765. Frequency domain identification of the active 3D mechanical structure for the vibration control system

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Abstract. Nowadays structures are light and compliance therefore such structures are opened to the influence of external and internal excitations which in results lead to the structure vibrations and cause a loss of the energy which is used in the process realized by the structure. For example: arms, antennas, satellite solar batteries or slender skyscrapers are such plants. To damp the vibrations and save loss energy we develop design the active vibration control systems. To design such control system we should realized very important stages beginning from analytical investigations through process identification of the dynamical system.

The 3D bar structure with sticked parallel piezo-stacks into chosen bars is considered in the paper. Piezo-elements play a role of piezo-actuators, while two eddy-current sensors located in free plane the structure are used to measurement displacement in directions X and Y. Such control plane will be considered as a two input and two output (TITO) system.

As a result of analytical and numerical investigations such system was divided to two single input single output (SISO) subsystems. Such the coupled system was used in the process of the full model identification. The chirp signal was applied in identification process. The structure was excited according to single input single output controlling force while outputs signals were measured in perpendicular direction X and Y. In such way we have confirmed that for control purposes the plant can be decupled.

Keywords: vibration control, piezo-stack, process identification, active bar structure.

Introduction

Active damping of mechanical vibration has been a field of extensive research during the past decades [1, 2, 3]. Each structure should realize its tasks with low level of vibration amplitude. Therefore we more and more, introduce active vibration control systems. Especially, it is important in the case a structures which are light and compliance. The control system procedure is realized in many steps. The structure dynamic identification is one with important steps because we need sufficiently exact model of the structure dynamic to design correctly the control law. Real structure has many degrees of freedom. Same parameters of the structure like eigenvalues can be obtained during numerical analysis of the structure dynamics but other parameters like damping coefficient, parameters of rheological connections are difficult for numerical modeling. For the control purposes we should reduced the model to model with small number of parameters. To fulfill such requirements we are force to use any identification procedure. During the procedure we can for example reduce the model to the desired range of the frequencies. To identification procedure we have used frequency response function [4] as a starting point. Next the left matrix-fraction description (LMFD) [4] was applied to obtain a state space models. To reduce the order of the polynomials of the transfer function the balance reduction method [5] was used. In our case we have used this method to find the reduced order transfer functions of the local subsystems and the full system.

Finite Element Modeling of the Active Structure

The steel space structure which consists of 126 steel bars and 44 aluminium joints is a subject of our investigation. Bars, joints and piezo-stacks have got glue connections. Complex structure with above elements has dimensions 0.12×0.12×1.2 [m] (length × width × high), respectively. The structure has one end free and second end fixed to the flat horizontal surface. Designed in above way framework is more flexible along vertical axis. Structure without and with piezo-stacks was analyzed with help of FEM [6, 7] in two perpendicular directions *X* and *Y*. For both cases the natural frequencies (are presented in Table 1) and structure modes were obtained.

Values of the natural frequencies for active structure (with piezo-stacks) was calculated for piezo-stacks located at high $z = 0.648$ [m] of structure.

Comparing values in Table 1 we see the natural frequencies of active structure have smaller values than the natural frequencies of bar structure. It is caused by the decrease of local stiffness in "active bar" with glued piezo-element. To design the control system we need the mathematical model. Therefore in the next section we have to carry out problem of identification of the model.

Table. 1. First four natural frequencies of the structure

Natural frequency	Bar structure [Hz]	Active structure [Hz]	
		Z-X	*Z-Y*
1st	16.4	14.95	16.2
2nd	80.1	77.9	88.9
3rd	187.0	146.5	157.5

Identification of dynamic parameters

A process of the model identification is very important for the proper choice of the control system [4, 8, 9]. In our case we have considered the identification of state space model in the frequency domain. The whole process of identification is divided on few steps beginning from measurement input/output data, through by determination FRF and proper curve fitting of the FRF using LMFD method and ending to determination appropriate of transfer function of estimation model. Before we deal with above procedure of identification in the first order we prepared the laboratory stand to obtain experimental results. To do this the full MIMO system with two inputs and two outputs was decoupled into two separate SISO systems. Two eddy current sensors were located near the free end the structure along two perpendicular directions *X* and *Y*. Two piezo-actuators were used to generate force torques around the same axes. In this way there were two local subsystems for further investigations.

