689. Analysis of acoustic field in the cylindrical shape with membrane

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Abstract. Directional characteristics of the acoustic field in a cylindrical shape with membrane are investigation in this paper. The main objective of the study is to determine the impulse response of the sound sources for calculating acoustic parameters in a closed cylindrical space. The obtained results demonstrate that the proposed analytical model enables evaluation of the sound field in the cylinder-shaped space with membrane.

Keywords: acoustic field, analytical method, estimation, cylindrical shape, membrane.

Introduction

A model for sound wave propagation in a cylindrical shape with heterogeneous medium, for example, a membrane in the pipe leads to more or less efficient method for solving the wave equation. In particular for noise propagation, the work [1] contains many useful solutions of wave equations applicable to specific technical sound insulation tasks of cylindrical pipes and shell using Bessel and Hankel functions. It is possible to solve wave equation using the boundary element method (BEM) and determine an approximate solution of the system of equations resulting from discretizing the surfaces into patches. The second possibility is to use the finite element method (FEM), where the wave equation is being solved by dividing the enclosure into the elements [2]. Then the wave equation is expressed by the discrete set of linear equations for these elements. On the other hand, FEM also allows modeling energy transmission between the separate surfaces. The third possibility is to describe the sound field in a cylindrical shape by sound particles moving around along sound rays. Such a geometrical model is using the simulation of sound in large rooms, for example the Ray tracing method and the Image source method. Ray tracing methods [3] find propagation paths between a source and receiver by generating rays emanating from the source position and following them individually as they propagate through the environment. Although this method is very general and simple to implement, it is subject to aliasing artifacts as the space of rays is sampled discretely. For instance, receiver position and diffracting edges are often approximated by volumes of space (in order to admit intersections with infinitely thin rays), which can lead to false hits and paths counted multiple times. More often, important propagation paths may be missed by all samples. Image source methods compute specular reflection paths by considering virtual sources generated by mirroring the location of the sound source over each surface of the environment. This method is simple for rectangular shape [4]. However, for every new receiver location, each of the virtual sources must be checked to see if it is visible to the receiver, since the specular reflection path might be blocked by a polygon or intersect a mirroring plane outside the polygon [5].

This paper is intended to present a new possibility to describe the propagation of sound in the cylindrical shape with membrane and using analytical method for the calculation of relative displacement [6] of air points under the action of the sound source in a known place in the cylindrical shape.

Investigation

First task is to construct a model of a cylindrical shape with membrane, for example, shown in Fig. 1.

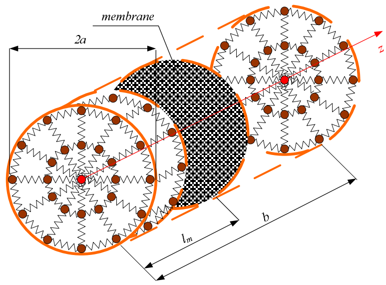

Fig. 1. Model of air in a cylindrical shape with membrane

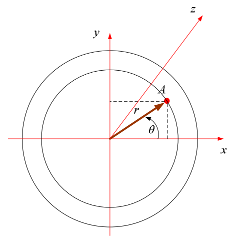

Fig. 2. Displacement counting diagram

Air particles are present within the cylindrical shape. They are represented as circles and a distance between particles is half of a wavelength. The motion of the particles is relative with respect to the frame. A membrane is located at a distance l_m from cylinder beginning. The number and location of particles depend on known frequency ν_k and speed of sound in air c, because λ_k – known wavelength:

$$\lambda_k = \frac{c}{\nu_k} \tag{1}$$

We shall choose a system of axes for derivation of the mathematical model and displacements (see Fig. 2). The analytical model of cylindrical acoustic field is based on the mathematical model derived by the calculation of relative displacements of particles of air under the action of sound source in the cylindrical system of axes [6]. The series of calculations are shown schematically in Fig. 3.

