778. Kelvin Voigt's model of single piezoelectric plate

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Abstract. Assemblies and subassemblies based on a piezoelectric effect, in construction and operation of machines, have been used for decades. Among important examples of the piezoelectric elements can be distinguish non-destructive testing methods of a various objects, such as an ultrasonic diagnostic technique, in which a piezoelectric film is glued on a surface of a tested object. In this article an impact of these phenomena on the dynamic characteristics of a modeled system will be examined. It is a base for a study of complex piezoelectric plates by other methods. Moreover, a damping of the systems in a single piezoelectric plate will be considered.

Keywords: analysis, piezoelectric, damping, characteristic.

Introduction

Modelling and testing of a simple and a complex piezoelectric systems is related to the multitude of phenomena occurring in them, and because of that, it is difficult and demanding to carry out a complex mathematical calculations. However, their correct implementation is a key element of the design process of an object under examination in which the piezoelectric transducer is used as a sensor or as an actuator. To develop functional system comprising piezoelectric transducers, it is required a selection of physical and geometrical parameters of the individual plate. It is also extremely important for the effect of dumping on the course (flow) of the system characteristics and impact of the implemented electric circuits. The primary objective of the undertaken work is to formulate and formalize the method of analysis of the piezoelements taking into account the internal damping of a piezoelectric material and the damping resulting from applied external electrical circuits.

Literature review

More and more, a development in a field of new technologies based on the phenomenon of piezoelectricity becomes clear. The piezoelectric phenomenon was widely described in many positions in the literature. A lot of practical applications in a various technical means were presented in [1, 2]. Piezoelectric can be also used for measurement of displacement in composites materials as [3]. In the national and foreign studies, in which the authors raised the issue of stability and vibration damping of mechatronic systems, containing piezoelectric transducers, most commonly used classical, continuous mathematical model of the form of individual plates or piezoelectric-beam systems [4]. In such, rich literature in the field of vibration analysis was not dealt with the application of the piezoelectric plate itself with damping.

Physics of piezoelectric phenomenon

Under consideration is a vibrating piezoelectric plate with parameters distributed in a continuous way. The model has a section A, a thickness d and is made of a uniform material with a density ρ. It was assumed, that one of the size - the thickness is much smaller than the other two. The model is shown in a Fig. 1.

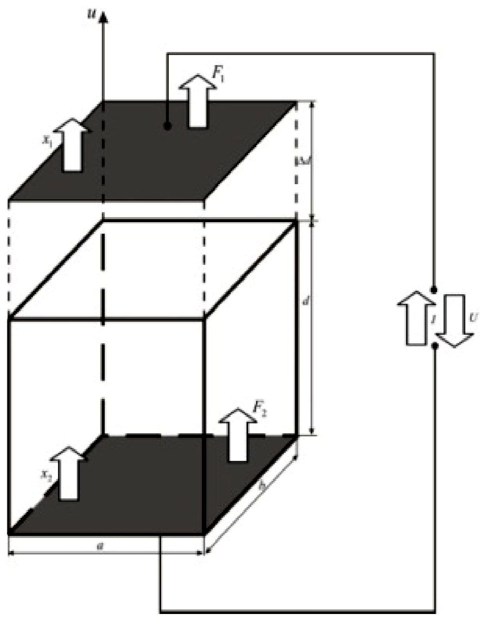

Fig. 1. Continuous and limited piezoelectric model

The paper also accepted that the investigated object will be the vibrating, piezoelectric plate, changing only its thickness – the one-dimensional system. The piezoelectric plate constitutive equation is as follows [5]:

$$\begin{cases} \sigma = E\dfrac{\partial u}{\partial x} - \varepsilon E_p, \\ D = \varepsilon^S E_p + \varepsilon \dfrac{\partial u}{\partial x}, \end{cases} \tag{1}$$

where: E - the modulus of elasticity, E_p - the value of electric field intensity, ε^S - the electric permeability, D - a dielectric induction.

Applying the boundary conditions:

$$\begin{cases} u = u_1, & \text{when} \quad x = x_1, \\ u = u_2, & \text{when} \quad x = x_2, \\ \sigma = -\sigma_1, & \text{when} \quad x = x_1, \\ \sigma = -\sigma_2, & \text{when} \quad x = x_2, \end{cases} \tag{2}$$

to Eq. (1), and after calculations presented in the previous publications of the author [6, 7] it was obtained:

$$F_1 = Ack\left[\frac{x_1}{tg(kd)} - \frac{x_2}{\sin(kd)}\right] + \frac{e}{\varepsilon^S}D, \tag{3}$$

$$-F_2 = -Ack\left[\frac{x_1}{\sin kd} - \frac{x_2 \cos kd}{\sin kd}\right] - \frac{e}{\varepsilon^S}D, \tag{4}$$

$$U = \frac{h}{\omega}(x_1 - x_2) + \frac{1}{\omega C_0} i. \tag{5}$$

Physics of piezoelectric phenomenon taking into account damping

In terms of the energy, deformation of the bodies under the influence of vibrations is a partially irreversible process. As a result of the internal damping part of used energy is dissipated. A very large impact on a level of the internal damping has a material structure, which is deformed. The research subject of this article is the piezoelectric plate with taking into account the internal material damping. It was performed by a description of the rheological properties of the Kelvin-Voigt plastic model. The standard modulus of elasticity from Eq. (1) was replaced by expression:

$$E_C = E\left(1 + \eta_p \frac{\partial}{\partial t}\right), \tag{6}$$

where: E - themodulus of elasticity, E_C - a substituted modulus of elasticity of Kelvin-Voigt's model, η_p - the relaxation time.

