744. The self-resonance effect of the planetary vibration excitation systems

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Abstract. Planetary type vibration excitation systems are analyzed in the paper. It is shown that resonance vibrations in such systems can occur in the regime of self-resonance, which is understood as a mode of motion when the angular frequency of a driving shaft is high, but the generated vibration frequency is relatively low. Theoretical investigations of the effect of self-resonance enabled to define the conditions of existence and stability of such motion modes in planetary vibroexciter. Such principle of vibration excitation can be successfully exploited in different areas of engineering as it eliminates the necessity of complex and expensive vibration control and stabilization equipment.

Keywords: rotating machinery, vibrations, nonlinear vibrations, vibroexciter.

Introduction

Various types and modifications of vibration excitation systems are widely applied in different areas of engineering and technology. Every technological process has its own optimal regimes of vibration characterized by range of frequencies and amplitudes. Moreover, every vibration excitation system possesses individual characteristics describing the relationship between the transferred vibration power and frequency - amplitude characteristics.

The operation of vibration excitation systems is most effective in the regime of resonance. Such regimes are usually maintained by automatic control systems. The stability of the systems without control usually is poor. On the other hand, the closed-loop control systems are relatively expensive. Another possibility for generating stable resonance vibrations without closed-loop control systems was found and investigated in a number of research reports on nonlinear dynamics [1], [2], [3]. In study [4], [5] the dynamics of eccentric vibroexciter with complex disbalance is investigated. It is shown that resonance vibrations can be excited without adaptation of frequency of forced oscillations – effect of self-resonance can be applied for this purpose. The main driving shaft in such vibroexciters can rotate with any angular velocity. The eccentric non-linear forces generate stable vibrations with frequency corresponding to the fundamental frequency of the non-linear system. The aim of this work is to perform theoretical studies of a self-resonance effect in the planetary vibration excitation systems.

Theoretical background

The scheme of the ordinary planetary vibration excitation system is presented in Fig. 1. Vibroexciter is presented as double ring system: the first ring contains diameter $2R$ and has possibility to rotate about axis O; second ring with the radius r, moves without sliding on the surface of the first ring. The non-sliding regime of the rings is arranged by introducing the tooth on the surface of the rings. The disbalance B is placed on the ring 2 at the point B.

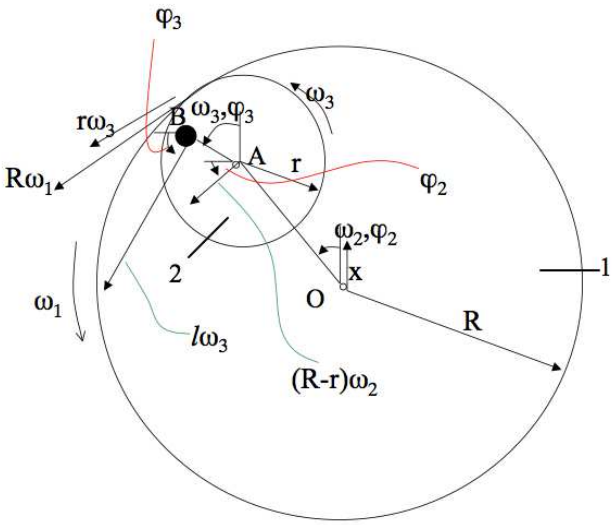

Fig. 1. Scheme of the planetary vibroexciter system: 1 – external ring; 2 – internal ring

Angular velocities can be expressed in the form:

$$\dot{\phi}_3 = \frac{R}{r}(\dot{\phi}_1 - \dot{\phi}_2) + \dot{\phi}_2 = \lambda(\dot{\phi}_1 - \dot{\phi}_2) + \dot{\phi}_2, \tag{1}$$

where φ_1 - rotational angle of the ring 1; φ_2 - rotational angle OA from horizontal line; φ_3 - disbalance rotational angle according to line OA; $\lambda = \frac{R}{r}$.

