696. Dynamic flexibility of the supported-clamped beam in transportation

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Abstract. The paper concerns the problem of vibrations of the beam in rotational transportation. The beam is the supported-clamped one. The beam is fixed on a rotational disk. The disk is treated as the rigid one. The method of dynamical flexibility is used for the dynamic analysis of the system. The beam is considered in terms of local vibrations transferred to the global reference frame where the interaction between the local displacement and the transportation movement is taken into consideration. The analyzed system can be treated as the model of many technical systems such as blades of pumps, rotors etc. Nowadays such a type systems are very rarely considered with taking into account the so called transportation effect. The analyzed case is the particular one, where the special boundary conditions are applied.

Keywords: dynamic analysis, vibrations, beamlike systems, transportation effect, supported-clamped system, forms of vibrations.

Introduction

The problem of supported-clamped vibrating beam is considered in this work. The paper is a part of series works concerning problems of vibratory beams in transportation. The paper concerns the well-known problem in the literature [1-9], but the presented model is the specific case of the beam fixed in a rotational disk. In considered model the local vibrations of the beam are in relation with the main motion. There are many technical applications where the beams fixed on the rotation disk are implemented. For instance the systems can be put into practice in many types of turbines, pumps or rotors. This analysis can be also used for an analysis of complex systems where one of the components of such a complex system is an analyzed beam. In this paper the method used for dynamic analysis is the dynamic flexibility one. This method is the one of the very popular ways of analyzing dynamics of systems. The dynamic flexibility is used for the analysis of the beam systems in rotational motion and gives an opportunity to specify the stability or instability zones. These zones are very important to control such a type of systems, taking into account the optimizing for the sake of the minimal amplitude of local and global vibrations criterion. In this way it is also possible to derive the modes of vibrations and zeros of the dynamic characteristics. Many publications in the literature concern the subject area of vibrating systems in motion as distinguished from the ones concerning stationary systems. These aspects are the reasons for widening of the dynamic analysis [1-9]. As a starting point of the dynamic flexibility, derivation algorithm of the mathematical model is assumed in the form of equations of motion. Considerations are done by the Galerkin's method. There are beamlike systems in rotational motion considered, treated in this paper as the main working motion. Considered motion is limited to the plane one. The dynamic characteristics in the form of dynamic flexibility as function of frequency and mathematical models are presented in this work.

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Modelling of the vibratory beam on a rotational disk

A model of the supported-clamped homogeneous beam is considered in this section (Fig. 1). The beam is supported at one end and clamped at the second one. The beam is fixed on a rotational disk with the support as a point of mounting to the disk. The rigid disk rotates with the angular velocity ω. The system is described in the two reference frames. The local vibrations are transferred to the global reference frame. The beam has the well-defined geometric parameters: the symmetric cross-section, the given external dimensions, the given geometric momentum and material parameters such as a material type, the Young's modulus, a mass density. It is assumed that the beam is supported at one end (for $x = 0$) and clamped at the second end (for $x = l$) (Fig. 1). The analyzed cross-section of the beam is loaded by a harmonic force with the unitary amplitude in the direction perpendicular to the centre line of the beam. The forces and the moments of forces at the ends of the beam are assumed as equal zero and also the displacements are equal zero.

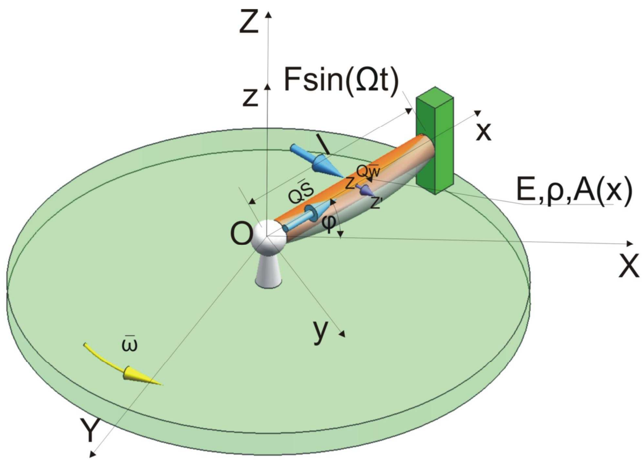


Fig. 1. Analyzed system, where: ρ – mass-density, $A(x)$ – cross-section, l – length of the beam, x – location of the analyzed cross-section, ω – angular velocity, Ω – frequency, **Q** – rotation matrix, **S** – position vector, **w** – vector of displacement, F – harmonic excitation force, E – Young modulus

In Fig. 1 the analyzed beam model – the supported-clamped one is presented.

