# 768. Determining the torsional natural frequency of underframe of off-road vehicle with use of the procedure of operational modal analysis

**Krzysztof Jamroziak[1], Mariusz Kosobudzki[2]**
Faculty of Security Sciences, Military Academy of Land Forces, Poland
**E-mail:** [1]*krzysztof.jamroziak@wso.wroc.pl*, [2]*mariusz.kosobudzki@poczta.onet.pl*

*(Received 10 September 2011; accepted 14 May 2012)*

**Abstract.** The paper presents the example of use of the operational modal analysis procedure (OMA) to set the torsional natural frequency of underframe of off-road vehicle. Determining the natural frequency for the object of this shape is possible with the output-only data and using the peak picking technique.

**Keywords:** underframe, off-road vehicle, torsional natural frequency, operational modal analysis.

## Introduction

The first step in modal analysis is the identification of natural frequencies of output-only system [1, 2]. In the classical approach of experimental modal analysis (EMA) it is necessary to simultaneously measure input and output signals in laboratory conditions, e.g. in the works [3, 4]. But in many practical applications it is difficult or even impossible to measure the input signal. Moreover, artificial excitation is normally conducted to measure the frequency response function (FRF) or impulse response function (IRF), which are typically used as primary data for the extraction of subsequent modal parameter. In this way we can test components rather than complete system and the boundary conditions need to be simulated [5, 6]. These limitations are minimized in OMA. This kind of analysis is cheap and easy to conduct; it doesn't need any sophisticated excitation equipment and boundary conditions simulations. Therefore, EMA is reduced to the measurement of response. The dynamic characteristics of the whole structural systems can be obtained under real operational conditions [7]. Additionally, the characteristics of the system under real loading can be linearized due to broadband random excitations. In OMA, all of the measurement coordinates can be used as references and the identification algorithm used for OMA ought to be multi-input multi-output – type (MIMO).

The technique presented in this paper is the example of classical Basic Frequency Domain approach which is frequently called the Peak Picking Technique. This approach uses the fact that frequencies can be estimated from the spectral densities calculated with the assumption of white noise input.

The relationship between the input (unknown) $x(t)$ and the measured output $y(t)$ can be expressed as [7]:

$$G_{yy}(j\omega) = H(j\omega)^{*} G_{xx}(j\omega) H(j\omega)^{T} \quad (1)$$

where: $G_{xx}(j\omega)$, $G_{yy}(j\omega)$ – are, respectively, the power spectral densities (PSD) of input and output, $H(j\omega)$ – is the Frequency Response Function (FRF), * and $T$ – denote, respectively, conjugate and transpose.

The FRF can be written in pole/residue form [1]:

$$H(j\omega) = \frac{G_{yy}(j\omega)}{G_{xx}(j\omega)} = \sum_{k=1}^{n} \frac{A_k}{\lambda_k^2 - \omega^2} \quad (2)$$

where: $A_k$ – is the modal constant, $\lambda_k$ – is the eigenvalue of the $k^{th}$ mode (it consists of natural frequency and damping factor).

If we suppose that the input signal is white noise (its PSD is constant $C$), the equation (2) can be transformed into:

$$G_{yy}(j\omega) = C\sum_{k=1}^{n} \frac{A_k}{\lambda_k^2 - \omega^2} \tag{3}$$

The estimation of the output PSD $G_{yy}(j\omega)$ known at discrete frequencies $\omega = \omega_i$ is then decomposed by taking the Singular Value Decomposition (SVD) of the matrix [7]:

$$\hat{G}_{yy}(j\omega_i) = U_i^* S_i U_i^T \tag{4}$$

where: $U_i$ – is the unitary matrix holding the singular vectors $u_i$, $S_i$ – is the diagonal matrix holding the scalar values $s_i$.

Near a peak corresponding to the $k^{th}$ mode in the PSD, this mode or possibly the close mode will dominate. So, the singular value $s_i$ is the auto power spectral density function of the corresponding single degree of freedom system. As long as a singular vector is found and has a high Modal Assurance Criteria (MAC) value, it belongs to the single degree of freedom density function. It allows us to obtain the natural frequency.

## Alternative way to extract the torsional natural frequencies

The procedure described above which extracts natural frequencies, mode shapes and damping coefficients requires in practice specially dedicated computer program. In many cases, on the first stage of the analysis it is sufficient to determine only the torsional natural frequency of the system. As an example, the shorten method is presented. The measurements were conducted during road tests of off-road vehicle (Fig. 1).

**Fig. 1.** The vehicle there was tested of-road

The different road surfaces were used: asphalt and cobbled. The output signals generated by piezoelectric accelerometers were recorded with 3 kHz frequency sampling (Fig. 2).

