681. Design of elastomeric shock absorbers with a “soft” stiffness characteristics of type “force-settlement”

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Abstract. In the design of vibration isolator it is often required to ensure a reduction or increase of the value of frequency η_r, which is associated with the ability to calculate the stiffness characteristics of elastomeric vibration isolator in the final low- and medium-level deformations. A similar problem arises in the design of equi-frequent rubber-metal compensating devices, which find application in various fields of engineering and construction industries, effectively replacing the hydro/pneumo-spring compensating devices, working under axial stress-strain. In this case, the stiffness characteristic of “force-settlement” $P = P(\Delta)$, even for small final deformations, will be non-linear (or piecewise linear). In this paper we propose a method for determination of rigidity dependence of “force-settlement” for shock-absorbing elements with absolutely rigid moving (parallel to the vertical axis z) vertical side stops being under pressure, which allows to take into account low compressibility of material of rubber layers. Obtained solutions can be used for establishing the dependence “force-settlement” for cylindrical shock absorbers during their design stage. Thereby it is possible to design a shock absorber with a given non-linear (“hard” or “soft”) stiffness characteristics.

Keywords: rubber, shock-absorber, weak compressibility, side stops, stiffness.

Introduction

Creating a high-speed vehicles, an increase in hardware power accompanied by an increase in the intensity and range of vibration of engineering structures to reduce the harmfulness of widely used rubber-metal vibration isolators of different geometry, with unquestionable advantages [2] to vibration isolators from other materials. In the theory of vibration the isolation attenuation coefficient transfer power from the source of vibration through the vibration isolators on the base is introduced for the frequency $\eta > 2^{0,5}\ \eta_r$, where η_r – mass resonance frequency of vibration source [1]:

$$\eta_r = \frac{(c/M)^{0.5}}{2\pi} \tag{1}$$

where: c - stiffness of vibration isolator; M - mass of mechanism of vibration source.

The main task of the design of vibration isolator is providing basic reduction of the resonance frequency below η_r frequency range vibro-active disturbing forces. At the same time static and dynamic displacement vibration source shall not exceed the permissible values.

For below resonance modes:

$$\eta_r = \frac{(g/\Delta_0)^{0.5}}{2\pi} \tag{2}$$

where: g - acceleration of gravity; Δ_0 – static settlement of the isolator elastic elements, if we assume that the stiffness of the isolator does not depend on the frequency and the load value.

From (2) it is necessary, what only for small deformations increase permissible settlement /Δ_0/ decreases the fundamental frequency η_r (since there is a linear relationship for the rigidity characteristics of the “force – settlement” elastomeric isolator [1, 4]. For large deformations stiffening behavior “force – settlement” of elastomeric vibration isolator is significantly non-linear dependence of the hard type and from a formula (2) it is necessary, what the frequency η_r

may again increase. For elastomeric vibration isolator minimum attainable frequency η_r, depending on the physical and mechanical properties of elastomeric material, is [2]: unreinforced elastomers - 8 Hz, rubber-metal elements at work on compression - 5 Hz, rubber-metal elements at work on a shift - 3 Hz. In the design of vibration isolator it is often required to ensure a reduction (or increase) of the value of frequency η_r, which is connected with the ability to calculate the stiffness characteristics of elastomeric vibration isolator in the final low- and medium-level deformations. A similar problem arises in the design and calculation of equi-frequent rubber-metal compensating devices, which find application in various fields of engineering and construction industries, effectively replacing the hydro/pneumatic-spring compensating device, working under axial stress-strain. In this case, the stiffness characteristic of "force - settlement" $P = P(\Delta)$, even for small final deformations, will be non-linear (or piecewise linear). To solve this problem for the function $P = P(\Delta)$, in the general case, it is necessary to design shock absorber ensuring the condition:

$$\frac{dP(\Delta)}{d\Delta}\cdot\frac{1}{P(\Delta)} = f(P(\Delta)) \tag{3}$$

For the special case f($P(\Delta)$) = const = C, then from equation (3) we obtain the solution for $P(\Delta)$:

$$P(\Delta) = B\cdot\exp(C\cdot\Delta) \tag{4}$$

The constant of integration B is determined from additional conditions. For example, if shock absorber is multi-layered, an additional condition can be minimization of weight of elastomeric material, providing the required initial settlement Δ_0 of shock absorber. In this case, from (4):

$$B = P(\Delta_0)\cdot\exp(-C\cdot\Delta_0) \tag{5}$$

and to write down the shock absorber stiffness characteristics $P(\Delta)$:

$$P(\Delta) = P(\Delta_0)\cdot\exp\left[C(\Delta-\Delta_0)\right] \tag{6}$$

will provide the desired balance between rigidity and the level of shock absorber loading.