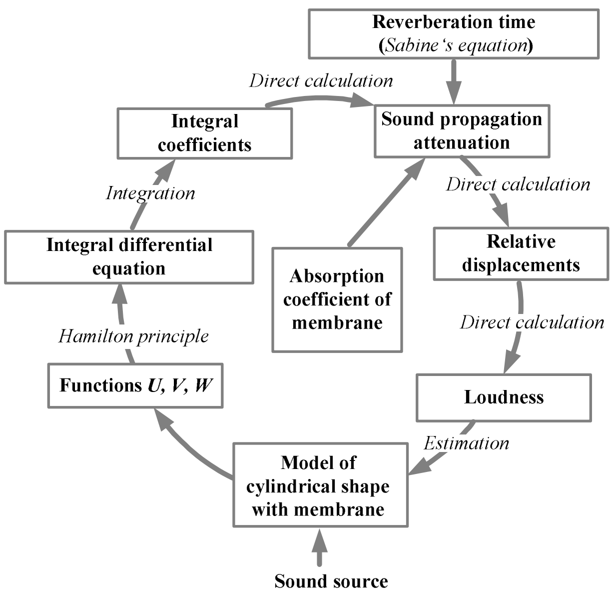

Fig. 3. Cycle of the analytical model

The mathematical model is obtained from Hamiltonian principle and giving the following equation:

$$m_o \frac{dq}{dt} = -k_o \int_0^{\tau} q dt + \int_0^{\tau} P dt \ , \tag{2}$$

where:

$$m_o = \rho_o \ddot{q} \iiint\limits_{(V)} \left(U^2 + V^2 + W^2 \right) r dr d\theta dz \ , \tag{3}$$

$$k_o = \frac{\rho_o}{c^2} \iiint\limits_{(V)} \left[U_r^2 + \left(V_\theta + \frac{U}{r} \right)^2 + W_z^{\ 2} \right] r dr d\theta dz \tag{4}$$

$$P = \iint\limits_{(S)} \left(\bar{X}U + \bar{Y}V + \bar{Z}W \right) dS \ . \tag{5}$$

$$\tau_i = \frac{1}{4\nu_i} \ . \tag{6}$$

$$U = U\left(r, \theta, z\right); \quad V = V\left(r, \theta, z\right); \quad W = W\left(r, \theta, z\right); \quad q = q\left(t\right) \tag{7}$$

$$\frac{\partial U}{\partial r} = U_r \ , \ \frac{\partial V}{r \partial \theta} = V_\theta \ , \ \frac{\partial W}{\partial z} = W_z \ , \ \frac{\partial U}{\partial z} = U_z \tag{8}$$

$$u = Uq \ ; \quad v = Vq \ ; \ w = Wq, \tag{9}$$

where ρ_o is density of air; c is speed of sound in air; $\bar{X}, \bar{Y}, \bar{Z}$ is projections of external surface force (line unit is subjected to that force) on coordinate axes. In the case of sound source, taking into account that the pressure of sound source is the same in all directions $(\bar{X} = \bar{Y} = \bar{Z})$:

$$I = \int_0^{\tau} \bar{X} dt = \int_0^{\tau} \bar{Y} dt = \int_0^{\tau} \bar{Z} dt \ . \tag{10}$$

$$\int_0^{\tau} P dt = IL = I \iint\limits_{(S)} (U + V + W) dS \tag{11}$$

Equation (9) can be solved approximately by means of an iteration method:

$$q = \frac{LI}{m_o} t - \frac{k_o LI}{6 m_o^2} t^3 + \frac{k_o^2 LI}{120 m_o^3} t^5 - \frac{k_o^3 LI}{5040 m_o^4} t^7 + \frac{k_o^4}{362880 m_o^4} t^9 \tag{12}$$

Knowing m_o, k_o, L and taking into account that sound intensity (pressure) is inversely proportional to the square distance from the point of sound source, we can calculate an approximate value q according to the equations (12). Finally, having applied equation (9), we can calculate approximate relative displacements of air particles.

The functions U, V and W are selected on the basis of the boundary conditions, i.e. it should fit for the cylindrical shape presented in Fig. 1. The duration of the sound source equals reverberation time T by Sabine's reverberation formula [7]:

$$T = 0.161 \frac{V}{A} \tag{13}$$

where V is the volume of cylindrical shape in cubic meters; A is the total absorption in square meters.

