766. Method of active synthesis of discrete fixed mechanical systems

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Abstract. In this work the method of active synthesis of mechanical systems in accordance with the desired frequency spectrum has been formulated and formalised. Active synthesis of a proportional regulation system has been performed in accordance with the method formulated and a verification of the correctness of the results has been carried out.

Keywords: mechanical impedance, passive synthesis, distribution of the characteristic.

Introduction

The harmful effect of vibration during machine operation, especially in the area of the resonant frequencies, can lead to a loss of stability or damage. To prevent this, it is advantageous to design a machine to work in the desired resonant frequency range. A correct model of the machine allows for proper optimisation of its control, making the application of appropriate control in the light of the adopted criteria possible [4-7]. One of the conditions for the adoption of such a machine model is a synthesis of mechanical systems with the desired dynamic properties [1-3]. The goal of this paper is to formulate and formalise a direct method of active synthesis and to demonstrate its use in combination with various methods of passive synthesis. The paper presents the application of active synthesis in the case of different boundary conditions of structures of obtained systems that meet the same dynamic properties.

Class of Considered Systems

The researched system should meet the desired dynamic properties. On the basis of the adopted dynamic properties in the form of a series of resonant and anti-resonant frequencies and the adopted resonance frequency drop value, also called in literature the stability reserve, the characteristic function is built (1). The resultant characteristic is a real rational function and the difference between the degree of the polynomial in the denominator and numerator should equal 1:

$$U(s)=H\frac{L(s)}{M(s)}=H\frac{\prod_{n=1}^{i}\left(s^2+\left(\omega_{2n-1}+h_{(2n-1)}\right)^2\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)}=H\frac{\prod_{n=1}^{i}\left(s^2+2h_{(2n-1)}s+\left(\omega_{2n-1}^2+2\omega_{2n-1}h_{(2n-1)}+h_{(2n-1)}^2\right)\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)}=$$
$$=H\frac{\prod_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)}+H\frac{\sum_{n=1}^{i}\prod_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)\left(2h_{(2n-1)}s+\left(\omega_{2n-1}^2+2\omega_{2n-1}h_{(2n-1)}+h_{(2n-1)}^2\right)\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)} \quad (1)$$
$$-H\frac{\sum_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)\left(2h_{(2n-1)}s+\left(\omega_{2n-1}^2+2\omega_{2n-1}h_{(2n-1)}+h_{(2n-1)}^2\right)\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)}=U_U(s)+U_F(s),$$

where: H - constant of proportionality, s - Laplace operator, $\omega_1, \omega_3, \ldots, \omega_{2n-1}$ - resonant frequencies, $\omega_0, \omega_2, \ldots, \omega_{2n}$ - anti-resonant frequencies, $h_1, h_3, \ldots, h_{(2n-1)}$ - decline in natural vibration angular frequency, $U_U(s)$ - dynamic characteristic of the desired system, $U_F(s)$ - dynamic characteristic of control force sought. A characteristic function (1) obtained in this way is the mechanical impedance of the sought system. Mechanical impedance as a sum of two functions is subjected to distribution using the methods of passive synthesis and the method of direct determination of the value of the active force.

Passive Synthesis of Mechanical Systems

The presented methods are used for distribution of characteristic functions in the form of impedance (2) describing fixed systems. The dynamic characteristic subjected to synthesis is a rational function resulting from formula (1), namely:

$$U_U(s) = H \frac{\prod_{n=1}^{i}\left(s^2 + \omega_{2n-1}^2\right)}{s\prod_{n=0}^{j}\left(s^2 + \omega_{2n}^2\right)} = H \frac{s^l + d_l s^{l-2} + \ldots + d_1}{s^k + c_k s^{k-2} + \ldots + c_0 s}, \tag{2}$$

where: l – odd degree of the numerator for $l - k = 1$, k – degree of denominator, ω_{2n-1} – resonant frequency, ω_{2n} – anti-resonant frequency, H – any positive real number.

a) Method of characteristic function distribution into a continued fraction

The impedance under consideration (2) is presented as the continued fraction (3):

$$U_U(s) = m_1 s + \cfrac{1}{\cfrac{s}{c_1} + \cfrac{1}{m_2 s + \ldots + \cfrac{1}{\cfrac{s}{c_k} + \cfrac{1}{m_k s}}}}. \tag{3}$$

As a result of the distribution of the characteristic function (2) into a continuous fraction (3), the discrete mechanical system (Fig. 1) and values of elastic and inertial type elements are obtained.

