742. Time-varying systems identification using continuous wavelet analysis of free decay response signals

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Abstract. This paper proposes a time-varying approach used for identification of time-varying systems and presents a simulation example of a simple vibration system with time-varying mass, stiffness and damping characteristics, which is a five-storey shear-beam building model. Free decay acceleration response signals are analyzed to reveal time-varying nature of the system. Wavelet analysis is used for system identification. The method is based on a recently developed direct identification algorithm. Numerical results confirm that the proposed method is accurate and effective in identification of the time-varying system.

Keyword: time-varying, parameter identification, continuous wavelet, free decay response signals, instantaneous frequencies.

1. Introduction

Linear Time-Invariant (LTI) models are usually appropriate and commonly used to describe the dynamic behavior of most structural systems. Since the identification of dynamic parameters is of prime importance in vibration analysis, a number of different time and frequency domain methods have been developed for vibration analysis of LTI systems. However, many engineering structures and systems exhibit time-varying dynamic properties. This includes aerospace, automotive and civil structures. Therefore major research effort has been put recently to develop modeling and system identification techniques for Linear Time-Varying (LTV) systems. Various time-varying system identification methods - based on different signal processing techniques - have been proposed, such as: state-space identification algorithms [1-2], Continuous Wavelet Transform (CWT) techniques [3-6], Empirical Mode Decomposition (EMD) and Hilbert Transform (HT) based methods [7-9], adaptive tracking methods [10-11], and Functional Series Time-dependent Auto-Regressive Moving Average (FS-TARMA) methods [12-13].

More recently a new identification algorithm for time-invariant systems has been proposed [14]. The method is based on continuous wavelet analysis. The CWT can be used for functional integration to obtain velocities and displacements from acceleration responses. Then vibration differential equations of motion are transformed to linear algebraic equations with wavelet expressions. Finally, time-invariant system parameters (that is mass, stiffness and damping) and frequency response functions (FRF) are estimated directly by solving these algebraic equations for each moment of time.

The current paper implements and applies the method to time-varying systems to reveal time-varying systemic natures (time-dependent stiffness and damping parameters, and instantaneous frequencies). A simple model of a five-storey shear-beam building is simulated. The work presented illustrates the performance of the method with respect to noise contamination. The results are compared with analytical solutions to demonstrate the ability of the method for time-varying system identification.

2. Continuous wavelet transform

The continuous wavelet transform of a function of time $y(t)$ can be defined as following:

$$\{W_\psi y\}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} y(t)\overline{\psi(\frac{t-b}{a})}dt \tag{1}$$

where a is a scale parameter, typically a positive real number; b is a shift parameter, indicating locality of transformation; $\psi(t)$ is a mother wavelet function, and the overbar indicates complex conjugate. The notation $\{W_\psi y\}(a,b)$ indicates that the function $y(t)$ is mapped to the (a,b) plane by the wavelet transform with the mother wavelet function $\psi(t)$.

In this work, we assume that the mother wavelet function $\psi(t)$ satisfies the two following conditions:

1) the mother wavelet function has at least two vanishing moments, i.e.

$$\int_{-\infty}^{\infty} t^i \psi(t)dt = 0, \qquad i = 0,1 \tag{2}$$

2) the mother wavelet has the first and second integrals $\Psi_1(t)$ and $\Psi_2(t)$ decaying fast and vanishing at infinity, i.e.

$$\psi(\pm\infty) = \Psi_1(\pm\infty) = \Psi_2(\pm\infty) = 0 \tag{3}$$

3. Continuous wavelet transform algorithm for functional integration

Assuming that the first integral $Y_1(t)$ of the function $y(t)$ exists, the wavelet transform of this integral is defined as

$$\{W_\psi(\int y(t)dt)\}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \{Y_1(t) + y_0\} \cdot \overline{\psi(\frac{t-b}{a})}dt \tag{4}$$

where y_0 is a constant. Partial integration of the right-hand side of equation (4) leads to the following result

$$\int_{-\infty}^{\infty} \{Y_1(t) + y_0\}\overline{\psi(\frac{t-b}{a})}dt = \frac{a}{\sqrt{a}} \cdot \left[Y_1(t)\overline{\Psi_1(\frac{t-b}{a})} \right]_{-\infty}^{\infty} - \frac{a}{\sqrt{a}} \int_{-\infty}^{\infty} y(t)\overline{\Psi_1(\frac{t-b}{a})}dt$$

$$+ \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} y_0 \overline{\psi(\frac{t-b}{a})}dt \tag{5}$$