Accelerometers were attached to the frame in four points over the front and rear axle in $Z$-direction with use of the magnetic mat (Fig. 3). This attachment method allowed us to measure the acceleration signals in the range to about 2 kHz [7].

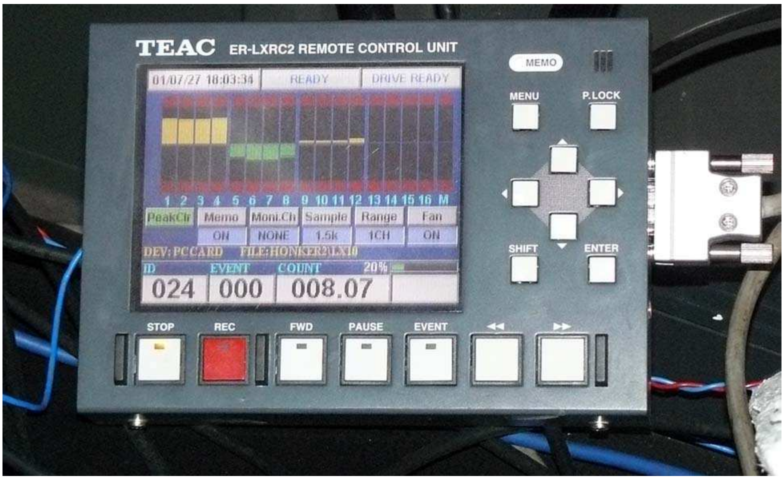


**Fig. 2.** The control panel of the recorder

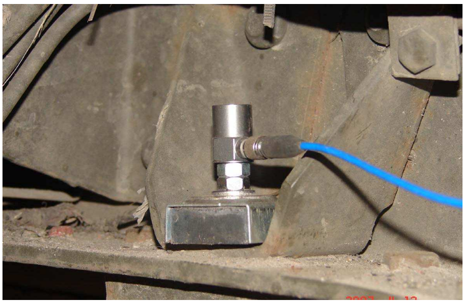

**Fig. 3.** Accelerometer attached with use of magnetic mat

The recorded signals were firstly corrected to the zero-mean value. The spectral analysis was performed using the periodogram method with an overlap of 67 % and Hanning weighting function. This ensures that all signals were equally weighted in the averaging process of minimizing leakage. Using the averaged spectrum for frequency Peak-Picking reduces the possibility of misinterpretation of spectral components [8].

With the use of measured data the signal of vibration $SUM_1$ was calculated according to the formula:

$$SUM_1 = aLP + aLT + aPT + aPP \quad (5)$$

where: *aLP*, *aLT*, *aPT*, *aPP* – are the acceleration signals of front left, rear left, rear right and front right respectively.

The vibration range was intentionally limited to 100 Hz, because upper vibration could be contaminated by the noise and random disturbances. For signal $SUM_1$ there was carried out the Fourier Transform (PSD) using windowing method described above. The results are presented below (Fig. 4).

It is difficult, or even impossible, to point on all the natural vibrations of the underframe based on the plot presented above. The well separated a few-hertz wide pick is visible for low frequency. Nevertheless, this range of frequency and the shape of the curve can be correlated with the input signal rather than with underframe natural frequency. The rest of the plot contains a lot of peaks which are difficult to interpret.

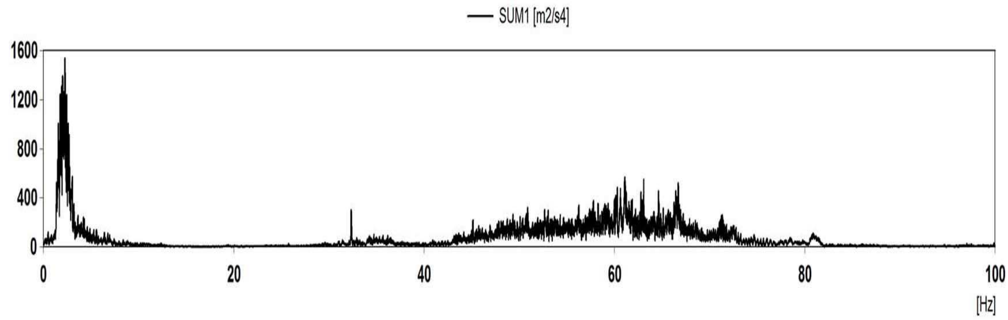


**Fig. 4.** Power Spectral Density of vibration of the underframe within range 0÷100 [Hz]

However, taking into consideration that the biggest influence on total life of underframe have torsional vibrations [9], measured signals were calculated in the different way to extract this kind of vibrations, according to the formula:

$$SUM_2 = aLP - aLT + aPT - aPP \tag{6}$$

For signal $SUM_2$ the Power Spectral Density was calculated (like previously) and presented below (Fig. 5).