Kinetic energy of the system takes the following form:

$$\begin{aligned} T &= 0,5m_1\dot{x}^2 + 0,5I_1\dot{\phi}_1^2 + 0,5m_A\{[-(R-r)\dot{\phi}_2\cos(\phi_2)]^2 + \\ &+[\dot{x}-(R-r)\dot{\phi}_2\sin(\phi_2)]^2\} + 0,5I_2\dot{\phi}_3^2 + 0,5m_B\{[-l\dot{\phi}_3\cos\phi_3 - (R-r)\dot{\phi}_2\cos\phi_2]^2 + \\ &+[\dot{x}-(R-r)\dot{\phi}_2\sin(\phi_2) - l\dot{\phi}_3\sin\phi_3]^2\}, \end{aligned} \tag{2}$$

where x - displacement of the vibration exciter; I_1 - inertia moment of the ring1 according to point O; I_2 - inertia moment of the ring 2 according to point A (without disbalance mass); m_A - mass of the ring 2 (without disbalance mass); m_B ir l - disbalance mass and eccentric; m_1 - the mass of the casing.

From Eq. (2) and taking into account Eq. (1), kinetic energy T of the system takes the following form:

$$\begin{aligned} T &= 0,5(m_1 + m_A + m_B)\dot{x}^2 + 0,5(I_1 + \lambda^2 I_2 + m_B l^2\lambda^2)\dot{\phi}_1^2 + 0,5\{(1-\lambda)^2 I_2 + \\ &+(m_A + m_B)(R-r)^2 + (1-\lambda)m_B[l^2(1-\lambda) + 2(R-r)l\cos\lambda(\phi_1 - \phi_2)]\}\dot{\phi}_2^2 + \\ &+[\lambda(1-\lambda)I_2 + m_B l^2\lambda(1-\lambda) + m_B(R-r)l\lambda\cos\lambda(\phi_1-\phi_2)]\dot{\phi}_1\dot{\phi}_2 - \\ &-m_B l\lambda\dot{x}\dot{\phi}_1\sin[\lambda\phi_1 + (1-\lambda)\phi_2] - \{(m_A + m_B)(R-r)\sin\phi_2 + \\ &+m_B l(1-\lambda)\sin[\lambda\phi_1 + (1-\lambda)\phi_2]\}\dot{x}\dot{\phi}_2, \end{aligned} \tag{3}$$

Differential equations of motion, taking into account Eq. (3), acquire the form:

$$\begin{aligned}
&a_1\ddot{x} + c\dot{x} + kx = a_7\ddot{\phi}_1 \sin[\lambda\phi_1 + (1-\lambda)\phi_2] + \{a_4 \sin\phi_2 + 0,5a_5 \sin[\lambda\phi_1 + (1-\lambda)\phi_2]\}\ddot{\phi}_2 + \\
&+\{a_4 \cos\phi_2 + 0,5a_5(1-\lambda)\cos[\lambda\phi_1 + (1-\lambda)\phi_2]\}\dot{\phi}_2^2 + \\
&+a_7\lambda\dot{\phi}_1^2 \cos[\lambda\phi_1 + (1-\lambda)\phi_2] + [a_7(1-\lambda) + 0,5a_5\lambda]\cos[\lambda\phi_1 + (1-\lambda)\phi_2]\dot{\phi}_1\dot{\phi}_2, \\
&a_2\ddot{\phi}_1 = M_1(\dot{\phi}_1) + \lambda M_3(\dot{\phi}_3) + a_7\ddot{x}\sin[\lambda\phi_1 + (1-\lambda)\phi_2] - [a_6 + a_7(R-r)\cos\lambda(\phi_1 - \phi_2)]\ddot{\phi}_2 - \\
&-\dot{\phi}_2^2(a_7 + 0,5a_5)(R-r)\lambda\sin\lambda(\phi_1 - \phi_2), \\
&[a_3 + a_4(R-r)]\ddot{\phi}_2 = M_2(\dot{\phi}_2) - \lambda_1 M_3(\dot{\phi}_3) + \{a_4 \sin\phi_2 + 0,5a_5 \sin[\lambda\phi_1 + (1-\lambda)\phi_2]\}\ddot{x} + \\
&+0,5a_5(R-r)\lambda\dot{\phi}_2^2 \sin\lambda(\phi_1 - \phi_2) - a_5(R-r)\ddot{\phi}_2 \cos\lambda(\phi_1 - \phi_2) + \\
&+a_7(R-r)\lambda\dot{\phi}_1^2 \sin\lambda(\phi_1 - \phi_2) - [a_6 + a_7(R-r)\cos\lambda(\phi_1 - \phi_2)]\ddot{\phi}_1 + \\
&+a_5(R-r)\lambda\dot{\phi}_1\dot{\phi}_2 \sin\lambda(\phi_1 - \phi_2),
\end{aligned} \tag{4}$$