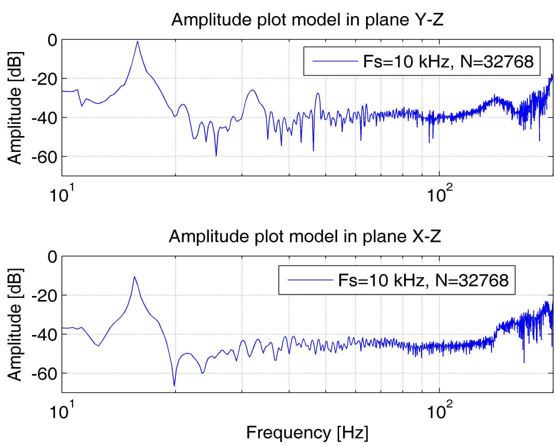


Fig. 1. Amplitude-frequency characteristics in plane: upper – *Y* - *Z*, bottom – *X* - *Z*

In the first subsystem a voltage signals as an excitation signals (chirp) were simultaneous and parallel applied to piezo-stacks ($u(t) = 5\sin(\omega t)$, ω = 10-200 Hz) Voltage signals from sensors in directions X and Y were the output signals. In the second subsystem the excitation signals were applied to piezo-stacks in another way. They were applied simultaneous but in opposite directions. In this way there was generated bonding moment which generated vibration the system mainly in direction X and output signals was recorded from the same both sensors.

On the base of the input and output signals and their discrete Fourier transforms (DFT) the frequency response function $\mathbf{G}(z_k)$ were determined. As a result the plots of the amplitude in plane Y-Z and X-Z versus frequency in the range 10 Hz to 200 Hz were obtained (Fig. 1). In upper plot we can notice that system has three resonance frequencies: 16.2; 82.1; 143 [Hz] and also three anti-resonance frequencies 30.8; 91.5; 165 [Hz]. Also the system in bottom plot has three resonance frequencies: 15.8; 72.5; 156.6 [Hz] and three anti-resonance frequencies 20.1; 89.2; 192.3 [Hz].

Further calculations allowed us to fit a linear transfer function to the frequency characteristics obtained during the each experiment. Since the values of the frequency response function $\mathbf{G}(z_k)$ for all considered frequencies z_k are known, therefore we may determine coefficients of the numerator polynomial $\mathbf{L}(z_k)$ and denominator $\mathbf{M}(z_k)$. The frequency response maybe written as:

$$\mathbf{G}(z_k) = \mathbf{M}^{-1}(z_k)\mathbf{L}(z_k) \tag{1}$$

where:

$$\mathbf{M}(z_k) = \mathbf{I} + \mathbf{M}_1 z_k^{-1} + \mathbf{M}_2 z_k^{-2} + \ldots\ldots + \mathbf{M}_p z_k^{-p}$$
$$\mathbf{L}(z_k) = \mathbf{L}_0 + \mathbf{L}_1 z_k^{-1} + \mathbf{L}_2 z_k^{-2} + \ldots\ldots + \mathbf{L}_p z_k^{-p}$$

and: p - assumed order the of the polynomials in the identified model (in our case: $p = 220$).

