746. Damage detection based on pattern-matching method of response signal

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Abstract. Using response data as the feature to extract the damage, this study proposes an analytical method to detect damage by matching the response pattern between a damage-expected beam system and the database. Response pattern database is composed of the response variation before and after the damage, prescribed by various damage scenarios. Pattern-matching is evaluated by the correlation coefficients between two data sets and the damage corresponds to high correlation coefficient. Response variations at a full set of DOFs are estimated by the expansion of a few measured response data. Numerical applications illustrate the validity and applicability of the proposed method.

Keywords: pattern-matching, damage detection, feature vector, correlation, constraint force.

1. Introduction

Structural damage detection technique indicates how to locate and detect damage in a structure by observing changes in its dynamic and static characteristics. Damage deteriorates the dynamic and static performances of intact structures, and leads to responses that deviate from the initial findings. The response deviation reveals that the response of a damaged system can be used as an important index to detect damage.

Generally, the measured data are less than the full set of DOFs of the system. The analysis is performed by reducing the DOFs of the system or by expanding the measured data. This work expands the measured response data to complete the unmeasured DOFs from the experimental model.

Signal-based damage detection method is a widely used damage detection method. It is performed in two stages, feature extraction and pattern recognition. Feature extraction incorporates the most relevant information from the raw data. It identifies and selects damage-sensitive features derived from the measured dynamic response, to quantify the damage to a structure [1].

Pattern recognition may use different vectors that have been classified or described. The classification or description scheme uses statistical and structural approaches. Statistical pattern recognition is based on statistical characterizations of patterns. It assumes that the patterns are generated by a probabilistic system. Syntactic pattern recognition is based on the structural interrelationship of features. Signal-based methods are classified into time-domain methods, such as the auto-regressive moving average (ARMA) model, the extended Kalman filter (EKF) [1-6], frequency-domain methods like Fourier transform [7], and time-frequency methods such as wavelet transform [8-10].

The pattern recognition in damage detection distinguishes between different classes of patterns, presenting these conditions based on prior knowledge or statistical information extracted from the patterns. Qiao et al [11] utilized two types of patterns formed by normalized FFT (Fast Fourier Transform) magnitudes and CWT (Continuous Wavelet Transform) coefficients of the signal with three statistical algorithms of correlation, least square distance and Cosh spectral distance. Cheung et al [12] presented the analysis of a progressive damage field test on the Z24 bridge in Switzerland, using an autoregressive time series modeling and statistical pattern recognition techniques for damage detection. Riveros et al [13] presented a statistical pattern recognition technique based on time series analysis of vibration data to identify structural damage. Lam and Ng [14] compare the performance of two pattern features of modal parameters and Ritz vectors in structural damage detection using pattern recognition.

Modal displacement response in the frequency domain corresponding to the first resonance frequency can be utilized as the feature to extract the pertinent information for damage detection. This work proposes pattern-matching algorithm in the frequency-domain based on the response variation before and after damage. The feature extraction and pattern recognition are implemented by the modal displacement variation, based on measured response data and their expansion to the full DOFs in the frequency domain. The damage scenarios include single and multiple damages as well as damage rate. Numerical simulations on a fixed-supported beam illustrate the validity of the proposed pattern-matching method.

2. Formulation

2. 1 Expansion of FRF matrix using measured response data

The dynamic behavior of a structure that is assumed to be linear and approximately discretized for n DOFs can be described by the equations of motion as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ denote the $n \times n$ analytical mass, damping and stiffness matrices, respectively, $\mathbf{u} = [u_1 \quad u_2 \quad u_3 \quad \cdots \quad u_n]^T$, and $\mathbf{f}(t)$ is the $n \times 1$ excitation vector. Dynamic responses can be expressed in the time domain and frequency domain. For linear systems, there is little loss of information going from the time domain to the frequency domain. In the frequency domain, the FRFs are measured directly from the system instead of measuring the displacement and the force individually.