where $a_1 = m_1 + m_A + m_B$; $a_2 = I_1 + \lambda^2 I_2 + m_B l^2\lambda^2$; $a_3 = (1-\lambda)^2 I_2 + m_B l^2(1-\lambda)^2$; $a_4 = (m_A + m_B)(R-r)$; $a_5 = 2m_B l(1-\lambda)$; $a_6 = \lambda(1-\lambda)(I_2 + m_B l^2)$; $a_7 = m_B l\lambda$; c – coefficient of viscous friction; k – stiffness; $M_1(\dot{\varphi}_1)$, $M_2(\dot{\varphi}_2)$ and $M_3(\dot{\varphi}_3)$ – generalized moments according to the coordinates φ_1, φ_2 and φ_3 (driving and resistance forces) and these moments are linear functions:

$$\begin{aligned}
&M_1(\dot{\varphi}_1) = A - B\dot{\varphi}_1, \\
&M_2(\dot{\varphi}_2) = -K_2\dot{\varphi}_2, \\
&M_3(\dot{\varphi}_3) = -K_3\dot{\varphi}_3,
\end{aligned} \tag{5}$$

A, B, K_2, K_3 – constants.

In Eq. (4) new variables are introduced:

$$\begin{aligned}
&\alpha = \frac{x}{R},\ p^2 = \frac{k}{a_1},\ \nu = \frac{c}{2\sqrt{ka_1}},\ A_7 = \mu_B\lambda\lambda_2, \\
&\mu_B = \frac{m_B}{m_1 + m_A + m_B},\ \lambda_2 = \frac{l}{R},\ A_4 = (\mu_A + \mu_B)\lambda_1, \\
&\mu_A = \frac{m_A}{m_1 + m_A + m_B},\ \lambda_1 = \frac{R-r}{R},\ A_5 = 2\mu_B(1-\lambda)\lambda_2, \\
&A_2 = \frac{a_2}{a_1R^2},\ A_6 = \frac{a_6}{a_1R^2},\ A_3 = \frac{a_3}{a_1R^2}, \\
&a = \frac{A}{a_1R^2},\ b = \frac{B}{a_1R^2},\ k_2 = \frac{K_2}{a_1R^2},\ k_3 = \frac{K_3}{a_1R^2}.
\end{aligned} \tag{6}$$

Taking into account Eq. (4-6) the differential equations of motion take the form:

$$\begin{aligned}
&\ddot{\alpha} + 2\nu p\dot{\alpha} + p^2\alpha = F_\alpha, \\
&A_2\ddot{\varphi}_1 = \Phi_1, \\
&[A_3 + A_4\lambda_1]\ddot{\phi}_2 = \Phi_2,
\end{aligned} \tag{7}$$

where:

$$F_\alpha = A_7\ddot{\phi}_1 \sin[\lambda\phi_1 + (1-\lambda)\phi_2] + \{A_4 \sin\phi_2 + 0{,}5A_5 \sin[\lambda\phi_1 + (1-\lambda)\phi_2]\}\ddot{\phi}_2 +$$
$$+\{A_4 \cos\phi_2 + 0{,}5A_5(1-\lambda)\cos[\lambda\phi_1 + (1-\lambda)\phi_2]\}\dot{\phi}_2^2 +$$
$$+A_7\lambda\dot{\phi}_1^2 \cos[\lambda\phi_1 + (1-\lambda)\phi_2] + \{[A_7(1-\lambda) + 0{,}5A_5\lambda]\cos[\lambda\phi_1 + (1-\lambda)\phi_2]\}\dot{\phi}_1\dot{\phi}_2,$$
$$\Phi_1 = a - b\dot{\phi}_1 - k_3\lambda^2 - k_3\lambda(1-\lambda)\dot{\phi}_2 + A_7\ddot{\alpha}\sin[\lambda\phi_1 + (1-\lambda)\phi_2] -$$
$$-[A_6 + A_7\lambda_1 \cos\lambda(\phi_1 - \phi_2)]\ddot{\phi}_2 - \dot{\phi}_2^2(A_7 + 0{,}5A_5)\lambda_1\lambda \sin\lambda(\phi_1 - \phi_2),$$
$$\Phi_2 = -k_2\dot{\phi}_2 + k_3\lambda\lambda_1\dot{\phi}_1 + k_3\lambda_1(1-\lambda)\dot{\phi}_2 + \{A_4 \sin\phi_2 + 0{,}5A_5 \sin[\lambda\phi_1 + (1-\lambda)\phi_2]\}\ddot{\alpha} +$$
$$+0{,}5A_5\lambda_1\lambda\dot{\phi}_2^2 \sin\lambda(\phi_1 - \phi_2) - A_5\lambda_1\ddot{\phi}_2 \cos\lambda(\phi_1 - \phi_2) +$$
$$+A_7\lambda_1\lambda\dot{\phi}_1^2 \sin\lambda(\phi_1 - \phi_2) - [A_6 + A_7\lambda_1 \cos\lambda(\phi_1 - \phi_2)]\ddot{\phi}_1 +$$
$$+A_5\lambda_1\lambda\dot{\phi}_1\dot{\phi}_2 \sin\lambda(\phi_1 - \phi_2).$$