Forms of vibrations

For deriving the forms of vibrations the boundary condition for the beam should be written as follows:

$$\begin{cases} w(0,t)=0,\ EI\dfrac{\partial w^2(0,t)}{\partial x^2}=0,\ w(l,t)=0,\ \dfrac{\partial w(l,t)}{\partial x}=0, \\ \dfrac{\partial}{\partial x}\left[EI\dfrac{\partial w^2(s,t)}{\partial x^2}\right]+2\int_0^l F_0\delta(x-0,5l)W(t)\,dx=0, \quad s\in(0,l), \end{cases} \tag{1}$$

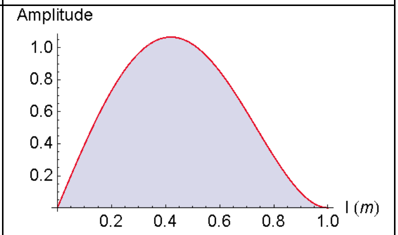
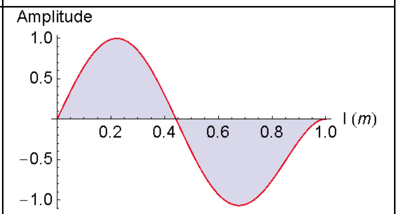
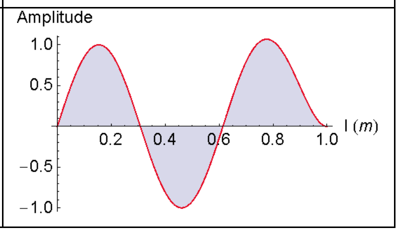
in every time moment $t \geq 0$. After solving the boundary problem there can be derived the eigenfunction of displacement in the form:

$$X(x) = \sin kx - \sinh kx \frac{\sin kl}{\sinh kl} \cong \sin kx - \sinh kx \frac{\cos kl}{\cosh kl}, \tag{2}$$

where: $k \cong \frac{4n+1}{4l}\pi,$ (3)

where: n is a mode of vibrations of the supported-clamped beam.

Table 1. Forms of vibrations of the supported-clamped beam

n	Exact calculated eigenvalues k	Eigenvalues approximated with the formulae (3)	Rounded relative error	The form of vibrations
1	3,92660231	3,92699082	-0,01%	
2	7,06858275	7,06858347	0,00%	
3	10,2101761	10,2101761	0,00%	

In Table 1 the three modes of vibrations for the supported-clamped beam are presented. The estimated relative error between the calculated exact eigenvalues and the approximated eigenvalues is very small and can be neglected in further calculations. The charts of forms of vibrations for two presented eigenfunctions of displacement are concurrent.

Mathematical model – equations of motion

The kinetic energy in accordance with the Koenig's law defined by the generalized coordinates and the generalized velocities is:

$$T=\frac{1}{2}M\mathbf{Q}\left[\bar{\boldsymbol{\omega}}\times\left(\bar{\mathbf{S}}+\bar{\mathbf{w}}\right)\right]^T\left[\bar{\boldsymbol{\omega}}\times\left(\bar{\mathbf{S}}+\bar{\mathbf{w}}\right)\right]+\frac{1}{2}M\mathbf{Q}\dot{\bar{\mathbf{w}}}^T\dot{\bar{\mathbf{w}}}=$$
$$=\frac{1}{2}M\left(\bar{\mathbf{i}}\dot{r}_X\right)^2+\frac{1}{2}M\left(\bar{\mathbf{j}}\dot{r}_Y\right)^2=\frac{1}{2}M\dot{r}_X^2+\frac{1}{2}M\dot{r}_Y^2, \tag{4}$$

where $\bar{\mathbf{i}}$, $\bar{\mathbf{j}}$, $\bar{\mathbf{k}}$ are versors in the global reference frame.