The dynamic characteristics can be investigated by the modal data of the natural frequency and its corresponding mode shape, or by its corresponding FRFs data. The relationships between the FRF and the modal parameters should be established for a successful modal testing. Inserting $\mathbf{u} = \mathbf{U}e^{j\Omega t}$ and $\mathbf{f} = \mathbf{F}e^{j\Omega t}$ into Eq. (1) and expressing it in the frequency domain, it follows that

$$\left(\mathbf{K} - \Omega^2\mathbf{M} + j\Omega\mathbf{C}\right)\mathbf{U}(\Omega) = \mathbf{F}(\Omega) \tag{2}$$

where $j = \sqrt{-1}$, Ω denotes the excitation frequency, and $\mathbf{F}(\Omega) = [F_1 \quad F_2 \quad \cdots \quad F_n]^T$ and $\mathbf{U}(\Omega) = [U_1 \quad U_2 \quad \cdots \quad U_n]^T$ represent the Fourier transform of the force and response vectors $\mathbf{f}$ and $\mathbf{u}$, respectively. Equation (2) is valid for an excitation frequency Ω. Defining the FRF matrix $\hat{\mathbf{H}}$,

$$\hat{\mathbf{H}} \equiv \left[\mathbf{K} - \Omega^2\mathbf{M} + i\Omega\mathbf{C}\right] \tag{3}$$

where $\hat{\mathbf{H}}(\Omega) = \begin{bmatrix} \hat{H}_{11} & \hat{H}_{12} & \cdots & \hat{H}_{1n} \\ \hat{H}_{21} & \hat{H}_{22} & \cdots & \hat{H}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{H}_{n1} & \hat{H}_{n2} & \cdots & \hat{H}_{nn} \end{bmatrix}$

$\hat{H}_{ij}$ denotes the displacement response measured at location i due to the unit force input at location j.

Assuming the initial system is unexpectedly damaged, in this case the dynamic response in the frequency domain will not satisfy Eq. (2). As a result of the damage, the physical parameter matrices or the dynamic stiffness matrix should be changed to describe the damaged system. To estimate the displacement response of a full set of DOFs, it is necessary to expand the measured data, because it is rarely possible to measure the full set of displacements. This section introduces an analytical method to expand the measured FRF data to a full set of FRF matrix [15].

Assume that the FRF data of the damaged system were measured at m different positions. The measured $m \times n(m < n)$ FRF matrix $\mathbf{G}$ and the response vector $\mathbf{U}_d$ have the relationship of

$$\mathbf{AU}_d = \mathbf{GF} \tag{4}$$

where $\mathbf{A}$ is an $m \times n$ Boolean matrix to define the measured locations, $\mathbf{U}_d$ represents the updated displacement vector, including the measured displacement vector, $\mathbf{U}_d = \begin{bmatrix} \mathbf{U}_{d,m} \\ \mathbf{U}_{d,u} \end{bmatrix}_{n \times 1}$, and $\mathbf{F} = \begin{bmatrix} \mathbf{F}_m \\ \mathbf{F}_u \end{bmatrix}_{n \times 1}$. The subscripts m and u represent the measured and unmeasured DOFs, respectively. $\mathbf{G}$ is $m \times n$ coefficient matrix representing the measured FRF data, $\mathbf{U}_{d,m}$ and $\mathbf{F}_m$ are $m \times 1$ displacement and force vectors corresponding to the measured locations, and $\mathbf{U}_{d,u}$ and $\mathbf{F}_u$ are $(n - m) \times 1$ displacement and force vectors corresponding to the unmeasured locations. The relation of Eq. (4) can be regarded as constraint conditions to describe the damaged system. The newly updated dynamic equation subjected to linear constraints can be derived from the initial dynamic equation in the frequency domain of Eq. (2) and the constraint equations of Eq. (4).

The updated response vector $\mathbf{U}_d$ is derived as

$$\mathbf{U}_d = \mathbf{U} + \Delta\mathbf{U} \tag{5}$$

where $\mathbf{U} = \hat{\mathbf{H}}\mathbf{F}$ and

$$\Delta\mathbf{U} = \left[\mathbf{D}^{-1/2}\left(\mathbf{AD}^{-1/2}\right)^{+}\left(\mathbf{G} - \mathbf{AD}^{-1}\right)\right]\mathbf{F} = \left[\hat{\mathbf{H}}^{1/2}\left(\mathbf{A}\hat{\mathbf{H}}^{1/2}\right)^{+}\left(\mathbf{G} - \mathbf{A}\hat{\mathbf{H}}\right)\right]\mathbf{F} \tag{6}$$