If constant $C = \rho^2/g$ (where: ρ - natural frequency of equipment that is installed with the shock absorber; g - acceleration of gravity), the equation (6) describes the stiffness characteristics $P(\Delta)$, which provides a description of equi-frequent characteristic of compensative shock absorber. Several design solutions are possible. In [1, 5] for one of the features (cylindrical shock absorbers with fixed absolutely rigid side stops) the technique of calculation of such shocks absorbers is presented. In all of value (4) can be implemented using special characteristics of $P(\Delta)$, which provides a choice of a particular geometry of the shock absorber. Several design solutions may be proposed. Most often it is desirable to use shock absorbers with absolutely rigid side stops. In this paper we propose a design shock absorber with rigid side boards which can move parallel to the direction of the force compressing the shock absorber. Since, due to the lateral movement stops changing free surface of elastomeric layer in a shock absorber, the stiffness and shock absorber changes. In contrast to the shock absorbers with fixed side stops (fixed stops help to implement only the increase of stiffness in the process of loading), the moving side stops make it possible to increase or decrease stiffness (from initial stiffness) in the process of loading it. Stiffness of shock absorber can vary from hard shock (free lateral surface of the elastomeric layer of shock absorber is zero, the precipitate is only due to the weak compressibility of the elastomer) to the stiffness of the shock absorber with no side stops (height of the free elastomeric layer of a shock absorber is the height of the elastomeric layer). In this case there can be three versions of moving the side supports. In the first version, the side stops moving at the time when the shock absorber is not loaded (Fig. 1). In this case we obtain a linear stiffness characteristic "force-settlement".

In this case we obtain a linear stiffness characteristic "force-settlement" (Fig. 2).

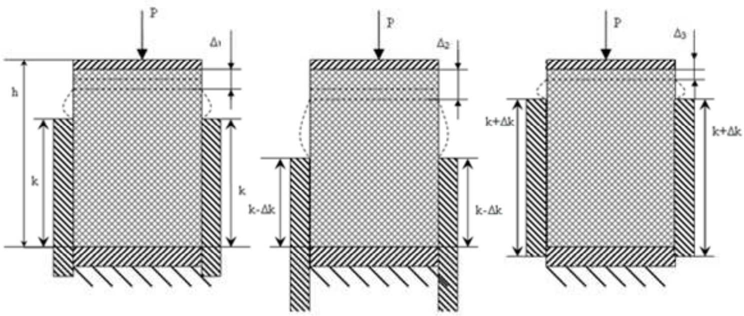

Fig. 1. Shock absorber with moving side stops

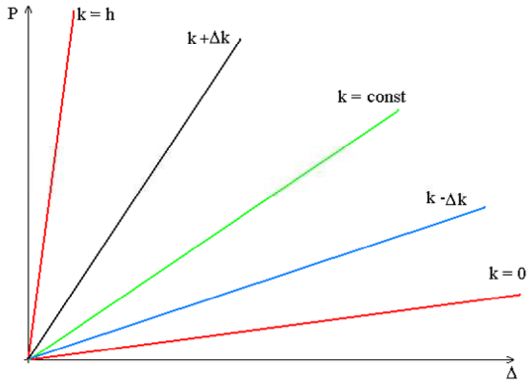

Fig. 2. Dependence "force – settlement" for shock absorber with moving side stops. First version

In the second version the side stops moving step by step depending on the loading of the shock absorber (Fig. 3).

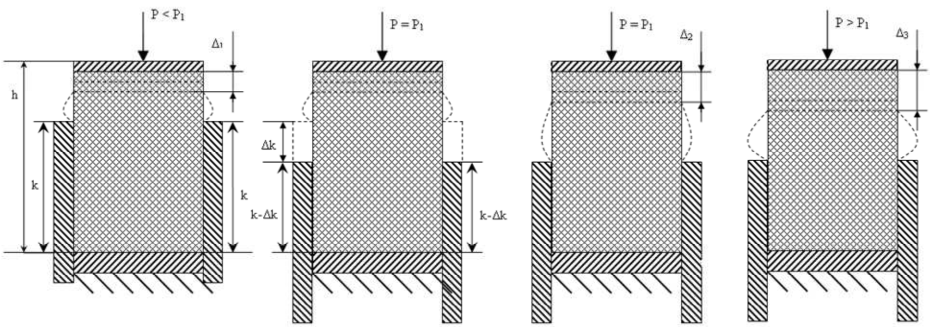

Fig. 3. Shock absorber with moving side stops. Side stops move "step by step"

In this case, a piecewise nonlinear stiffness characteristics "force-settlement" is shown in Fig. 4. "Jumps" along the axis of settlement correspond to the time change of height of the side stops at this point force is constant, but by decreasing the free surface of the elastomeric layer settlement increases.