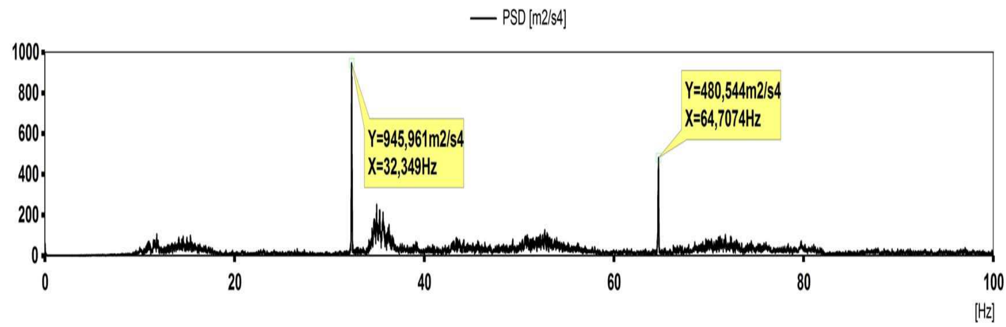


**Fig. 5.** Power Spectral Density of torsional vibration within range 0÷100 [Hz]

This simple calculation allowed to set the plot and to point much easier on the torsional natural frequencies. The low-frequency pick well visible in the Fig. 4 is not present in the Fig. 5. Instead, the two separated picks for frequencies 32,3 and 64,7 Hz are marked. To display shapes of the underframe for these frequency the three dimensional deformation is presented below (Fig. 6). Table 1 presents the coefficients of MAC matrix.

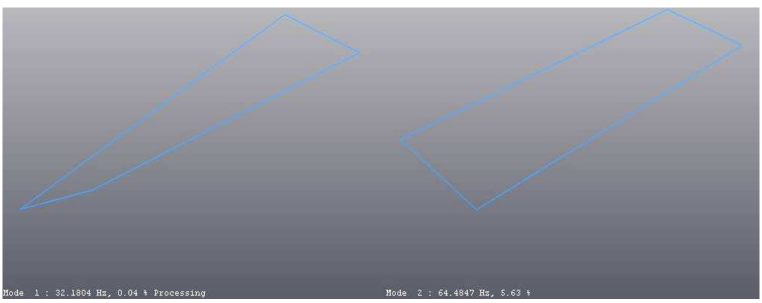


**Fig. 6.** Deformation on the underframe for frequency 32,2 Hz and 64,5 Hz

**Table 1.** Coefficients of MAC matrix

| Frequency | Mode Phase Collinearity | Mode Phase Deviation | Mode Participation |
|---|---|---|---|
| 32,2 Hz | 90,25 % | 24,6 % | 19,47 % |
| 64,5 Hz | 99,77 % | 2,78 % | 80,5 % |

## Conclusions

The paper presents quite easy and simple method of determining the torsional natural frequencies of underframe of off-road vehicle. The way of arrangement and the number of the accelerometers on the frame allows us to draw conclusions about the global, rather than local, torsional deflections. Gained information could be helpful on the first stage of dynamic analysis of the frame and the whole vehicle.

## References

**[1]** **Ewins D. J.** Modal Testing: Theory, Practice and Applications (Second Edition). Research Studies Press Ltd., Baldock, Hertfordshire, England, 2000, p. 562.

**[2]** **Uhl T., Lisowski W.** Eksploatacyjna Analiza Modalna i Jej Zastosowanie. Wydawnictwo Katedry Dynamiki Maszyn i Robotyki, AGH, Kraków, 1999.

**[3]** **Zolkiewski S.** Damped vibrations problem of beams fixed on the rotational disk. International Journal of Bifurcation and Chaos, DOI No: 10.1142/S0218127411030337, Vol. 21, Issue 10, 2011, p. 3033-3041.

**[4]** **Zolkiewski S.** Dynamic flexibility of the supported-clamped beam in transportation. Journal of Vibroengineering, Volume 13, Issue 4, 2011, p. 810-816.

**[5]** **Lingmi Z., Tang W., Yukio T.** A frequency – spatial domain decomposition (FSDD) method for operational modal analysis. Mechanical Systems and Signal Processing, 24, 2010, p. 1227-1239.

**[6]** **Uhl T.** Komputerowo Wspomagana Identyfikacja Modeli Konstrukcji Mechanicznych. WNT, Warszawa, 1997.

**[7]** **Brincker R., Lingmi Z., Andersen P.** Output-Only Modal Analysis by Frequency Domain Decomposition. Proceedings of ISMA, 25, 2000.

**[8]** **Batel M.** Operational modal analysis – another way of doing modal testing. Sound and Vibration, August 2002, p. 22-27.

**[9]** **Rusinski E.** The Principles of Designing the Underframe of Vehicles. Publishing House of Wroclaw University of Technology, Wroclaw, 2002, p. 455.