where the superscript '+' indicates the Moore-Penrose inverse and $\mathbf{D} = \hat{\mathbf{H}}^{-1}$. $\Delta\mathbf{U}$ of Eq. (6) indicates the variation in the displacement caused by the damage. This means that the damage can be evaluated by investigating the variation in the displacement responses. Pre-multiplying both sides of Eq. (5) by the matrix $\mathbf{D}$, it becomes:

$$\mathbf{DU}_d = \mathbf{F} + \mathbf{D}^{1/2}\left(\mathbf{AD}^{-1/2}\right)^{+}\left(\mathbf{G} - \mathbf{AD}^{-1}\right)\mathbf{F} \tag{7}$$

The second term in the right-hand side of Eq. (7) indicates the additional force vector required for obtaining the measured FRF data in the frequency domain, and the response of the damaged system can be described by the additional action of the calculated forces:

$$\mathbf{F}^c = \mathbf{D}^{1/2}\left(\mathbf{AD}^{-1/2}\right)^{+}\left(\mathbf{G} - \mathbf{AD}^{-1}\right)\mathbf{F} = \hat{\mathbf{H}}^{-1/2}\left(\mathbf{A}\hat{\mathbf{H}}^{1/2}\right)^{+}\left(\mathbf{G} - \mathbf{A}\hat{\mathbf{H}}\right)\mathbf{F} \tag{8}$$

The coefficient matrix of the force **F** in Eq. (8) represents the variation in the FRF matrix $\Delta\mathbf{H}$ due to the mechanical change of the dynamic system. This equation can be utilized in determining the FRF matrix of the dynamic system subjected to linear constraints. Thus, the updated FRF matrix $\mathbf{H}$ can be predicted by:

$$\mathbf{H} = \hat{\mathbf{H}} + \Delta\mathbf{H} \tag{9}$$

where $\Delta\mathbf{H} = \hat{\mathbf{H}}^{-1/2}\left(\mathbf{A}\hat{\mathbf{H}}^{1/2}\right)^{+}\left(\mathbf{G} - \mathbf{A}\hat{\mathbf{H}}\right)$.

Equation (9) represents the entire FRF matrix of the damaged dynamic system obtained from the expansion of incomplete FRF data. The $\Delta\mathbf{H}$ indicates the variation in FRF matrix and provides information on the displacement variation of the damaged system. The estimated displacement variation is matched with the prescribed displacement pattern database and the damage is detected by evaluating the matching rate.

2. 2 Correlation coefficient

Correlation is one of general techniques for pattern recognition and is used in many applications. It is a statistical measurement of the relationship between two vectors. Possible correlations range from +1 to –1. A zero correlation indicates that there is no relationship between them and a correlation of +1 indicates a perfect positive correlation. The correlation coefficient $\rho_{x,y}$ between two random variables *x* and *y* with expected values μ_x and μ_y and standard deviations σ_x and σ_y is defined as:

$$\rho_{x,y} = corr\left(x, y\right) = \frac{cov\left(x, y\right)}{\sigma_x \sigma_y} = \frac{E\left[\left(x - \mu_x\right)\left(y - \mu_y\right)\right]}{\sigma_x \sigma_y} \tag{10}$$

where *E* is the expected value operator, *cov* means covariance, and, *corr,* a widely used alternative notation for Pearson’s correlation.

Damage detection method presented in this study is done by calculating the correlation coefficients to match the pattern database with the response vector of the expected structure damage (Fig. 1). Large coefficient values indicate high possibility of damage scenarios and vice versa.

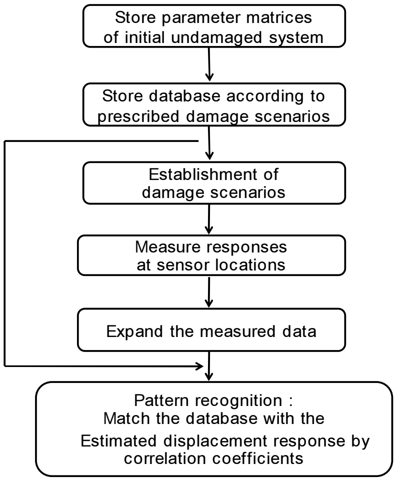

Fig. 1. Flow chart of damage detection method by pattern-matching

3. Application

Displacement variations in the frequency domain are estimated from incompletely measured FRF data on the dynamic system where damage is expected. Considering the displacement variation as the sensitive feature, this application detects damage by matching vector sets.