The equations of motion of the non-damped beam in transportation are derived in the matrix form. This system of equations of motion is the fourth order partial differential equations and in each point of the range $D=\left\{(x,t),x\in(0,l),t>0\right\}$ coincides with the boundary conditions and the initial conditions. It can be expressed as follows:

$$\rho A\begin{bmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ \dfrac{\partial^2 w}{\partial t^2}\\ 0\end{bmatrix}-\rho A\begin{bmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\omega^2 s\\ \omega^2 w\\ 0\end{bmatrix}+$$
$$-2\rho A\begin{bmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\omega\dfrac{\partial w}{\partial t}\\ 0\\ 0\end{bmatrix}+\rho A\begin{bmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-\dot{\omega}w\\ \dot{\omega}s\\ 0\end{bmatrix}= \tag{5}$$
$$=-EI\begin{bmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ \dfrac{\partial^4 w}{\partial x^4}\\ 0\end{bmatrix}.$$

Every periodic motion can be expressed as complex motion compound from series of harmonic motions. The description of displacements in the global reference frame is as follows:

$${}_1s_X=w_X=\sum_{n=1}^{\infty}A_X X(x,n)e^{j\Omega t}, \tag{6}$$

$${}_1s_Y=w_Y=\sum_{n=1}^{\infty}A_Y X(x,n)e^{j\Omega t}, \tag{7}$$

where A_X and A_Y are the searched amplitudes, $X(x)$ is the eigenfunction for displacement, Ω is the frequency and j is the imaginary unit, n for displacement, Ω is the frequency and j is the imaginary unit.

In accordance to the definition, the mathematical form of the modulus of dynamic flexibility of the considered systems can be obtained as:

$$|Y|=\frac{2}{\rho A\gamma_n^2}\sum_{n=1}^{\infty}\frac{X(l)X(x)}{\rho A\gamma_n^2\sqrt{a^4k^8+\left(\omega^2-\Omega^2\right)^2-2a^2k^4\left(\omega^2+\Omega^2\right)}}. \tag{8}$$

If the angular velocity of the rotational disk equals zero then the dynamic flexibility is:

$$|Y|=\frac{2}{\rho A\gamma_n^2}\sum_{n=1}^{\infty}\frac{X(l)X(x)}{a^2k^4-\Omega^2}. \tag{9}$$

Where the norm equals:

$$\gamma_n^2 = \int_0^l X^2(x)\,dx = \\ = \frac{kl - 3\sin(2kl) + 6\cos^2(kl)\tanh(kl) + kl\left[\tanh^2(kl) - \cos(2kl)\,\mathrm{sech}^2(kl)\right]}{4k}. \tag{10}$$

The derived (9) dynamic flexibility is the same as the dynamic flexibility of the stationary beams.

Numerical examples

Numerical examples in the form of dynamic characteristics are presented. In Figures 2-3 the samples of dynamic flexibilities are presented in the chart form. Figure 2 presents the attenuation-frequency characteristic, the dynamic flexibility both in the function of the frequency and the angular velocity (the transportation velocity) is presented.

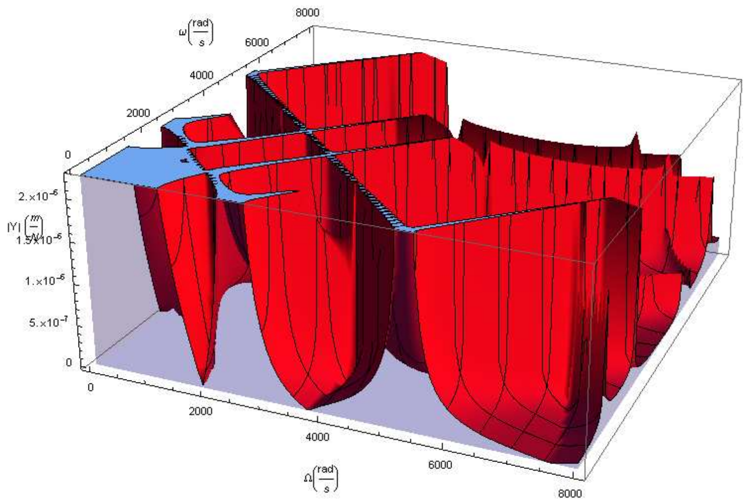


Fig. 2. Dynamic characteristic of dynamic flexibility of the beam in relation to angular velocity and frequency