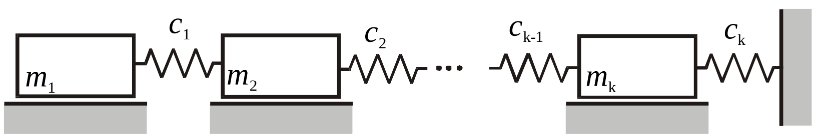


Fig. 1. System obtained through distribution of the characteristic into a continuous fraction

b) Algorithm for determining characteristics of elastic one-ports

Fixed systems described through dynamic characteristics have an anti-resonant value for zero, regardless of the number of restraints imposed on the system. The starting point for the presented method is a characteristic function (2) describing the dynamic properties in the form of resonant and anti-resonant frequencies of the desired system. To designate values of stiffness for the restraint, the coefficient standing closest to the lowest numerator power is to be divided by the coefficient of the lowest denominator power $U_U(s)$ (2) yielding:

$$H\frac{d_0}{c_1 s}=\frac{c}{s}, \tag{4}$$

where: $\frac{c}{s}$ corresponds to a flexible element in the impedance set. The resulting value is the sought stiffness of the restraint ascertained from the dynamic characteristic in the form of impedance (2). This method brings the characteristic function to a form, which permits the use of known methods of synthesis. In this case, the denominator of the impedance in question must be written in the following form:

$$s\left(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c_1\right). \tag{5}$$

The polynomial (5) is multiplied by the value of the ascertained stiffness (4) as follows:

$$H\frac{d_0}{c_1}\left(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c_1\right). \tag{6}$$

Function (6) is subtracted from the numerator of the considered impedance (2) as follows:

$$H\left(d_l s^l+d_{l-1}s^{l-1}+\ldots+d_0\right)-H\frac{d_0}{c_1}\left(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c_1\right), \tag{7}$$

as a result, the following expression is obtained:

$$H\left(d_l s^l+d_{l-1}^1 s^{l-2}+\ldots+d_1^1 s\right). \tag{8}$$

After performing the calculations (4 – 8), the impedance $U_U(s)$ (2) can be written as:

$$U_U(s)=H\frac{d_0}{c_1 s}+H\frac{s(d_l s^{l-1}+d_{l-1}^1 s^{l-2}+\ldots+d_1^1)}{s(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c)_1}=\frac{c}{s}+U_U^{'}(s), \tag{9}$$

where: $U_U'(s)$ - resulting impedance, which is subjected to further synthesis through a method consisting of its distribution into a continued fraction or simple fractions. The numerical value $\frac{d_0}{c_1}$ is the upper limit for the range of numbers greater than zero, from which the sought stiffness is determined. The stiffness depends on the number of restraints on the system. In the present case the author has assumed that the system has one restraint. The number of restraints imposed does not affect the algorithm, but only the value of sought elements. In the case of determining p elements of the $\frac{c}{s}$ type, the considered characteristic in the form of (2) should be multiplied by $\frac{s^{p-1}}{s^{p-1}}$. This procedure is there to obtain additional zeroes within the inverse impedance function, which signify the number *p*-1 of restraints on the synthesised system. In addition to this, figures should be specified for sequentially designated *p* elastic elements, resulting from the synthesis of the impedance function $U_U(s)$ for the interval $\left(0, H\frac{d_0}{c_1}\right\rangle$. When the value of elasticity does not belong to the interval $\left(0, H\frac{d_0}{c_1}\right\rangle$, then the continued use of this method is impossible because the inertial elements obtained in subsequent steps of the synthesis take negative values.