3. 1 Feature extraction

Feature extraction is carried out for the reduction of the raw data and the damage identification by the parameter to be sensitive to the response. The responses of a damaged system are closely related to the damage position and degree. Variation in response before and after the appearance of damage is described by the action of additional forces required to obtain the measured displacements. Additional forces are affected by measured response data and the parameter matrices of the initial system. The observation means that the displacement variation caused by the constraint force can be used as a feature to extract damage location and degree.

Consider the dynamic system shown in Fig. 2 to evaluate the sensitivity of constraint force to act on the damaged system. The constraint forces obtain the actual responses at the measurement positions. Fig. 3 represents the magnitude of the constraint forces to act at mass 3 due to the deterioration of K_1 and K_4. The solid line indicates the 20% deterioration of K_1, the dashed line - the 40% deterioration of K_1 and the dotted line - the 20% deterioration of K_4. The existence of damage leads to the change in the resonance frequency as well as the magnitude of the constraint forces. It is shown that the resonance frequencies of the intact and damaged systems do not coincide. The inconsistency gradually increases with the increase in resonance frequency. This study considers the displacement variation at the minimum resonance frequency to represent the least inconsistency. It is found that the constraint force is more sensitive to the damage than the frequency. As the result, this study uses the displacement variation corresponding to the action of the constraint force as the feature for the pattern recognition because the displacement variation is expressed by the constraint forces.

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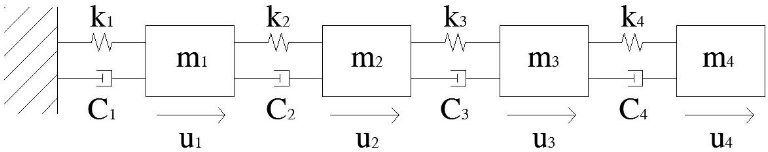

Fig. 2. A dynamic system of four DOFs

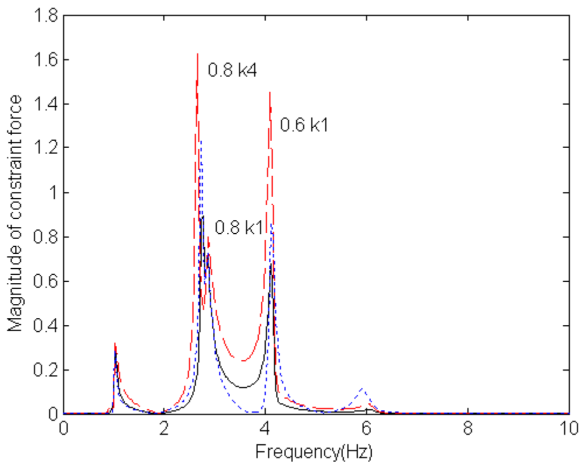

Fig. 3. Constraint force curve according to frequency

3. 2 Pattern-matching

Pattern-matching algorithm detects damage by evaluating the matching rate between the estimated responses data and the prescribed response database. Let us consider the pattern recognition on the fixed-supported beam shown in Fig. 4. The nodal points and the members are numbered as shown in the figure. The beam of 1 m length was modeled as five beam elements. Each node has two degrees of freedom of vertical deflection and rotation, and its cross-section was specified as $b \times h = 75 \times 9\text{mm}$. Mass density per unit length was 0.005kg/mm . It is impossible to make the database of all displacement response sets depending on all damage levels and locations, and match them with the actual values. The prescribed database is sorted based on the 30 damage scenarios, depending on the damage location and damage rates of 10% and 20% as shown in Table 1. The displacement variation in the database was numerically obtained by calculating the displacement difference between intact and damaged states subjected to an impulse force at node 3. It is not easy to obtain the deflection and rotation data at the full set of DOFs. Expanding the measured FRF displacements measured at two nodal positions, the estimated displacements at the full set of DOFs are matched with the displacement data in the database and the damages are evaluated by correlation coefficients between two vectors.