Because the transfer function is known for all k, $z_k = e^{j\frac{2\pi k}{l}}$ (k = 0,1,….l-1) we have N (N = 32768) equations available. Therefore, the formula (1) leads to linear regression matrix form:

$$\mathbf{\Psi} = \mathbf{\Theta}\boldsymbol{\varphi} \tag{2}$$

where:

$$\boldsymbol{\varphi} = \begin{bmatrix} \mathbf{G}(z_0)z_0^{-1} & \mathbf{G}(z_1)z_1^{-1} & \ldots & \ldots & \mathbf{G}(z_{l-1})z_{l-1}^{-1} \\ \mathbf{G}(z_0)z_0^{-2} & \mathbf{G}(z_1)z_1^{-2} & \ldots & \ldots & \mathbf{G}(z_{l-1})z_{l-1}^{-2} \\ \mathbf{G}(z_0)z_0^{-p} & \mathbf{G}(z_1)z_1^{-p} & \ldots & \ldots & \mathbf{G}(z_{l-1})z_{l-1}^{-p} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\ z_0^{-1}\mathbf{I} & z_1^{-1}\mathbf{I} & \ldots & \ldots & z_{l-1}^{-1}\mathbf{I} \\ z_0^{-2}\mathbf{I} & z_1^{-2}\mathbf{I} & \ldots & \ldots & z_{l-1}^{-2}\mathbf{I} \\ z_0^{-p}\mathbf{I} & z_1^{-p}\mathbf{I} & \ldots & \ldots & z_{l-1}^{-p}\mathbf{I} \end{bmatrix}_{[(mk+rk)p+rk\ \ (rkl)],}$$

$$\mathbf{\Theta} = \begin{bmatrix} -\mathbf{M_0} & -\mathbf{M_1} & \ldots\ldots & -\mathbf{M_p} & \mathbf{L_0} & \mathbf{L_1} & \ldots\ldots & \mathbf{L_p} \end{bmatrix}_{[(mk+rk)p+rk)\ \ mk]}$$

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{G}(z_0) & \mathbf{G}(z_1) & \ldots & \mathbf{G}(z_{l-1}) \end{bmatrix}_{[rkl\ \ \ mk],}$$

where: *mk* - amount of output, *rk* - amount of input.

The vector $\boldsymbol{\Theta}$ which is an estimator of the coefficients in the polynomials of the transfer function can be obtained by the solving of the equation (2). The least square sum method was used to calculate the estimated parameters:

$$\widehat{\boldsymbol{\Theta}} = \boldsymbol{\Psi}\boldsymbol{\varphi}^{*} \tag{3}$$

where: $\widehat{\boldsymbol{\Theta}}$ - estimated values of numerator and denominator polynomial coefficients, $\boldsymbol{\varphi}^{*}$ - pseudo-inverse matrix to the matrix $\boldsymbol{\varphi}$.

In the next step taking into account the estimated values (3) we may describe our system in the form of the state space model. Using the LMFD the observable canonical-form (as triplet matrices **A**, **B**, **C**) was derived. Matrix **D** equals the first coefficient L_0 of the estimated numerator. State space model has the following form:

$$\begin{aligned} \mathbf{x}(z_k)z_k &= \mathbf{A}\mathbf{x}(z_k) + \mathbf{B}\mathbf{u}(z_k) \\ \mathbf{y}(z_k) &= \mathbf{C}\mathbf{x}(z_k) + \mathbf{D}\mathbf{u}(z_k) \end{aligned} \tag{4}$$

where:

$$\mathbf{x}(z_k) = \begin{bmatrix} x_1(z_k) \\ x_2(z_k) \\ x_3(z_k) \\ . \\ . \\ x_{p-1}(z_k) \\ x_p(z_k) \end{bmatrix}_{(1xp)} \qquad \mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & -M_p \\ 1 & 0 & 0 & \dots & 0 & -M_{p-1} \\ 0 & 1 & 0 & \dots & 0 & -M_{p-2} \\ . & . & . & & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & 0 & -M_2 \\ 0 & 0 & 0 & \dots & 1 & -M_1 \end{bmatrix}_{(mkp\ x\ mkp)}$$

$$\mathbf{B} = \begin{bmatrix} L_p - M_p D \\ L_{p-1} - M_{p-1} D \\ L_{p-2} - M_{p-2} D \\ . \\ . \\ L_2 - M_2 D \\ L_1 - M_1 D \end{bmatrix}_{(rk\ x\ mkp)} \qquad \mathbf{C} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ 0 \\ 1 \end{bmatrix}^{T}_{(mkp\ x\ mk)} \tag{5}$$