In the third version the height of the side stops does not change continuously, depending on the load (Fig. 5). In this case, a piecewise nonlinear stiffness characteristics "force-settlement" is shown in Fig. 5.

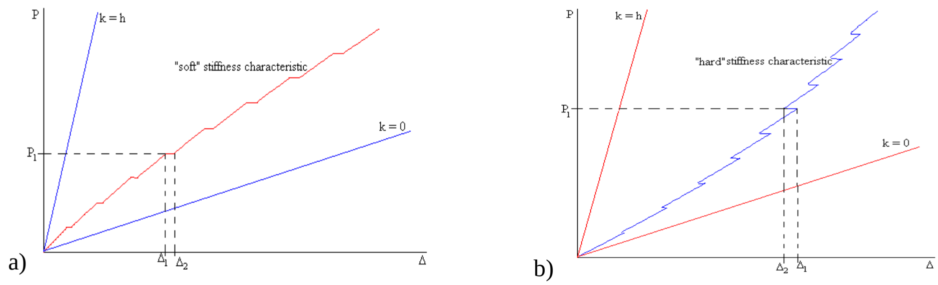

Fig. 4. Dependence "force – settlement" for shock absorber with moving side stops. Second version: (a) – "soft" stiffness characteristic, (b) – "hard" stiffness characteristic

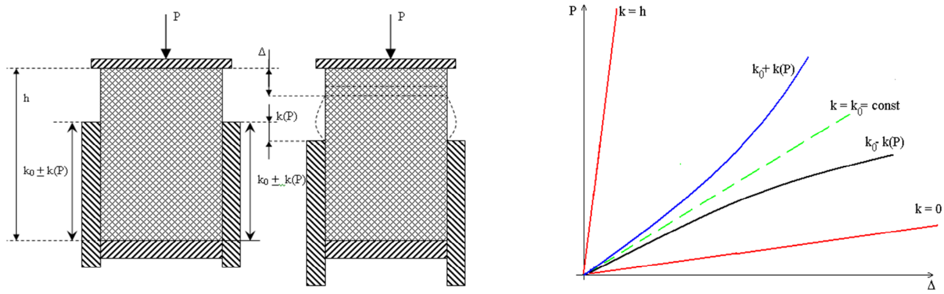

Fig. 5. Dependence "force – settlement" for shock absorber with moving side stops. Third version

To obtain the analytic dependence of P (Δ) "force-settlement" for small finite strains (up to 10-15%) the shock absorber is divided into two parts: Part I $((h - k(P)) \leq z \leq h)$, we have an axi-symmetric compression in Part II $(0 \leq z \leq (h - k(P))$ volume compression. The solution is obtained by taking into account the weak compressibility of the elastomer.

For the first part of the shock absorber the dependence of the "force – settlement" $P(\Delta)$ in the light of weak compressibility of rubber is found by using the principle of minimum total potential energy of deformation $U(u_1,w_1)$ [1, 2]:

$$U(u_1,w_1) = J(u_1,w_1) - P\Delta_1 \tag{7}$$

where:

$$J = 2\pi G \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^b \left[\begin{matrix} \left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{2}\left(\left(\frac{\partial u}{\partial z}\right) + \left(\frac{\partial w}{\partial r}\right)\right)^2 + \\ + \frac{3\mu}{1+\mu} s \left(\left(\frac{\partial u}{\partial r}\right) + \frac{u}{r} + \left(\frac{\partial w}{\partial z}\right)\right) - \frac{9\mu(1-2\mu)}{4(1+\mu)^2} s^2 \end{matrix} \right] r\,dr\,dz - P\Delta_1 \tag{8}$$

where: G - is modulus of rigidity, μ - is Poisson's ratio.