Presented characteristics make also possible analysis of the influence of the angular velocity treated as the transportation velocity on the modes bifurcation process. In figure 3 the bottom view of the characteristic from figure 2 is presented. In figure 3 the top of the characteristic (Fig. 2) is presented. After assuming the angular acceleration equals zero the relation between angular velocity and the modes of the dynamic flexibility is the linear function.

Conclusions

This paper is the consideration of the vibration problem of beams fixed on a rotational rigid disk. The beam was located onto the rotational disk that rotates with a constant angular velocity. The beam moves in terms of the plane motion and the model presented here makes possible to consider local and global vibrations, taking into consideration the transportation effect (acting of Coriolis and centrifugal forces). The way of modelling of supported-clamped vibrating beam in transportation was presented in this abstract. The model considers mutual relations between the main motion treated as the transportation and the local displacements treated as vibrations. The dynamic flexibility formula is presented as well and the solution is proposed as the sum of the eigenfunctions products. The presented model can be used for dynamic analysis of simple

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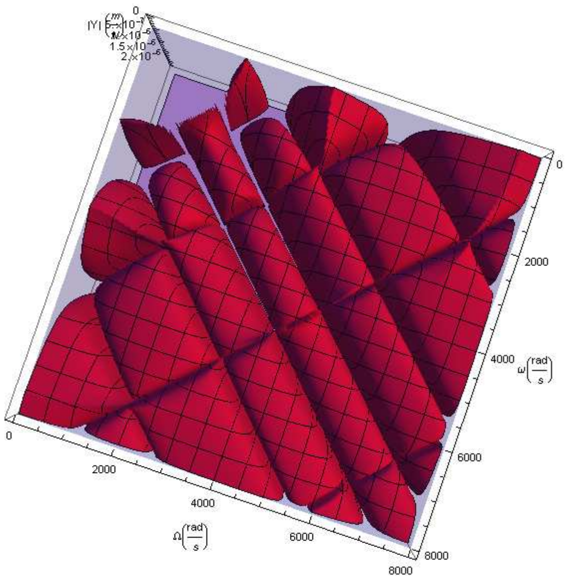
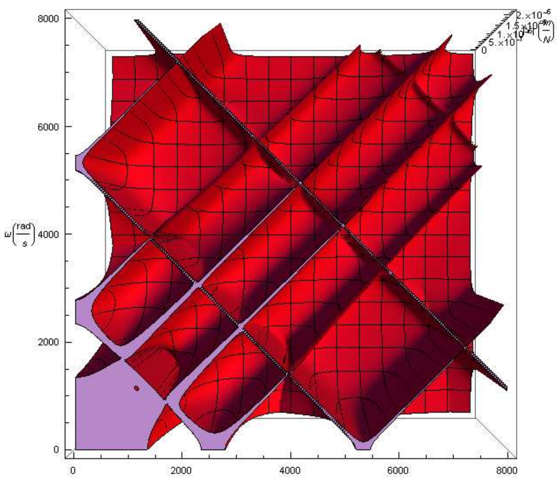
beamlike systems with the specific boundary conditions. The numerical examples are presented in this work. The derived mathematical formulae (8) makes possible to determine the dynamic flexibility for different working parameters. In future works the damping forces analysis will be also provided and the analysis of systems with geometrical and physical nonlinearities.



a)



b)

Fig. 3. a) Bottom view of the characteristic (Fig. 2). b) Relation between angular velocity and modes of dynamic flexibility

Acknowledgements

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