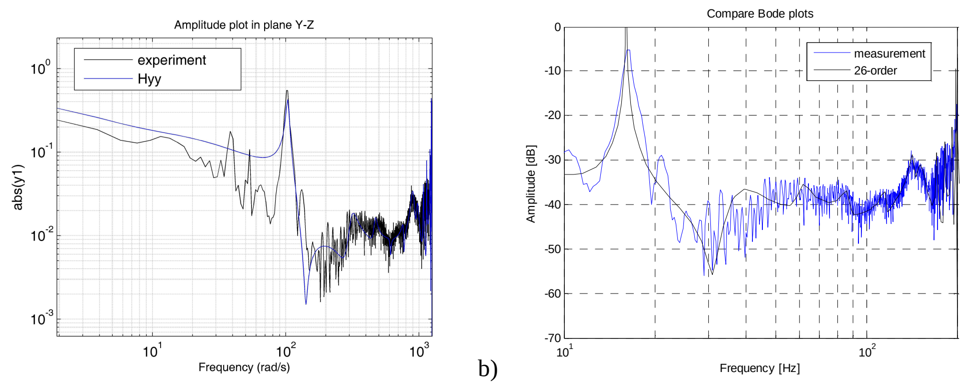
The determination of the model transfer function is the last step in the identification procedure. Assumed order of estimated model is firmly high, so there is a need to reduce its (Fig. 2a). To do it the discrete model was changed into continuous one. With help of the balance method the model was reduced to the one with 42-order polynomials. Because in this transfer function some zeros and poles are out of consider frequency range, therefore order model transfer function decrease to 26-order. It is appeared that such model has very good curve-fitting to the experimental results as it is shown in Fig. 2b.

Obtained transfer function is still high. Therefore we may cancel these zeros and poles which frequencies are close each other. Finally, obtained 6th order transfer function is a minimal realization of structure *Y-Z* model:

$$H_{YY}(s) = 0{,}003595\frac{\left(s^2+6{,}165s+2{,}001e4\right)\left(s^2+15{,}49s+3{,}739e5\right)\left(s^2+59{,}8s+1{,}151e6\right)}{\left(s^2+4{,}828s+1{,}073e4\right)\left(s^2+27{,}74s+2{,}19e5\right)\left(s^2+53{,}92s+8{,}127e5\right)} \tag{6}$$

In very similar way there identified was input-output model between the same excitation (in plane *Y-Z*) and vibrations in perpendicular direction (plane *X-Z*). Also in this case assumed order model was 220. Of course this order is firmly high in comparison to characteristics FRF in consider frequency range. In the next step model was reduced to 26-order model by used balance method. Finally the model $H_{XY}(s)$ of 6-order was obtained by cancellation same zeros and some poles in frequency range 10-200 [Hz]. The final transfer function is as follows:

$$H_{YX}(s) = -0{,}014163\frac{\left(s^2+35{,}84s+2{,}104e4\right)\left(s^2+33{,}18s+3{,}575e5\right)\left(s^2+38{,}03s+1{,}459e6\right)}{\left(s^2+14{,}47s+9383\right)\left(s^2+38{,}99s+1{,}827e5\right)\left(s^2+86{,}64s+9{,}581e5\right)} \tag{7}$$



a) b)

Fig. 2. Comparison FRF for H_{YY} transfer function with a) model of 220-order, b) model of 26-order

Finally above transfer function models H_{XY} and H_{YY} was compared. As we can see from Fig. 3 both models have different resonance and anti-resonance frequencies what is correct because we analyze vibrations the structure in two different planes *X-Z* and *Y-Z*. Furthermore, we can notice that excitation in one direction has much smaller influence on the vibrations in perpendicular direction. The amplitude of the transfer function $H_{XY}(s)$ is about 10 [dB] lower than the amplitude of the transfer function $H_{YY}(s)$ in considered range of frequencies. So, we can consider the system as a really decoupled.