Analytical investigation of the effect of self-resonance

Steady state motion can be analyzed when the angular velocities of the ring 1 and 2 are average and different and

$$\varphi_1 = \omega_1 t, \quad (8)$$

where ω_1 – angular velocity of the ring 1.

It is assumed that the deflections of $\dot{\varphi}_2$ around the average value are small and therefore:

$$\Phi_2 = \varepsilon\Phi_2, \quad (9)$$

where ε is a fictitious small parameter, which is assumed to be equal to one at the end of the calculations.

Steady state regime of motion can be expressed in the form:

$$\alpha = \alpha_0 + \varepsilon\alpha_1 + \dots,$$
$$\varphi_2 = \varphi_{20} + \varepsilon\varphi_{21} + \dots, \quad (10)$$

where

$\alpha_0, \alpha_i, \varphi_{2i}$, i=1, 2, … – periodic functions of time.

From Eq. (7) and keeping in mind Eq. (9-10), the first approximation of the system of differential equations produces:

$$\phi_{20} = \omega_{20}t + C, \quad (11)$$

where ω_{20}, C – constants determined from approximation of higher orders.

Equation for calculation of α_0, taking into account Eq. (8) – (11), takes the form:

$$\ddot{\alpha}_0 + 2\nu p\dot{\alpha}_0 + p^2\alpha_0 = (\mu_A + \mu_B)\lambda_1\omega_2^2 \cos(\omega_2 t + C) +$$
$$+\mu_B\lambda_2[\lambda\omega_1 + (1-\lambda)\omega_2]^2 \cos\{[[\lambda\omega_1 + (1-\lambda)\omega_2]t + (1-\lambda)C\}. \quad (12)$$

From Eq. (12):

$$\alpha_0 = \frac{(\mu_A + \mu_B)\lambda_1\omega_2^2}{(p^2 - \omega_2^2)^2 + (2\nu p\omega_2)^2}[(p^2 - \omega_2^2)\cos(\omega_2 t + C) + 2\nu p\omega_2 \sin(\omega_2 t + C)] +$$
$$+\frac{\mu_B\lambda_2[\lambda\omega_1 + (1-\lambda)\omega_2]^2}{(p^2 - [\lambda\omega_1 + (1-\lambda)\omega_2]^2)^2 + (2\nu p[\lambda\omega_1 + (1-\lambda)\omega_2])^2}\times \quad (13)$$
$$\times\{(p^2 - [\lambda\omega_1 + (1-\lambda)\omega_2]^2)\cos\{[\lambda\omega_1 + (1-\lambda)\omega_2]t + (1-\lambda)C\} +$$
$$+2\nu p[\lambda\omega_1 + (1-\lambda)\omega_2]\sin\{[\lambda\omega_1 + (1-\lambda)\omega_2]t + (1-\lambda)C\}.$$