When conditions given by equations (2) and (3) are fulfilled, the first and third terms of the right-hand side in equation (5) are equal to zero. Therefore the CWT algorithm for functional integration can be used by applying the wavelet transform to the function $y(t)$ with $\Psi_1(t)$

used as the mother wavelet

$$\{W_{\psi}(\int y(t)dt)\}(a,b)=\{W(Y_1(t)+y_0)\}(a,b)=-\frac{a}{\sqrt{a}}\int_{-\infty}^{\infty}y(t)\overline{\Psi_1(\frac{t-b}{a})}dt$$

$$=-a\{W_{\Psi_1}y\}(a,b) \tag{6}$$

By analogy, double functional integration can be performed using the same approach, i.e. the CWT is applied to $y(t)$ and $\Psi_2(t)$ is used as the mother wavelet

$$\{W_{\psi}(\iint y(t)dtdt)\}(a,b)=\{W(Y_2(t)+y_1t+y_2)\}(a,b)=\frac{a^2}{\sqrt{a}}\int_{-\infty}^{\infty}y(t)\overline{\Psi_2(\frac{t-b}{a})}dt$$

$$=a^2\{W_{\Psi_2}y\}(a,b) \tag{7}$$

where $Y_2(t)$ is the doubled integral of $y(t)$; y_1 and y_2 are arbitrary constants.

The algorithm presented in this section has been originally proposed by Sone et al. [14] and used for identifying the dynamic parameters and Frequency Response Function of linear time-invariant systems. In this study, the algorithm is implemented and used for identification of time-varying systems; the CWT and acceleration response data are needed to achieve that.

4. Time-varying parameters identification

4. 1 Free vibration differential equations of linear time-varying system

The free vibration differential equations of motion for a p degrees-of-freedom (DOF) linear time-varying system can be defined as:

$$\mathbf{M}(t)\ddot{\mathbf{x}}(t)+\mathbf{E}(t)\dot{\mathbf{x}}(t)+\mathbf{K}(t)\mathbf{x}(t)=0 \tag{8}$$

where $\mathbf{M}(t)$, $\mathbf{E}(t)$ and $\mathbf{K}(t)$ are ($p\times p$) time-varying mass, damping and stiffness matrices respectively; $\mathbf{x}(t)$ is ($p\times 1$) displacement vectors. The velocity vector $\dot{\mathbf{x}}(t)$ and the displacement vector $\mathbf{x}(t)$ can be obtained using functional integration of $\ddot{\mathbf{x}}(t)$ acceleration data

$$\dot{\mathrm{x}}(t)=\int_0^t\ddot{\mathrm{x}}(t)dt+\dot{\mathrm{x}}(0) \tag{9}$$

$$\mathrm{x}(t)=\int_0^t\dot{\mathrm{x}}(t)dt+\mathrm{x}(0) \tag{10}$$

where $\mathbf{x}(0)$, $\dot{\mathbf{x}}(0)$ and $\ddot{\mathbf{x}}(0)$ are constant vectors determined by initial conditions.

4. 2 Time-varying parameters identification method

When the mother wavelet function $\psi(t)$ is selected to satisfy both conditions given by equations (2) and (3), equations (9) and (10) can be substituted into the equation of motion

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given by Equation (8). Without loss of generality, assuming that system mass coefficients ($\mathbf{M}(t)$ matrix) are time invariant or their time-varying rules are known experimentally. Assuming the system $\mathbf{E}(t)$ and $\mathbf{K}(t)$ matrices are approximately constant in a very short time, then applying the CWT to the response signals of equation (8), and using equations (6) and (7), the following equation can be obtained

$$\mathbf{M}(b)\{W_{\psi}\ddot{\mathbf{x}}\}(a,b) - a\mathbf{E}(b)\{W_{\Psi_1}\ddot{\mathbf{x}}\}(a,b) + a^2\mathbf{K}(b)\{W_{\Psi_2}\ddot{\mathbf{x}}\}(a,b) = 0 \quad (11)$$

Rewriting the above equation leads to

$$-a\mathbf{E}(b)\{W_{\Psi_1}\ddot{\mathbf{x}}\}(a,b) + a^2\mathbf{K}(b)\{W_{\Psi_2}\ddot{\mathbf{x}}\}(a,b) = -\mathbf{M}(b)\{W_{\psi}\ddot{\mathbf{x}}\}(a,b) \quad (12)$$

or in a matrix form:

$$\begin{bmatrix}\mathbf{E}(b) & \mathbf{K}(b)\end{bmatrix} \cdot \begin{bmatrix} -a\{W_{\Psi_1}\ddot{\mathbf{x}}\} \\ a^2\{W_{\Psi_2}\ddot{\mathbf{x}}\} \end{bmatrix} = -\mathbf{M}(b)\{W_{\psi}\ddot{\mathbf{x}}\}(a,b) \quad (13)$$

