675. The vibrations of microbeams and nanotubes

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(Received 1 September 2011; accepted 4 December 2011)

Abstract. Temperature variations can significantly change the dynamic characteristics of macro-, micro- and nano-structures. In the presented article we have studied the microbeams and nanotubes under thermal effects. Microbeams and nanotubes will be very important in the future in the fields of MEMS and NEMS. For the physical explanation of vibrations of nanotubes classical mechanics is valid with some limitations. We have taken into account the influence of thermal force, axial force in rotating shaft and also gyroscopic effect. The effect of temperature-dependent material properties was considered primarily with respect to the temperature variations. On the basis of our analytical model it is possible to determine the vibrational characteristics in a very wide range of temperatures. In the presented paper it is shown for the first time in scientific literature the combined influence of temperature, gyroscopic effects and rotor speeds on shaft and beam vibrations.

Keywords: vibrations of microbeams, vibrations of nanotubes, rotor vibration, thermomechanics, nanomechanics.

Introduction - the difference between vibration of beams, rotors, microbeams and nanotubes

The vibrations of beams and microbeams are of vital importance in mechanical engineering. Machines very often operate under diverse thermal conditions. In internal combustion engines, rocket systems, movement of the satellites, MEMS and NEMS the conditions are particularly temperature-sensitive. Thermodynamic effects are frequently ignored in research, which may yield totally incorrect results. In [3] it is shown that even the slightest temperature change leads to huge alteration of the clamped beam vibration properties. Contrary as in papers [1-3], the present paper does not neglect the change in thermodynamic properties, which have to be taken into consideration at major temperature changes. Carbon nanotubes with respect to the chiral angle can be classified into three types: armchair, zigzag and chiral. Numerous studies are available on the physical properties of armchair and zigzag carbon nanotube. However, only a limited portion of the literature studied nanotubes in dependence of temperature field. This article develops a model that analyzes the frequency of the chiral single-walled carbon nanotubes (SWCNTs) subjected to a thermal vibrations by using Timoshenko beam model, including the effect of rotary inertia and shear deformation. The Timoshenko model was compared with the Euler model.

Carbon nanotubes could be classified into single wall nanotubes (SWNT) and multi wall nanotubes (MWNT). On the basis of molecular simulation many researchers found that modulus of elasticity is no more constant and depends on the diameter of a nanotube and its thickness [4, 5, 13]. On the basis of molecular dynamics calculation we could express the equations for surface Young's modulus and Poisson's ratio for the armchair SWNT:

$$Y_s = \frac{4\mu K_p}{\sqrt{3}(\lambda + 3\mu)} \qquad (1)$$

where:

$$v = \frac{\lambda - \zeta\mu}{\lambda + \zeta\mu} \qquad (2)$$

$$\lambda = \frac{7 - cos(\pi / n)}{34 + 2\,cos(\pi / n)} \tag{3}$$

$$\mu = \frac{K_\theta^2}{K_\rho r_0^2} \tag{4}$$

The above equations are obtained on the basis of continuous mechanics and molecular simulation [3, 4], where Y_s means surface Young's modulus [13] and ν – Poisson's ration. From the Figs. 1-3 we observe that material properties are temperature and also size dependent.