Periodicity of φ_{20} and Eq. (7), Eq. (10-13) lead to:

$$\bar{\Phi}_2 \Big|_{\substack{\alpha=\alpha_0 \\ \phi_2=\phi_{20}}} = \bar{\Phi}_2(\omega_2, C) , \tag{14}$$

where top line defines averaging by time *t.*

Assuming that $k\omega_1 \neq l\omega_2$, where *k*, *l* – integer numbers, equation (14) looks like:

$$F_1 = F_2 , \tag{15}$$

where:

$$F_1 = k_3\lambda\lambda_1\omega_1 - [k_2 - k_3\lambda_1(1-\lambda),$$
$$F_2 = \frac{0,5(\mu_A+\mu_B)^2\lambda\lambda_1\omega_2^5(2\nu p)}{(p^2-\omega_2^2)^2+(2\nu p\omega_2)^2} +$$
$$+\frac{0,5\mu_B^2\lambda_2(1-\lambda)[\lambda\omega_1+(1-\lambda)\omega_2]^5(2\nu p)}{(p^2-[\lambda\omega_1+(1-\lambda)\omega_2]^2)^2+(2\nu p[\lambda\omega_1+(1-\lambda)\omega_2])^2}.$$

The effect of self-resonance can exist if the equation (15) has a real solution in terms of ω_2. The graphical solution of this equation is presented in Fig. 2 when $\omega_1 = 38$, $p = 15$, $\nu = 0.15$, $\lambda = 5$, $\lambda_2 = 0.09$, $\mu_B = 0.01$, $\mu_A = 0.05$, $k_2 = 0.02$, $k_3 = 0.05$.

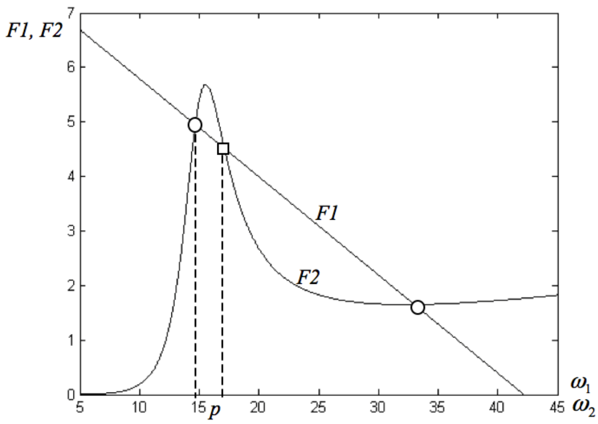

Fig. 2. The steady state motions: ○ – stable, □ – unstable

The formation of the linearized variational equations produces the conditions of stability of regime of self-resonance:

$$\frac{\partial F_1}{\partial \varpi_2} < \frac{\partial F_2}{\partial \varpi_2}. \tag{16}$$

The stable regime of motion near the resonance is described by approximate equation $\varpi_2 \approx p$. Thus the planetary vibroexciter with high speed motors can guarantee the non-linear dynamical effect when the generated self-resonance frequency automatically follows the natural frequency of the system.

The influence of system parameters to the steady state regime of operation is presented in Fig. 3.

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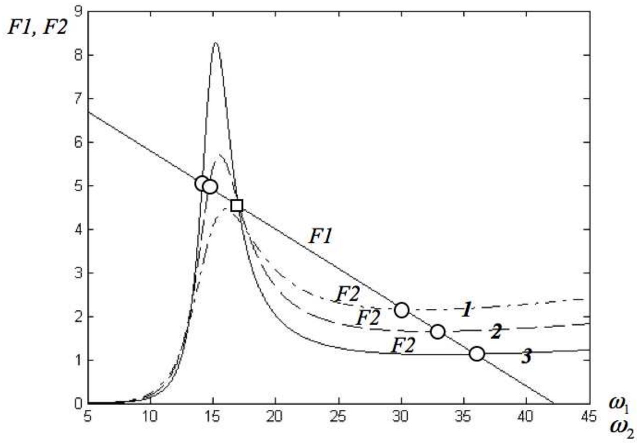

Fig. 3. The steady state motions: $1 - \nu = 0.1$; $2 - \nu = 0.15$; $3 - \nu = 0.2$; ○ – stable; □ – unstable

As shown in Fig. 3, effect of the self-resonance depends on systems parameters c, k, m_1, m_A, m_B and for example when $\nu = 0.1$ self-resonance phenomena do not occur.

Conclusions

The dynamics of planetary vibroexciters is investigated in the paper. It was determined that self-resonance mode can coexist with usual vibration modes at certain parameter values when the vibration exciter operates in automatic resonance mode. The conditions of existence and stability of regime of self-resonance in planetary vibroexciters is determined. Such vibration excitation systems have high practical value as there is no necessity for complex vibration control equipment – the stability of operation is guaranteed by non-linear dynamical interactions.

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