Equation (12) or (13) represents a set of linear algebraic equations given for different time instants. Under the assumption that system mass properties are known in advance, and by considering a sliding time window of proper length (in order to compromise parameter tracking with achievable accuracy), an over-determined set of equations may be set up and solved in a linear least squares in order to obtain estimates of the system parameters (stiffness and damping) corresponding to the central time instant of the sliding time window. Instantaneous frequency estimates may be subsequently obtained by solving the eigenvalue problem.

5. Numerical simulations

Numerical simulations are performed to validate the identification algorithm presented in Section 4 and to demonstrate the capability of the method for time-varying system identification. A five-storey shear-beam building model is shown in Figure 1. The mass coefficients are assumed as $m_1 = m_2 = m_3 = m_4 = m_5 = 5$ kg, the stiffness coefficients and damper coefficients are $K_1 = K_2 = K_3 = K_4 = K_5 = 80000$ N/m and $E_1 = E_2 = E_3 = E_4 = E_5 = 3$ N·s/m respectively. The first five natural frequencies of the system are 5.7 Hz, 16.7 Hz, 26.4 Hz, 33.9 Hz and 38.6 Hz. Free vibration responses data are calculated using Newmark-beta method. All initial displacements and velocities condition are set to zeros but the initial values of accelerations are assumed as 5000 m/s^2. The free decay acceleration signals are sampled at $f_s = 1000$ Hz and the duration of the signals is two seconds (T = 2s).

The Mexican hat mother wavelet function is used in the wavelet-based identification algorithm. The Mexican hat wavelet function can be defined as

$$\psi(t) = (-t^4 + 6t^2 - 3)e^{-0.5t^2} \quad (14)$$

It is easy to check that this function satisfies equations (2) and (3) needed for the performed analysis. The identification algorithm utilizes the following values of wavelet scale and shift parameter $a = 2^{0.01\cdot j}$, $b = 2^{0.01\cdot j}(16k+8)$, where $j = -680$ and $k = 0:0.001:12.999$. To estimate the time-varying system physical parameters and instantaneous frequency, the acceleration response signals of all five masses are needed. In this example 1000 samples are used for setting up the linear least squares problem.

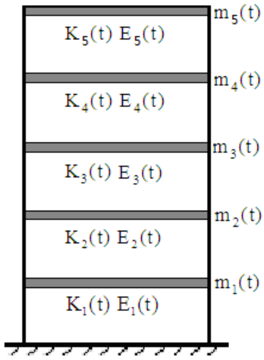

Fig. 1. A five-storey shear-beam building model

The identification algorithm, presented in Section 4, is applied to reveal the time-varying nature of the system. Three time-varying cases, i.e., the abruptly, smoothly and periodically varying scenarios, are studied to verify the ability and robustness of the identification algorithm.

The estimated physical parameters and instantaneous frequencies are shown in the following figures extracted for the LTV building model. The results demonstrate that all physical parameters and natural frequencies can be tracked relatively well using the acceleration responses.

Case 1: A linear time-varying system with abruptly varying damping and stiffness.

$m_5(t) = 5 - 0.2t$ known in advance.

$E_3(t) = E_5(t) = \begin{cases} 3 & t < 1.1s \\ 4 & t \geq 1.1s \end{cases}$; $K_2(t) = K_4(t) = \begin{cases} 80000 & t < 1.1s \\ 50000 & t \geq 1.1s \end{cases}$, the other mass, stiffness and damping coefficients are assumed as constant at their initial values.

Figures 2 - 3 illustrate the identification of abruptly varying stiffness and damping (dotted lines), which are compared with analytical solutions (solid lines). The estimated instantaneous frequencies are shown in Figure 4 and 5. Results indicate that the proposed identification algorithm has a good capability of tracking parametric abrupt changes of the LTV system.

It is easy to see the abrupt jump in the physical parameters and instantaneous frequencies of the system can be tracked using the proposed identification algorithm using free vibration acceleration response data. The estimated results with the dotted line shown in Figure 3 reveal an abrupt change in the stiffness from 80000 to 50000 N/m, which matches with the corresponding true changes denoted with the solid line at $t = 1.1s$. However, it is noted that there exists larger identification error at the time instance when the system stiffness has an abrupt change. This indicates that the proposed method has poor capability in tracking abrupt variations due to the assumption that the system exhibits the same time-varying properties during the time intervals in the development of the parameter identification equations in Section 4.