This numerical experiment considered four damaged beam cases: 1) a single damage of 17% stiffness deterioration at element 3; 2) a single damage of 8% stiffness deterioration at element 5; 3) multiple damages of 15% and 8% stiffness deterioration at elements 2 and 5; 4) multiple damages of 14% and 7% stiffness deterioration at elements 1 and 5. Assuming that the unit impulse acts at mass position 3, the dynamic responses at mass positions 2 and 4 within a frequency range of 2.5–3.5Hz in the frequency interval of 0.02 Hz were measured and the displacement variations were calculated using Eq. (7). The correlation coefficients between the vector in the database and the estimated displacement vector are evaluated for damage detection using Eq. (10).

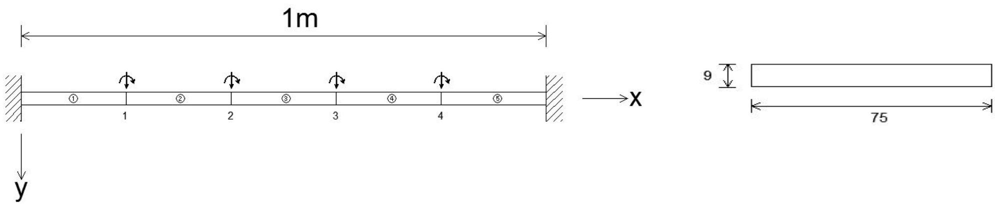

Fig. 4. Finite element modeling of a fixed-supported beam

Table 1. Summary of specimens in database

Specimens		No. of damages	Damage location	Specimens		No. of damages	Damage location
A1_1	B1_1	1	①	A3_1	B3_1	3	①②③
A1_2	B1_2		②	A3_2	B3_2		①②④
A1_3	B1_3		③	A3_3	B3_3		①②⑤
A1_4	B1_4		④	A3_4	B3_4		①③④
A1_5	B1_5		⑤	A3_5	B3_5		①③⑤
A2_1	B2_1	2	①②	A3_6	B3_6		①④⑤
A2_2	B2_2		①③	A3_7	B3_7		②③④
A2_3	B2_3		①④	A3_8	B3_8		②③⑤
A2_4	B2_4		①⑤	A3_9	B3_9		②④⑤
A2_5	B2_5		②③	A3_10	B3_10		③④⑤
A2_6	B2_6		②④	A4_1	B4_1	4	①②③④
A2_7	B2_7		②⑤	A4_2	B4_2		①②③⑤
A2_8	B2_8		③④	A4_3	B4_3		①②④⑤
A2_9	B2_9		③⑤	A4_4	B4_4		①③④⑤
A2_10	B2_10		④⑤	A4_5	B4_5		②③④⑤

* ”A” and “B” indicate the 10% and 20% stiffness reduction respectively.

The damage level and location should be enunciated by comparing the response pattern between the estimated displacement variation and the information of the database. Fig. 5 exhibits the correlation coefficients between two data sets. Figs. 5(a)-(b) represent the matching graphs with the damage case 1 to have 17% stiffness deterioration at the element 3. It is observed from the plots that the damaged responses are more properly matched with the 10% damage pattern database rather than the 20% damage pattern database. The sample with the damage at element 3 exhibits a high correlation factor at the element. Each element in the finite element model has the responses to be measured at both ends of the element, thus the displacement response at a measurement position relates with its two adjacent elements. The correlation factor corresponding to the database of 20% single damage (1-5 of data-base number) represents a high correlation factor at most database of multiple damages (6-30 of data-base number). The damage cannot be detected by such observation. This outcome is expected because the tendency is that the damage element affects the stiffness of the other elements through the nodal displacements in the neighborhood of the damage element.