In similar way were identified models the second subsystem. In this case the excitation signals were applied to piezo-stacks simultaneous but in opposite directions ($\mathbf{u_1}(t)$ = 5 sin(ωt), $\mathbf{u_2}(t)$ = -5 sin(ωt), ω = 10-200 Hz). So, the main vibrations of the structure are in plane *X-Z*. On the base of this assumption were recorded voltage excitation signals and also output signals with two eddy-current sensors in planes *X-Z* and *Y-Z*. Similar to previous subsystem also in this case in the first step amplitude characteristics were obtained by using DFT method. In the second step one more time by using equations (2), (3) and assumed order model equal 220 were built triplet matrices (**A**, **B**, **C**) in the state space model according to equation (5) and next the model were reduced by using balance method. Finally, the transfer function 6th described minimal realization of structure *X-Z* and *Y-Z* model were obtained order after necessary cancelation of some zeros and poles:

$$H_{XX}(s) = 0{,}014163\frac{\left(s^2+35{,}84s+2{,}104e4\right)\left(s^2+33{,}18s+3{,}575e5\right)\left(s^2+38{,}03s+1{,}459e6\right)}{\left(s^2+14{,}47s+9383\right)\left(s^2+38{,}99s+1{,}827e5\right)\left(s^2+86{,}64s+9{,}581e5\right)} \tag{8}$$

$$H_{YX}(s) = 0{,}011018\frac{\left(s^2+19{,}79s+2{,}572e4\right)\left(s^2+20{,}76s+3{,}163e5\right)\left(s^2+68{,}07s+1{,}033e6\right)}{\left(s^2+2{,}692s+9687\right)\left(s^2+22{,}4s+3{,}096e5\right)\left(s^2+116{,}2s+7{,}793e5\right)} \quad (9)$$

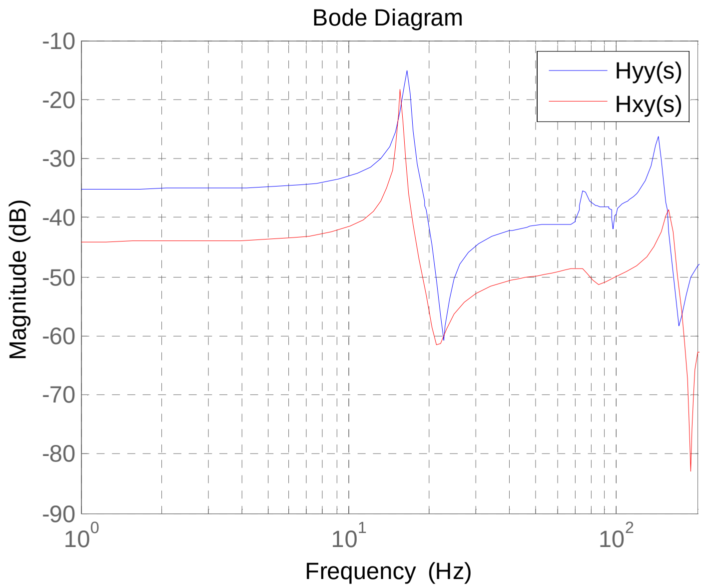


Fig. 3. The comparison the amplitude-frequency characteristics of both directions for the same excitation in plane *Y-Z*

The last step in this process was compared for 6th order transfer function on Bode plot, what is shown in Fig. 4.