Case 2: A linear time-varying system with smoothly varying damping and stiffness.

$m_5(t) = 5 - 0.2t$ known in advance; $E_2(t) = 3 + 0.5t$; $K_5(t) = 80000 - 20000t$, the other mass, stiffness and damping coefficients are assumed as constant at their initial values.

Figures 6 and 7 provide the identification of smoothly varying damping E_2 and stiffness K_5 (dotted lines), which are compared with analytical solutions (solid lines). The estimated instantaneous frequencies are shown in Figure 8 and 9. Results demonstrate that the proposed identification algorithm has a good capability of tracking the parametric smooth changes of the LTV system.

Case 3: A linear time-varying system with periodically varying damping and stiffness.

$E_2(t) = 3 - 0.5\cos\pi t$; $K_4(t) = 80000 - 30000\cos\pi t$, the other mass, stiffness and damping coefficients are assumed as constant at their initial values.

Figures 10 - 11 show the identified results for the periodically time-varying damping coefficient E_2 and stiffness coefficients K_4 respectively. It can be observed that the dotted lines (the identification values) are found varying around the solid lines (the true values) very well. The proposed method is shown to be capable of tracking the periodical change of stiffness during the whole time duration. It is noted that there is a little phase shift between the estimated result and the corresponding true value in Figure 10. This phase shift is due to some time instants used for calculation at each discrete time instant. It is noted that the instantaneous frequencies cannot be correctly identified for the beginning and end of the analyzed time records in Figure 12 and 13. The proposed algorithm has no capability of tracking the system parameter variations there due to the edge effects related to the CWT calculations.

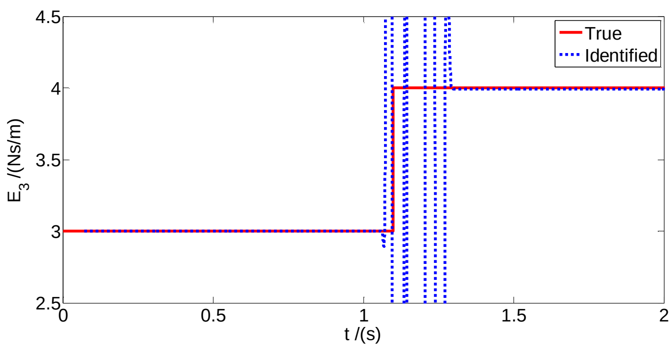

Fig. 2. Comparison of true value and the identified damping coefficient E_3 for the abruptly time-varying structure

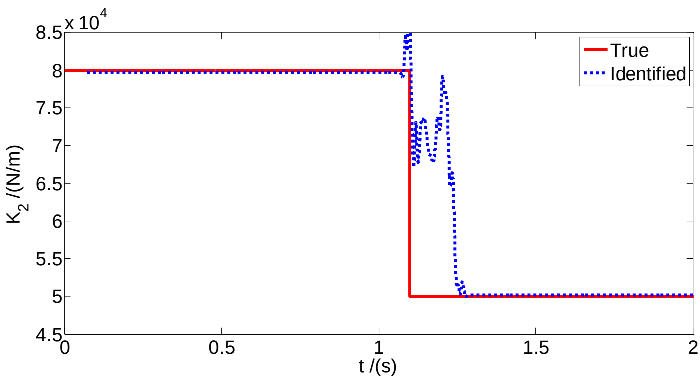

Fig. 3. Comparison of true value and the identified stiffness coefficient K_2 for the abruptly time-varying structure

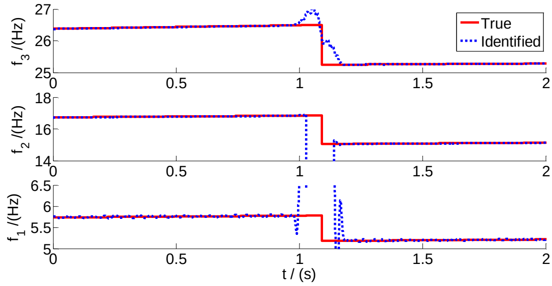

Fig. 4. Comparison of true value and the identified instantaneous frequencies for the abruptly time-varying structure