c) Synthesis of fixed systems using the method of proportional distribution

The presented method, as opposed to the ones described above, does not necessitate the value of the anti-resonant frequency to be specified. Dynamic properties to be satisfied by the system, with a cascade structure and fixed on both sides are given in the form of the resonant frequencies only. In case of application of such properties, the dynamic characteristic is built in the form of (2) assuming that every second resonant frequency becomes an anti-resonant frequency and obtaining the appropriate form of impedance:

$$U_U(s) = H\frac{\prod_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)}{s\prod_{n=1}^{j}\left(s^2+\omega_{2n-3}^2\right)} = H\frac{s^l + d_l s^{l-2} + \ldots + d_1}{s^k + c_k s^{k-2} + \ldots + c_0 s}, \tag{10}$$

where: $\omega_{2n-1}, \omega_{2n-3}$ – resonant frequency, H – any positive real number. Such a form of the dynamic characteristic is subjected to synthesis using the method of its distribution into a continuous fraction. The result of the synthesis carried out at this stage is a cascade system as presented in Figure 1. The resulting mechanical system is subjected to modification. The modification consists of proportional distribution of inertial and elastic parameters of the system. This results in a system with two restraints (Fig. 2) that meets the desired requirements in the form of resonant frequencies, which are the zeros and poles of the dynamic characteristic considered (10).

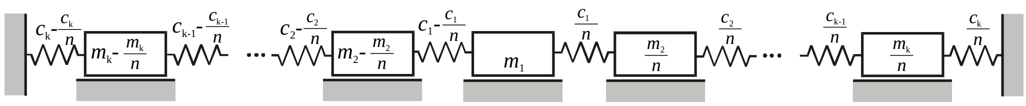


Fig. 2. Discrete system obtained through synthesis using the method of proportional distribution

Method of Determining the Active Force

The starting point for the presented method is the system obtained by applying the method of passive synthesis in the case of impedance $U_U(s)$ and the characteristic $U_F(s)$. On the basis of the system obtained through passive synthesis, the characteristic functions are determined in the form of stiffness, if the active force is exerted on a material point, corresponding to the impedance (2):

$$\frac{F(s)}{X_1(s)} = \frac{L(s)}{M_1(s)}, \frac{F(s)}{X_2(s)} = \frac{L(s)}{M_2(s)}, \ldots, \frac{F(s)}{X_k(s)} = \frac{L(s)}{M_k(s)}. \tag{11}$$

The values of polynomials $M_1(s), M_2(s), \ldots, M_k(s)$ from equation (11) will be used to determine the value of the force in the following way:

$$U_F(s)=\frac{L_1(s)}{M(s)},\ \frac{L_1(s)}{M_1(s)}=a_1s+b_1+\frac{L_2(s)}{M_1(s)},\ \frac{L_2(s)}{M_2(s)}=a_2s+b_2+\frac{L_3(s)}{M_2(s)},\ldots,$$
$$\frac{L_{k-1}(s)}{M_{k-1}(s)}=a_{k-1}s+b_{k-1}+\frac{L_k(s)}{M_{k-1}(s)},\ \frac{L_k(s)}{M_k(s)}=a_ks+b_k, \tag{12}$$
$$F(s)=a_1s+b_1+a_2s+b_2+\ldots+a_ks+b_k.$$

Finally, as the result of the active synthesis performed, that is, a combination of passive synthesis and determining active force, the system is obtained, with a dynamic characteristic consistent with the impedance (2) subject to synthesis.

Numerical Example

In the case of formalised methods for active synthesis, numerical calculations will be performed. The ambiguity of synthesis will be shown on the basis of the presented examples. This ambiguity consists of the fact that different systems are obtained, which meet the same dynamic properties. The conditions imposed on the desired system are presented in the form of a sequence of resonant frequencies, anti-resonant frequencies and the decline in natural vibration frequencies: $\omega_1=15\frac{rad}{s}$, $\omega_3=85\frac{rad}{s}$, $\omega_5=100\frac{rad}{s}$ - resonant frequencies, $\omega_0=0\frac{rad}{s}$, $\omega_2=30\frac{rad}{s}$, $\omega_4=90\frac{rad}{s}$ - anti-resonant frequencies, $h_1=0.75\frac{rad}{s}$, $h_3=1\frac{rad}{s}$ - resonance zone offset value. On the basis of such dynamic properties of the designed systems, the dynamic characteristic is elaborated. The resulting function is the impedance of vibrating fixed systems with three degrees of freedom with active force control. The considered impedance takes the form of a real rational function compatible with the class of the desired mechanical systems (2):