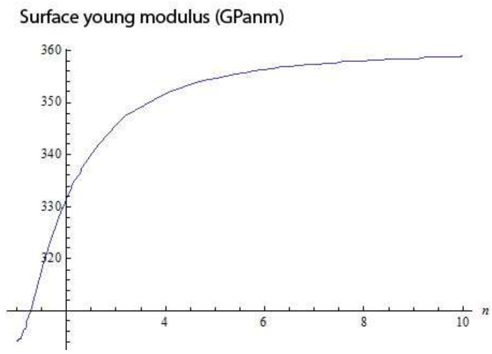

Fig. 1. Young modulus of armchair nanotubes

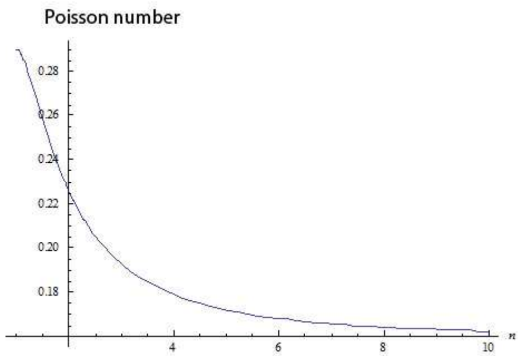

Fig. 2. Poisson number of armchair nanotubes

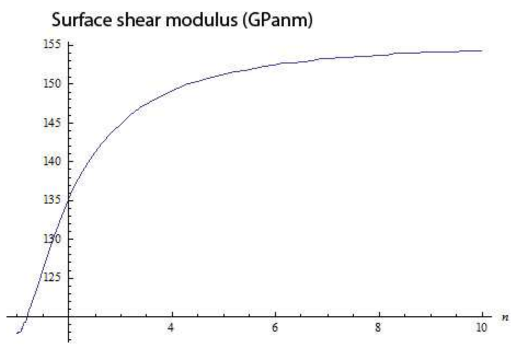

Fig. 3. Shear modulus of armchair nanotubes

Young's modulus of carbon nanotube is also dependent on a temperature field. On the basis of Prakash [6] molecular dynamics simulation we obtain the next relation for modulus of elasticity and linear expansion coefficient α for SWNT:

$$Y_s = Y_s(1 - 0.000075T) \tag{5}$$

$$\alpha = \frac{1}{l}\left(\frac{dl}{dT}\right) = \frac{10^{-18}T^2 - 2\cdot 10^{-15}T + 10^{-13}}{10^{-18}T^3 - 10^{-15}T^2 + 10^{-13}T + 3\cdot 10^{-8}} \tag{6}$$

Euler-Bernoulli model for beams and rotors under thermal stresses

Let us assume that the support is homogenous, having the same temperature over its entire length. As a result of thermal expansion, an additional axial force F_T occurs:

$$F_T = \alpha\theta EA \tag{7}$$

In equation (7) α is the linear thermal extension coefficient, θ is the temperature difference between the actual and initial or reference temperature. The equation by means of which we can solve the problem using the axial force is as follows according to Wear, Timoshenko and Young [4]:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + F_T\frac{\partial^2 w(x,t)}{\partial x^2} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0\,, \tag{8}$$

where E means Young's modulus, I - area moment of inertia, A - area, ρ - density of material, t - time and w - displacement. Using the method of separation of variables $w(x,t) = X(x)\Omega(t)$ and introducing the new functions, Equation (8) can be written down in a slightly less complicated way:

$$c^2\frac{X''''(x)}{X(x)} + 2\gamma\frac{X''(x)}{X(x)} - \frac{\ddot{\Omega}(t)}{\Omega} = \omega^2\,, \tag{9}$$

where the partial derivatives have been replaced with the total derivatives.

$$\ddot{\Omega}(t) + \omega^2\Omega(t) = 0 \tag{10}$$

$$X''''(x) + 2\gamma X''(x) - \beta^4 X(x) = 0 \tag{11}$$

In Equation (11), the new symbols represent the following functional relations:

$$\beta^2 = \frac{\omega}{c}, c^2 = \frac{EI}{\rho A}, \gamma = \frac{F_T}{2EI} \tag{12}$$