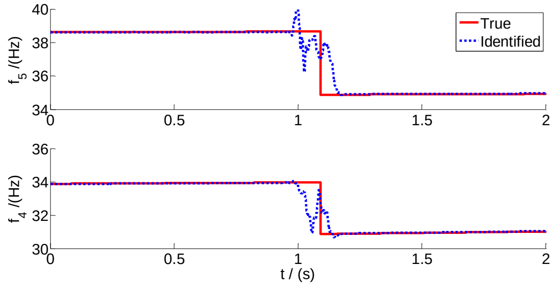

Fig. 5. Comparison of true value and the identified instantaneous frequencies for the abruptly time-varying structure

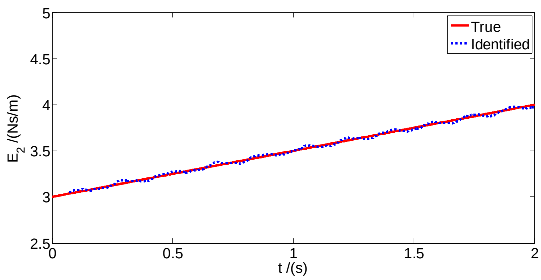

Fig. 6. Comparison of true value and the identified damping coefficient E_2 for the smoothly time-varying structure

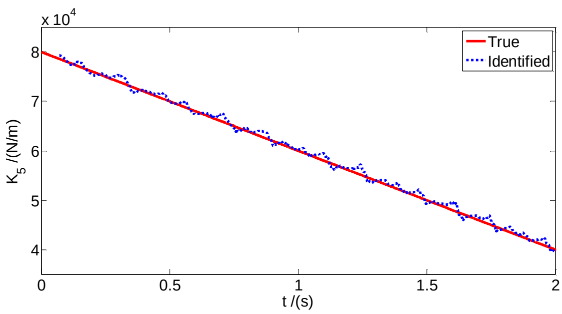

Fig. 7. Comparison of true value and the identified stiffness coefficient K_5 for the smoothly time-varying structure

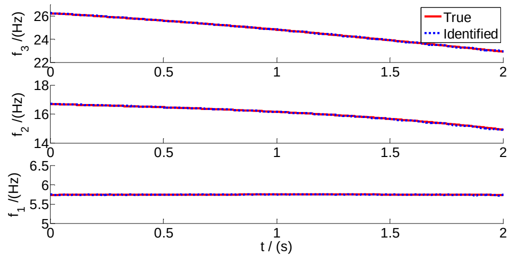

Fig. 8. Comparison of true value and the identified instantaneous frequencies for the smoothly time-varying structure

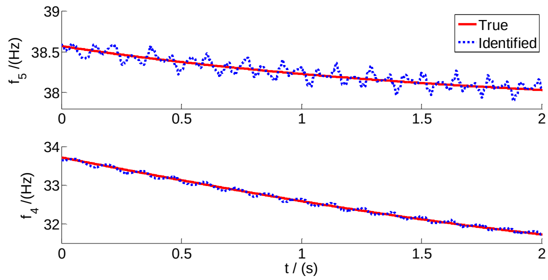

Fig. 9. Comparison of true value and the identified instantaneous frequencies for the smoothly time-varying structure

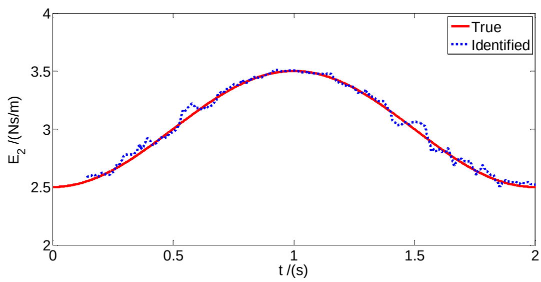

Fig. 10. Comparison of true value and the identified damping coefficient E_2 for the periodically time-varying structure

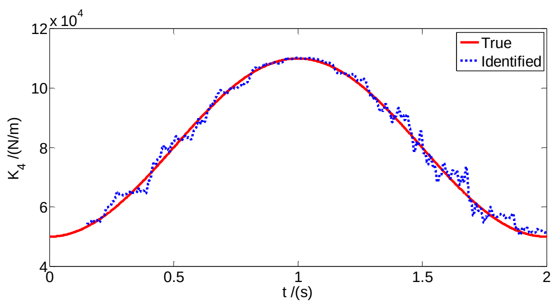

Fig. 11. Comparison of true value and the identified stiffness coefficient K_4 for the periodically time-varying structure