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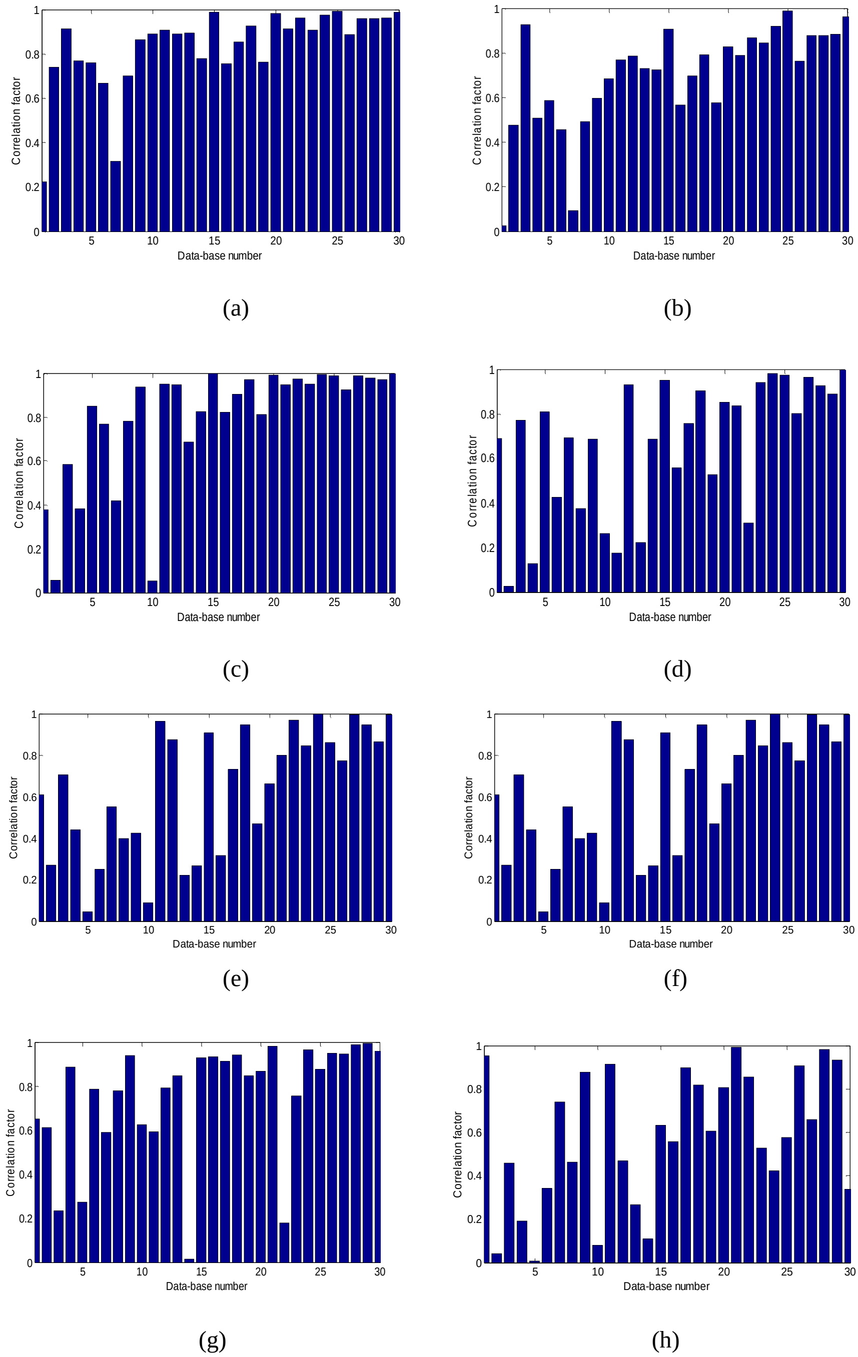

Fig. 5. Correlation coefficients between the displacement responses of damaged beams and the database: (a) damage case 1 with 20% damage database, (b) damage case 1 with 10% damage database, (c) damage case 2 with 20% damage database, (d) damage case 2 with 10% damage database, (e) damage case 3 with 20% damage database, (f) damage case 3 with 10% damage database, (g) damage case 4 with 20% damage database, (h) damage case 4 with 10% damage database

Figs. 5(c)-(d) exhibit the correlation factors of an 8% single damage beam at the element 5. The plots reveal that the damage can be detected using the 10% damage database rather than the 20% damage database because the damage is only 8% less than 10%. They represent high correlation factors at the multiple damages of the 20% damage database, such that the damage location can rarely be detected. It is shown that the correlation factors corresponding to the multiple damage patterns, including the element 5, are high, but it is not easy to prove that the damage solely locates at the element 5. Figs. 5(e)-(f) exhibit the correlation coefficients of the damaged beam with multiple damages at the elements 2 and 5. It is demonstrated that the correlation factors corresponding to the 10% single damage database are low compared to those of the multiple damage database, unlike the single damage database. The plots represent high correlation factors at the damage patterns, including the damage elements 2 and 5. The plots indicate that the beam has multiple damages by investigating the high correlation factors. Figs. 5(g)-(h) represent similar results as the multiple damages of the case 4.