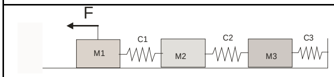
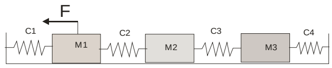
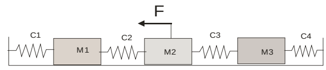
$$U(s)=H\frac{(s^2+15^2)(s^2+85^2)(s^2+100^2)}{s(s^2+30^2)(s^2+90^2)}$$
$$+H\frac{(2\cdot0.75s+2\cdot15\cdot0.75+0.75^2)(s^2+100^2)(s^2+85^2)+(2\cdot1s+2\cdot85\cdot1+1^2)(s^2+100^2)(s^2+15^2)}{s(s^2+30^2)(s^2+90^2)}$$
$$\frac{+(2\cdot0.75s+2\cdot15\cdot0.75+0.75^2)(s^2+100^2)(2\cdot1s+2\cdot85\cdot1+1^2)}{s(s^2+30^2)(s^2+90^2)}=U_U(s)+U_F(s)$$

As a result of the performed active synthesis of the characteristic function in the form of impedance, the structure of the sought system, its inertial and elastic value, as well as the value of the active force are obtained. The results of synthesis using the methods discussed are presented in Table 1.

Conclusions

The work proposes a combination of the passive synthesis method with the method of determining the active force as the method for active synthesis of discrete mechanical systems. The formalised method of active synthesis is a proposal for the selection of active force and a vibrating system as a system meeting the required dynamic properties in the form of a sequence of resonant frequencies, anti-resonant frequencies and the decline in natural vibration frequencies. The document is also an attempt at identifying new opportunities and a direction of research in the design of machine components with a desired frequency spectrum.

Table 1. The results of synthesis

The method of distribution into a continuous fraction	The algorithm for designating elastic one-ports	The method of proportional distribution of parameters
		
$m_1 = 1.00$ kg, $m_2 = 9.9$ kg, $m_3 = 3.5$ kg, $c_1 = 8450\frac{\mathrm{N}}{\mathrm{m}}$, $c_2 = 18418\frac{\mathrm{N}}{\mathrm{m}}$, $c_3 = 3626\frac{\mathrm{N}}{\mathrm{m}}$. $F(t) = H\begin{pmatrix} -3.5\dot{x}_1 - 197x_1 - 17.62\dot{x}_2 \\ -472.7x_2 + 1\dot{x}_3 + 420.9x_3 \end{pmatrix}$.	$m_1 = 1.00$ kg, $m_2 = 6.13$ kg, $m_3 = 1.67$ kg, $c_1 = 1800\frac{\mathrm{N}}{\mathrm{m}}$, $c_2 = 6650\frac{\mathrm{N}}{\mathrm{m}}$, $c_3 = 9990.7\frac{\mathrm{N}}{\mathrm{m}}$ $c_4 = 481\frac{\mathrm{N}}{\mathrm{m}}$. $F(t) = H\begin{pmatrix} -3.5\dot{x}_1 - 197x_1 - 14.6\dot{x}_2 \\ -413.23x_2 + 0.53\dot{x}_3 + 303x_3 \end{pmatrix}$.	$m_1 = 0.2315$ kg, $m_2 = 1$ kg, $m_3 = 0.2315$ kg, $c_1 = 173.75\frac{\mathrm{N}}{\mathrm{m}}$, $c_2 = 1500\frac{\mathrm{N}}{\mathrm{m}}$, $c_3 = 1500\frac{\mathrm{N}}{\mathrm{m}}$ $c_4 = 173.75\frac{\mathrm{N}}{\mathrm{m}}$. $F(t) = H\begin{pmatrix} -3.5\dot{x}_2 - 197x_2 + 0.6357\dot{x}_1 \\ +104x_1 + 0.6357\dot{x}_3 + 104x_3 \end{pmatrix}$.

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