Choosing the displacement functions $u_1(r, z)$, $w_1(r, z)$, respectively, along the axes r and z:

$$u_1 = A_1 r \left(1 - \frac{z}{h}\right)\left(1 - \frac{z}{h^*}\right), \quad h^* = h - k(P)$$

$$w_1 = -\frac{3\Delta_1}{2h^*\left(1 + 2\frac{h}{h^*}\right)}\left(z - \frac{z^2(h+h^*)}{hh^*}\right) + \frac{z^3}{3hh^*} - \frac{4h^{*2}}{3h} \tag{9}$$

that satisfy the geometric boundary conditions:

$$u_1(r,h) = 0, \qquad u_1(r,\ h - k(P)) = 0 \tag{10}$$

$w_1(r, h) = -\Delta_1, \; w_1(r, h - k(P)) = 0$

From the principle of minimizing the functional $U(u_1, w_1)$ the dependence of the "force – settlement" $P(\Delta_1)$:

$$\Delta_1 = \frac{P \cdot (h - k(P))}{\pi b^2 G} \left[1{,}8 + \frac{1{,}2 + 1{,}5\alpha^2}{\left(1 + 3\frac{1-2\mu}{2\mu}\alpha^2\right)} \right]^{-1} \quad \alpha = b/(h - k(P)) \tag{11}$$

For the second part of the shock absorber, assuming that the mobility of the side constraint is absolutely rigid, the dependence $P(\Delta_{II})$, without taking into account the frictional forces on the contact surface elastomer-metal, is defined as in the case volumetric compression [1]:

$$\Delta_{II} = \frac{3Ph}{2\pi b^2 G} \cdot \frac{(1 - 2\mu)}{(1 + \mu)} \tag{12}$$

For all the shock absorber, the dependence of "force – settlement" $P(\Delta)$, with (9) and (10) takes the form:

$$\Delta = \Delta_1 + \Delta_{II} \tag{13}$$

When using the dependences (9) - (13), note that they obtained for small finite deformations, when:

$$0 \le \Delta_1 / (h - k(P)) \le 0{,}1 \div 0{,}15 \tag{14}$$

The thickness of the free (without side boards) elastomeric layer h_0 ($P = 0$) in the process of loading can vary:

$$0 \le h_0(P = 0) = h - k(P = 0) \le h \tag{15}$$

Based on the requirements of exploitation the shock absorber can vary, the thickness of the elastomeric layer is free depending on the size of loading, or can be given any requirement to change the stiffness characteristics of the shock absorber. For the two algorithms the condition of smallness of final deformation must be satisfied (11). The most simple to constructing dependence of "force - settlement", if you specify the intervals for changing the load P and the thickness of the free layer of elastomeric shock absorber. In this case, the algorithm for constructing the function $P(\Delta)$ will be the following. Let the interval values of the compressive force P is split into N (not necessarily equal) intervals and set requirements for the change in the thickness of the free elastomeric layer, that is a function $k(P)$ for each interval of the force P. For *i-th* interval:

$$P_{i-1} \le P_i \le P_{i-1} + P_i^* \tag{16}$$

here: P_i^*- step of force P (if these steps are the same, $P_i^* = P / N$).

The thickness of the free (without side boards) elastomeric layer to the *i-th* interval of loading takes values:

$$h_{i-1}(P = P_{i-1}) \le h(P = P_i) = h(P = P_{i-1}) - k(P = P_i) \le h \tag{17}$$

The total settlement $\Delta_{\Sigma i}$ shock absorber on the *i-th* step of loading is given by:

$$\Delta_{\Sigma i} = \Delta_{\Sigma(i-1)} + \left(\Delta_I\left(h_0 - k_i(P_i), P_i^*\right) + \Delta_{II}(k_i(P_i), P_i^*)\right) \tag{18}$$

where the values of Δ_I $((h_0 - k_i(P_i), P_i^*)$ and $\Delta_{II}(k_i(P_i), P_i^*)$ calculated by formulas (11) and (12) and substituting the values of the required thickness of the free elastomeric layer is at each stage of loading. From (8) - (14) that if $k(P = P_i)$ is negative, the thickness of the elastomeric layer is free to grow and shock absorber stiffness will decrease, that is, the dependence of the "force – settlement" will be of "soft" type. For positive values of $k(P = P_i)$ elastomeric layer thickness of free will decreases and damping increases, that is dependence of the "force – settlement" will be of "hard" type.