4. Conclusions

This study presented an analytical damage detection method to locate damage by pattern-matching between the response data of a damaged beam system and the prescribed response pattern database. The database consists of the damage-expected response data in the frequency domain according to damage number and rate. This work considered response variation as the main feature. The database is composed of response data before and after the appearance of damage according to different scenarios, including the degree and rate of the damage. Measured response data are expanded to the response data of the full set of DOFs and the estimated data are matched with the response pattern database. The damage is detected by the correlation coefficients between two data sets. Numerical applications illustrated the validity and applicability of the proposed method.

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References

[1] **Sohn H., Czarnecki J. A., Farrar C. R.** Structural health monitoring using statistical process control. Journal of Structural Engineering, Vol. 126, Issue 11, 2009, p. 1356-1363.

[2] **Sohn H., Farrar C. R.** Damage diagnosis using time-series analysis of vibration signals. Smart Materials and Structures, Vol. 10, Issue 3, 2002, p. 446-451.

[3] **Nair K. K., Kiremidjian A. S., Law K. H.** Time series-based damage detection and localization algorithm with application to the ASCE benchmark structure. Journal of Sound and Vibration, Vol. 291, 2006, p. 349-368.

[4] **Liu Y., Li A., Fei Q., Ding Y.** Feature extraction and damage alarming using time series analysis. Journal of Southeast University, Vol. 23, Issue 1, 2007, p. 86-91.

[5] **Yan Y. J., Hao H. N., Yam L. H.** Vibration-based construction and extraction of structural damage feature index. International Journal of Solids and Structures, Vol. 41, 2004, p. 6661-6676.

[6] **Omenzetter P., Brownjohn J. M. W.** Application of time series analysis for bridge monitoring. Smart Materials and Structures, Vol. 15, 2006, p. 129-138.

[7] **Fasel T. R., Sohn H., Farrar C. R.** Damage detection using frequency domain arx models and extreme value statistics. LA-UR-02-6587, 2005.

[8] **Biemans C., Staszewski W. J., Boller C., Tomlinson G. R.** Crack detection in metallic structures using broadband excitation of acousto-ultrasonics. Journal of Intelligent Material Systems and Structures, Vol. 12, 2001, p. 589-597.

[9] **Hou Z., Noori M., Amand R. St.** Wavelet-based approach for structural damage detection. Journal of Engineering Mechanics, Vol. 126, Issue 7, 2000, p. 677-683.

[10] **Robertson A. N., Farrar C. R., Sohn H.** Singularity detection for structural health monitoring using Holder exponents. Mechanical Systems and Signal Processing, Vol. 17, Issue 6, p. 1163-1184.

[11] **Qiao L., Esmaeily A., Melhem H.** Structural damage detection using pattern-matching method. Structural Engineering Mechanics and Computation, Vol. 3, 2007, p. 1836-1841.

[12] **Cheung A., Cabrera C., Sarabandi P., Kiremidjian A., Wenzel H.** The application of statistical pattern recognition methods for damage detection to field data. Smart Materials and Structures, Vol. 17, 2008, p. 065023 (12 p.).

[13] **Riveros C. A., Utsunomiya T., Maeda K., Itoh K.** Damage detection in flexible risers using statistical pattern recognition techniques. International Journal of Offshore and Polar Engineering, Vol. 18, Issue 1, 2008, p. 35-42.

[14] **Lam H. F., Ng C. T.** The selection of pattern features for structural damage detection using an extended Bayesian ANN algorithm. Engineering Structures, Vol. 30, Issue 10, 2008, p. 2762-2770.

[15] **Lee E. T., Eun H. C.** FRF based substructuring and decoupling of substructures. Journal of Vibroengineering, Vol. 13, Issue 3, 2011, p. 386-398.