In addition, to determine the influence of the membrane in the cylindrical shape for sound radiation we can use the absorption coefficient α . Sound waves more or less are absorbed by a membrane. We can suggest using the multiplier φ and the following calculation technique, for example, where the membrane is located at $z = z_m$:

$$\varphi = 1 - \alpha \tag{14}$$

$$\int_0^\tau P dt = IL^* = I \iint_{(S_1)} (U + V + W) dS + \varphi I \iint_{(S_2)} (U + V + W) dS \tag{15}$$

$$q^* = \frac{L^* I}{m_o} t - \frac{k_o L^* I}{6 m_o^2} t^3 + \frac{k_o^2 L^* I}{120 m_o^3} t^5 - \frac{k_o^3 L^* I}{5040 m_o^4} t^7 + \frac{k_o^4}{362880 m_o^4} t^9 \tag{16}$$

$$u^* = Uq^* ; \quad v^* = Vq^* , \quad w^* = Wq^* \tag{17}$$

where:

S_1 – integration area before the membrane ($z < z_m$);

S_2 – integration area behind the membrane ($z > z_m$).

Finally, taking into account relationships of acoustic quantities associated with a plane progressive acoustic sound wave [8], loudness at any point of the cylindrical shape is calculated.

The problem simulated numerically is sketched in Fig. 1. For example, the geometrical values of the pipe are a=0.5 m, b=10 m and the membrane of 12 mm acoustic belt with the absorption coefficient α =0.5 is located in l_m=5 m. Let's suppose that density of air $\rho_o = 1.224$ kg/m^3, speed of sound in air $c = 343$ m/s. The functions U, V and W are selected on the basis of the boundary conditions, i.e. it should fit for the cylindrical shape presented in Fig. 1:

$$U = \frac{1}{a^3}\left(a^2 - x^2\right)\left(k_1 a \cos\theta - k_2 x \sin\theta + z\right) \tag{18}$$

$$V = \frac{1}{a^3}\left(a^2 - x^2\right)\left(k_2 a \cos\theta - k_1 x \sin\theta + z\right) \tag{19}$$

$$W = \frac{1}{a^3}\left(b - z\right)^2 \left(a \cos\theta - x \sin\theta + k_3 z\right) \tag{20}$$

and parameters for calculation of equations (2) – (20) are shown in Table 1.

Fig. 4 and 5 illustrate fragments of the calculation results for the acoustics field of frequency $\nu = 4000$ Hz when the sound source is located at the center of pipe at z=0 and the acoustic field is evaluated at z=6 m. Fig. 3 illustrates the fragments of acoustics field distribution in the pipe without membrane and Fig. 4 – with membrane in l_m=5 m in same distance from sound source z=6 m.

Table 1. Values of parameters

Parameters	Value
k_1	-34.5784
k_2	-11.9927
k_3	0.0562014
m_o	252763.344
k_o	0.753595063
φ	0.5

The obtained results for the acoustics field indicate that sound pressure in the pipe with the installed membrane is by several decibels lower in comparison to the pipe without the membrane. These numerical results may be compared to the experimental findings presented in [9].

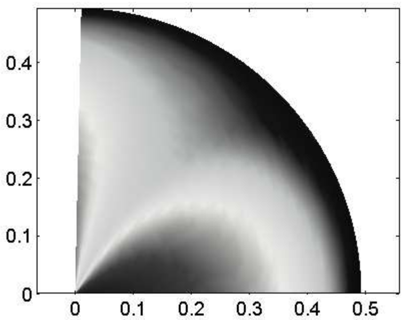

Fig. 4. The fragment of acoustic field in the pipe without membrane

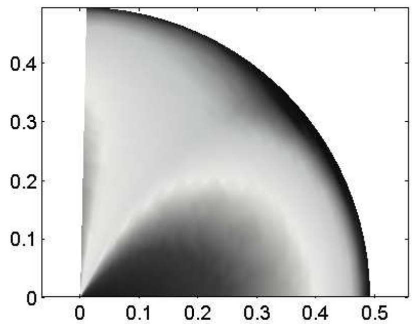

Fig. 5. The fragment of acoustic field in the pipe with membrane

Conclusions

The proposed and developed analytical method allows the analysis of sound field in the cylindrical shape with membrane. The analytical method enables:

1. to calculate approximately the displacements of air particles under sound impact at a specific location in the cylindrical shape with membrane;
2. to create a precondition for operation of the acoustic field in an enclosure taking into account a membrane positioned at a specific place of the cylindrical shape.

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