The longitudinal modulus of elasticity was changed on module of plastic plate of Kelvin-Voigt's model, by inserting Eq. (6) into Eq. (1). Assuming that the equations transformations is a similar to model without damping presented in previous author's articles, Eq. (7) for the mechanical stress takes the form:

$$\sigma = \left[E_C \left(1 + \eta_p \frac{\partial}{\partial t}\right)\right] \frac{\partial u}{\partial x} + \frac{\varepsilon^2}{\varepsilon^S} \frac{\partial u}{\partial x} - \frac{\varepsilon}{\varepsilon^S} D. \tag{7}$$

The derived equations of system describe the mechanical properties of piezoelectric plate, which vibrate longitudinal, and the material characteristics due to an introduction of the rheological properties description by using theKelvin-Voigt's model:

$$\begin{cases} F_1 = Ak \cdot \left(\left[E\left(1 + \eta_p \frac{\partial}{\partial t}\right)\right] + \frac{\varepsilon^2}{\varepsilon^S} \right) \left[\frac{u_1}{tg[kd]} - \frac{u_2}{\sin[kd]} \right] + \frac{\varepsilon}{\varepsilon^S} D, \\ F_2 = Ak \cdot \left(\left[E\left(1 + \eta_p \frac{\partial}{\partial t}\right)\right] + \frac{\varepsilon^2}{\varepsilon^S} \right) \left[\frac{u_1}{\sin[kd]} - \frac{u_2}{tg[kd]} \right] + \frac{\varepsilon}{\varepsilon^S} D. \end{cases} \tag{8}$$

By adding the electrical relation, the following dependences of system of the piezoelectric plate with internal damping were obtained:

$$\begin{cases} F_1 = Ak \cdot \left(\left[E\left(1 + \eta_p \frac{\partial}{\partial t}\right)\right] + \frac{\varepsilon^2}{\varepsilon^S} \right) \left[\frac{u_1}{tg[kd]} - \frac{u_2}{\sin[kd]} \right] + \frac{hi}{\omega}, \\ F_2 = Ak \cdot \left(\left[E\left(1 + \eta_p \frac{\partial}{\partial t}\right)\right] + \frac{\varepsilon^2}{\varepsilon^S} \right) \left[\frac{u_1}{\sin[kd]} - \frac{u_2}{tg[kd]} \right] + \frac{hi}{\omega}, \\ U = \frac{h}{\omega}(u_2 - u_1) + \frac{1}{\omega C_C} i. \end{cases} \tag{9}$$

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Chart of the simple piezoelectric plate with and without material damping

In this paragraph, three dimensional characteristic of the piezoelectric plate were shown. Fig. 2 contains the influence of a relaxation time on the characteristic. The best representation of damping in investigated model is the relaxation time in range from η_p = 0.0000005 [s] to η_p = 0.0007 [s]. In a scope of future research analysis of results obtained by using two others models – Maxwell's and standard models - which take into account damping, is proposed.

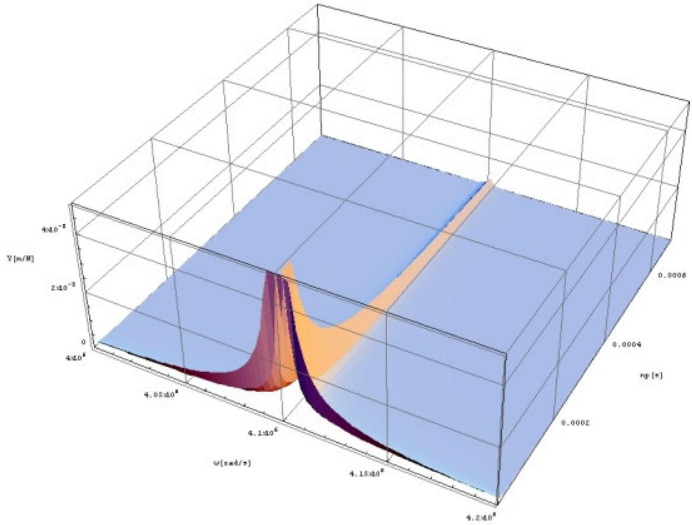

Fig. 2. 3D characteristics of the simple plate with a different relaxation time in a frequency domain

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References

[1] **Buchacz A., Wróbel A.** The investigation of the piezoelectric effect of bimorph system and his practical application in level sensors. The XII Scientific Conference of Vibroacoustics and Vibrotechnique, Jachranka, 2007, p. 63-66.

[2] **Shin H., Ahn H., Han D. Y.** Modeling and analysis of multilayer piezoelectric transformer. Materials Chemistry and Physics, Vol. 92, 2005, p. 616-620.

[3] **Jamroziak K., Bociany M.** Identification of composite materials at high speed deformation with the use of degenerated model. Journal of Achievements in Materials and Manufacturing Engineering, Vol. 28, Issue 2, International OCOSCO World Press, Gliwice, 2008, p. 171-174.

[4] **Buchacz A., Płaczek M.** Damping of mechanical vibrations using piezoelements, including influence of connection layer's properties on the dynamic characteristic. Solid State Phenomena, Vols. 147-149, 2009, Trans Tech Publications, Switzerland, p. 869-875.

[5] **Soluch W.** The Introduction to Piezoelectronics. WKIŁ, Warszawa, 1980.

[6] **Buchacz A., Wróbel A.** Homogenization of mechatronical systems in modeling piezoelectric layer. Machine Dynamics Problems, 2008, Vol. 32, No. 3, p. 25.

[7] **Buchacz A., Wróbel A.** Modeling of complex piezoelectric system by non-classical methods. Journal of Achievements in Materials and Manufacturing Engineering, Volume 35, July 2009, p. 63-70.