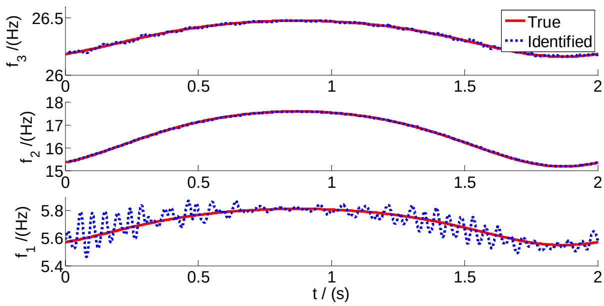

Fig. 12. Comparison of true value and the identified instantaneous frequencies for the periodically time-varying structure

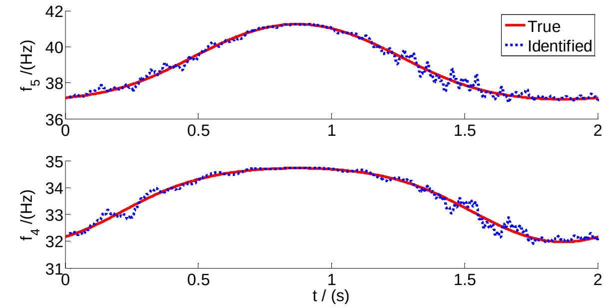

Fig. 13. Comparison of true value and the identified instantaneous frequencies for the periodically time-varying structure

In order to assess the capability of the method for system identification under noisy conditions, the response signals are contaminated with the zero-mean Gaussian white noise. The signal-to-noise ratio (SNR) is defined as:

$$\mathrm{SNR} = Srs/Sn \tag{15}$$

where Srs is the "pseudo" (aggregate over the time duration) standard deviation of the original response signal and Sn the "pseudo" standard deviation of the added noise. The simulated responses are calculated and their Srs levels estimated. The level of Sn is then determined from a given value of SNR. A standard Gaussian white noise with zero mean and a unit standard deviation is then generated, multiplied with the value Sn and added the response data in order to produce contaminated responses. The analyzed SNR values are equal to 100, 90, 80, 50 and 30.

Tables 1 and 2 compare the true (analytical) and identified instantaneous frequencies for different noise levels. The Mean Absolute Percentage Error (MAPE) defined as:

$$\mathrm{MAPE} = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|P_i - \hat{P}_i\right|}{P_i}\times 100\% \tag{16}$$

is used to assess the performance of the method. P_i, $\hat{P}_i$ designate the true and identified natural frequency, respectively, at the $i-$th time instant and N the total number of time instants. The results in Tables 1 and 2 demonstrate good performance of the method with respect to the considered noise levels. All five instantaneous natural frequencies are identified correctly; the estimated MAPE error in all the cases is smaller than 4.5%.

Table 1. Errors of identification (MAPE) using noisy data (different SNR values) with smoothly varying parameters

SNR	f_1 (%)	f_2 (%)	f_3 (%)	f_4 (%)	f_5 (%)
Without noise	0.066	0.840	1.139	0.788	0.585
100	0.791	0.838	1.141	0.792	0.605
90	0.974	0.840	1.138	0.786	0.615
80	4.249	2.057	1.139	0.784	0.622
50	4.440	1.832	1.139	0.791	0.642
30	4.497	1.591	1.136	0.808	0.747

Table 2. Errors of identification (MAPE) using noisy data (different SNR values) with periodically varying parameters

SNR	f_1 (%)	f_2 (%)	f_3 (%)	f_4 (%)	f_5 (%)
Without noise	0.873	2.072	0.244	1.219	1.588
100	1.314	2.075	0.243	1.223	1.598
90	1.329	2.069	0.245	1.228	1.601
80	1.414	2.068	0.246	1.218	1.585
50	2.963	2.451	0.246	1.229	1.620
30	3.626	2.116	0.248	1.240	1.687

6. Conclusions

A linear time-varying system identification algorithm was postulated using a CWT-based integration procedure that transforms the differential equations of motion into linear algebraic equations. Under the assumption of the systemic mass properties are know in advance, varying system parameters (stiffness and damping) are extracted by solving the linear algebraic equations via a Linear Least Squares procedure within a sliding time window and the instantaneous frequencies are subsequently identified by solving the eigenvalue problem.

The performance of the method was illustrated using a simulated LTV five-storey shear-beam building model. The results demonstrated good performance of the identification method. Time-varying stiffness and damper parameters and instantaneous frequencies were established correctly for various noise levels.

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