Example 1 ("Soft" characteristic "force - settlement")

As an example, consider a step change in stiffness of the shock absorber. Parameters of the shock absorber: b = 5 cm, h = 6 cm, μ = 0.495, G = 7 kg/cm^2, k_0 = 0.5h, $k = k_0 - k(P)$

$k(P)$ = if 0 N < P < 200 N, then k = 3 cm
if 200 N < P < 400 N, then k = 2.75 cm
if 400 N < P < 600 N, then k = 2.50 cm
if 600 N < P < 800 N, then k = 2.25 cm
if .... , then ....
if 2000 N < P < 2200 N, then k = 0

The dependence of "force-settlement" for cylindrical shock absorber with moving side stop calculated by formula (13). Results are shown in Fig. 6.

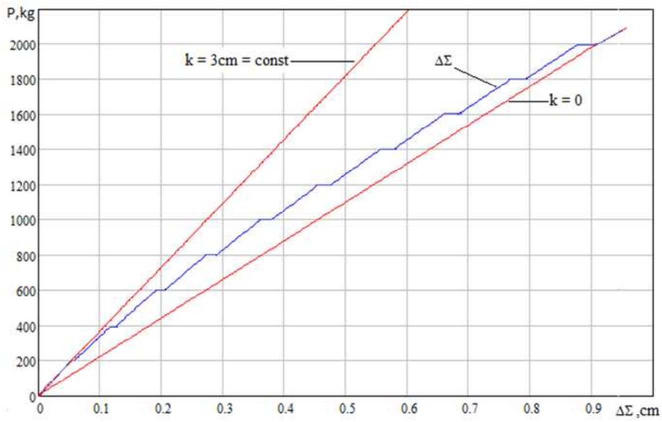

Fig. 6. "Soft" characteristic "force - settlement" for cylindrical shock absorber

As can be seen at the graph (Fig. 6) the stiffness of the shock absorber varies from 4000 N/cm to 2200 N/cm. "Jumps" along the axis of settlement correspond to the time change of height of the side stops at this point force is constant, but by decreasing the free surface of the elastomeric layer the settlement increases.

Example 2 ("hard" characteristic "force - settlement")

Parameters of the shock absorber:
b = 5 cm, h = 6 cm, μ = 0.495, G = 7 kg/cm^2, k_0 = 0.5h, $k = k_0 + k(P)$

$k(p)$ = if 0 N < P < 200 N, then k = 3 cm
if 200 N < P < 400 N, then k = 3.1 cm
if 400 N < P < 600 N, then k = 3.2 cm
if 600 N < P < 800 N, then k = 3.3 cm
if .... , then ....
if 2000 N < P < 2200 N, then k = 4 cm

The dependence of "force-settlement" for cylindrical shock absorber with moving side stop is calculated by formula (10). Results are shown in Fig. 7. As can be seen from the graph (Fig. 7) stiffness of shock absorber varies from 4000 N/cm to 5700 N/cm.

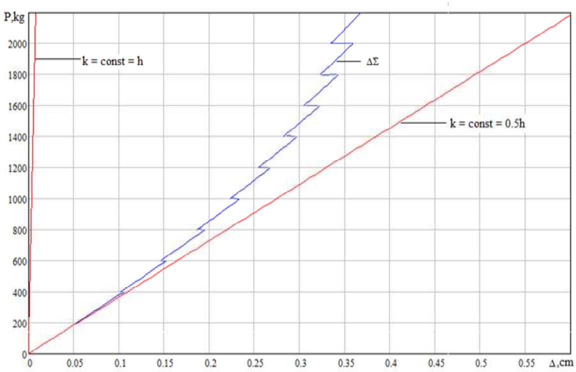

Fig. 7. “Hard” characteristic “force - settlement” for cylindrical shock absorber

Conclusion

In this paper we proposed a method for determination of rigidity dependence "force-settlement" for shock-absorbing elements with absolutely rigid moving (parallel to the vertical axis *z*) vertical side stops being under pressure. It allows to take into account low compressibility of the material of rubber layers. Obtained solutions can be used to determine the dependence "force-settlement" for cylindrical shock absorbers, as well as in designing of such shock absorbers. It enables to design a shock absorber with a given non-linear (“hard” or “soft”) stiffness characteristic. As an example the technique of designing a rubber shock absorber with variable stiffness is considered (the first example of a “soft” stiffness, the second with “hard” stiffness). The height of the side stops depends on the applied force and changes gradually in a “step-by-step” manner. The results are presented graphically.

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