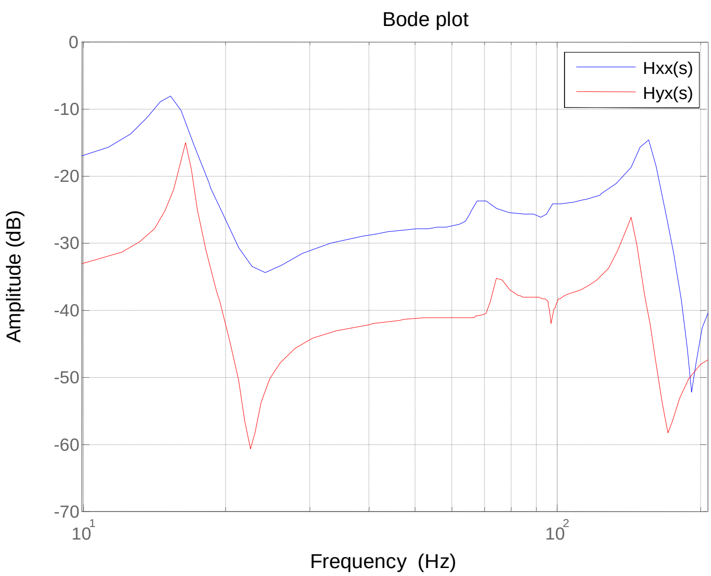


Fig. 4. The comparison of the amplitude-frequency characteristics of both directions for the same excitation in plane *X - Z*

Obtained results also proved that this subsystem is really decoupled. Both models $H_{XX}(s)$ and $H_{YX}(s)$ have different values frequencies resonance and frequencies anti-resonances. Of course it is correct, because above models described vibrations of the structure in two perpendicular planes. Furthermore amplitude model $H_{YX}(s)$ is about 20 [dB] less than amplitude model $H_{XX}(s)$.

Finally from these investigations we can notice that the global system is described by equation (10):

$$\mathbf{H}(s) = \begin{bmatrix} \mathrm{H}_{XX}(s) & \mathrm{H}_{XY}(s) \\ \mathrm{H}_{YX}(s) & \mathrm{H}_{YY}(s) \end{bmatrix} \tag{10}$$

Analysis of both subsystems proved that considered models are decoupled. Then the whole system also is decoupled. So to design the local control law we have used only transfer functions H_{YY} and H_{XX}, omitting cross-coupling transfer functions H_{YX} and H_{XY}.

Conclusions

In the paper we have described the identification procedure of the active vibration damping for the space bar structure. Sticked into structure piezo-stacks work parallel (in the same direction or in opposite ones) to generate force torques in two perpendicular directions in the plane parallel to the structure base. The measurements were realized in the same directions as force torques. This way the MIMO (two inputs, two outputs) system was divided into two SISO subsystems. Then the whole process identification was divided on two separate parts depending on type of excitation signals. It significantly simplifies the identification procedure and the design of control laws. By the sequence of the transformations of the experimental signals the local mathematical model was obtained. All obtained models have had a very good fit to the recorded results. Furthermore consider both SISO subsystems are really decoupled. Then the whole system is also really decoupled. In this way we can choose only diagonal transfer functions H_{YX} and H_{XY} to design appropriate control laws in further calculation.

References

[1] **Hansen C., Snyder S.** Active Control of Noise and Vibration. Champan & Hall, 1997.

[2] **Premount A.** Vibration Control of Active Structures: an Introduction. Second Edition, Kluwer Academic Publisher, 2002.

[3] **Graham Kelly S.** Advanced Vibration Analysis. CRC Press, Taylor & Francis, 2007.

[4] **Juang J. N.** Applied System Identification. Prentice-Hall Inc., 1994.

[5] **Laub A. J.** Computation of Balancing Transformations. Proc. ACC, San Francisco, Vol. 1, paper FA8-E, 1980.

[6] **Seto K.** Modeling strategy suited to motion and vibration control for flexible structures. 7th International Conference on Motion and Vibration Control, MOVIC, 2004.

[7] **Zagrajek K., Krzesiński G., Marek P.** Metoda Elementów Skończonych w Mechanice Konstrukcji: Ćwiczenia z Zastosowaniem Systemu Ansys, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa, 2005.

[8] **Stoica P.** Identyfikacja systemów, PWN,1997.

[9] System Identification Toolbox**,** the MathWorks Inc., 2010.