Fig. 3, is used for the experiment. The total height of this model is 1.15 m, and the total width of this model is 0.33 m. The model is made of aluminum with the material properties as follow: Young's modulus  $E = 70 \times 10^9 \text{ Pa}$ , Poisson's ratio  $\mu = 0.3$  and density  $\rho = 2700 \text{ kg/m}^3$ . A white noise excitation force is applied to the top of thin-walled structural midpoint (corresponds to the location of origin - 0 from Z axis in Fig. 3) horizontally using an exciter equipped with a force sensor (PCB2008C03). Each boundary node is installed with one acceleration sensor (PCB ICP 333B32) to measure the vibration data (corresponds to the location of nodes 1, 2, ..., 5 & 7, 8, ..., 11 from Z axis in Fig. 1). NI data acquisition system was used to achieve excitation and acquisition of the signal. The sampling frequencies of acceleration responses and the white noise excitation are 1000 Hz. Experimental setup is shown in Fig. 3.

##### 4.2 Experimental results

For the identification technique based on incomplete measurements in the aircraft thin-walled structure described previously, the following initial values were assumed:  $k_1 = k_2 = \dots = k_{20} = 15 \text{ kN/m}$ ,  $\mathbf{x} = 0$ ,  $\dot{\mathbf{x}} = 0$ ,  $P_0 = 10^4 I$  and  $\bar{P}_{00} = I$ . The equivalent stiffness of the beam elements is identified for which is presented in Fig. 4 as solid curves. Also,

the dashed curves are the FEM values for comparison. The difference between the estimated and FEM values has been expected due to the testing noise, including height condensation. It is observed from Fig. 4 and Table 2 that the identified values match the FEM ones well, the difference being less than 7 % to meet the needs of practical application. Besides, the convergence speed is very fast for a real-time SHM. Thus, the parameter identification approach based on incomplete measurements is able to describe the physical properties of aircraft thin-walled structures, leading to the detection of structural damages in practice.

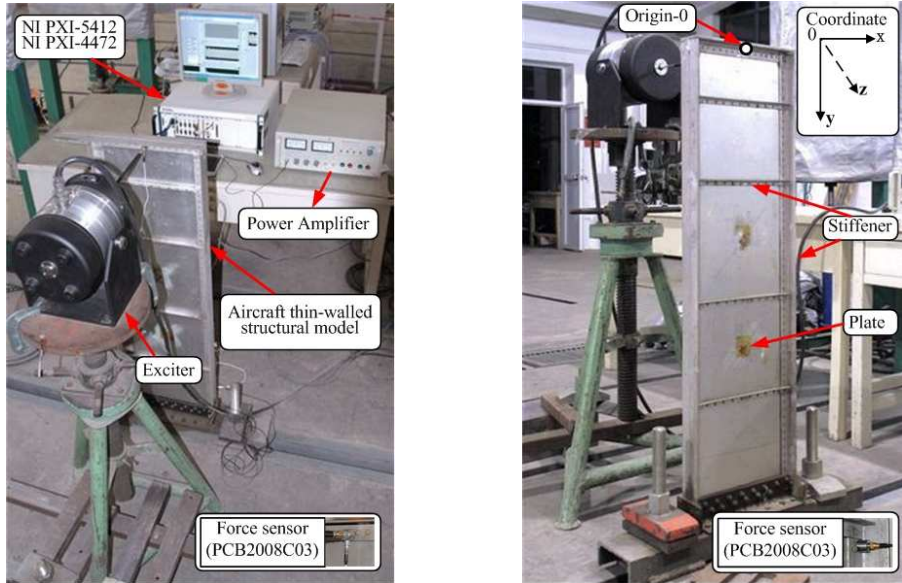


Fig. 3. Experimental setup

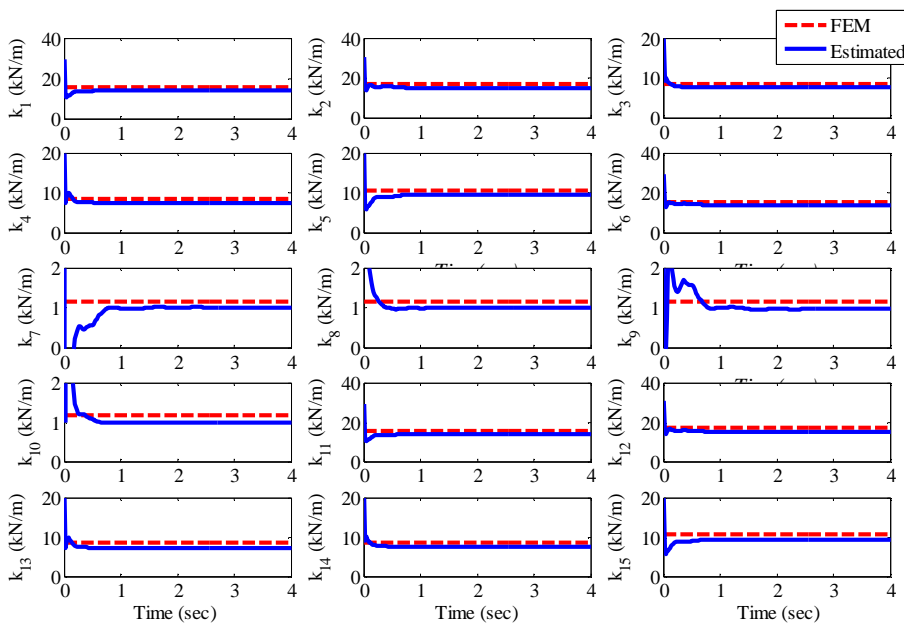


Fig. 4. Identified stiffness parameters in experiment

**Table 2.** Identified stiffness parameters in experiment

Stiffness	$k_1$	$k_2$	$k_3$	$k_4$	$k_5$	$k_6$	$k_7$	$k_8$	
Ref. values	15.82	16.87	8.44	8.43	10.54	15.34	1.16	1.15	
Predicted	15.39	16.33	8.19	8.15	10.18	14.89	1.10	1.09	
Difference (%)	2.72	3.20	2.96	3.32	3.42	2.93	5.17	5.22	
Stiffness	$k_9$	$k_{10}$	$k_{11}$	$k_{12}$	$k_{13}$	$k_{14}$	$k_{15}$		Unit: kN/m
Ref. values	1.22	1.16	15.82	16.87	8.43	8.44	10.54		
Predicted	1.14	1.09	15.39	16.44	8.13	8.21	10.00		
Difference (%)	6.56	6.03	2.72	2.55	3.56	2.73	5.12		

## 5. Conclusions

In this paper, a new time-domain analysis technique, the sequential nonlinear least-square estimation method, along with model reduction technique is proposed to identify the parameters of aircraft thin-walled structures using vibration data, which is referred to as the reduced order model based SNLSE approach. This technique could diminish the number of sensors and achieve efficient on-lone parameter identification. Simulation and experimental studies have been conducted for the equivalent stiffness identification of the aluminum thin-walled structure. As indicated by simulation and experimental results, the identified stiffness parameters based on the proposed approach correlate reasonably well with those obtained by FEM. Consequently, it is demonstrated that the proposed approach using incomplete measurements is very effective in parameter identification of aircraft thin-walled structures.

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