Thus, a general solution to Equations (4) and (5) are ($\lambda = \sqrt{\beta^4 + \gamma^2}$) [1-4]:

$$X(x) = C_1\cos\left(\sqrt{\lambda+\gamma}\,x\right) + C_2\cosh\left(\sqrt{\lambda-\gamma}\,x\right) + C_3\sin\left(\sqrt{\lambda+\gamma}\,x\right) + C_4\sinh\left(\sqrt{\lambda-\gamma}\,x\right) \tag{13}$$

$$\Omega(t) = A\sin(\omega t) + B\cos(\omega t) \tag{14}$$

In the equation (7) the value of λ (where the influence of angular frequency ω is hidden) and three of four constants of integration C_1, C_2, C_3 and C_4 are determined from the boundary conditions. The fourth constant is possible to find in the combination with the constants A and B in Equation (8). For a given beam at defined temperature the values by λ depend upon the boundary conditions [5-9]. Using boundary conditions, the following solutions can be analytically computed ($\Gamma = L^2\gamma, \Lambda = L^2\lambda$):

For the supported-simply supported beam we obtain the next equation:

$$\sin(\Lambda + \Gamma) = 0 \tag{15}$$

With the known angular frequencies ω_n of individual modes of vibration it is possibly to calculate X_n and Ω_n of individual modes of vibration. To determine the solution for the displacement we have to solve the equation [5-9]:

$$w(x,t) = \sum_{i=1}^{\infty} \left(A_n \, sin(\omega_n t) + B_n \, cos(\omega_n t)\right) X_n(x), \tag{16}$$

where the modal shapes can be shown to be orthogonal:

$$\int_0^l X_n(x) X_m(x) dx = 0 \quad for \; n \neq m \tag{17}$$

The model presented in our paper is fully analytical, but if compared with the measured results it points to a large deviation from reality [1, 2]. The biggest problem of this model is that the clamped wall can fully withstand the beam for the beam to have a constant length all the time. The above assumption is not realistic. As a result, a new model was designed to reduce to some extent the huge differences between the analytical results and the measured values.

The dynamic model for beams under thermal stresses

Fig. 3 and 4 illustrate a new rheological model for the clamped and the simply supported beam. To this end, a spring is added with the spring constant K. The model slightly differs from the model presented in paper [1], where the authors Marques and Inman integrated additional torsion springs into the rheological model.

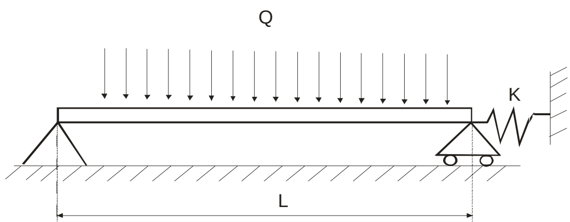

Fig. 4. New model of simply supported beam

In this case, force F_T can be computed in the following way [3]:

$$\varepsilon EA = \alpha\theta EA - K\delta \,, \; K\delta = EA\left(\alpha\theta - \frac{\delta}{L}\right), \; \delta = \frac{EAL\alpha\theta}{KL + EA} \tag{18}$$

The reaction force computation can be as follows:

$$F_T = K\delta = \frac{EAL\alpha\theta}{L + \frac{EA}{K}}, \tag{19}$$

where the modulus of elasticity E and linear expansion coefficient α are temperature dependent functions. By means of the boundary conditions, Equations (5) and (6) also apply now.

An aluminum beam with the dimensions indicated in Table 1 was used for the computation.

Table 1: Parameters of the considered aluminum beam

	Beam
Length (m)	$6.35 \cdot 10^{-2}$
Width (m)	$2.04 \cdot 10^{-2}$
Thickness (m)	$1.62 \cdot 10^{-3}$
Young modulus (N/m^2)	$6.9 \cdot 10^{10}$
Volume expansion coefficient (1/K)	$24 \cdot 10^{-6}$ K^{-1}
Spring constant (N/m)	$1.553 \cdot 10^{5}$
Density (kg/m^3)	2780

Vibration of cylindrical shafts under thermal effects

In the case of rotor vibration we have used the rheological model as it was presented for beams (Figure 9 and 10):

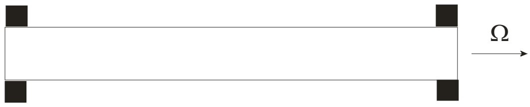

Fig. 5. Vibration of cylindrical shaft

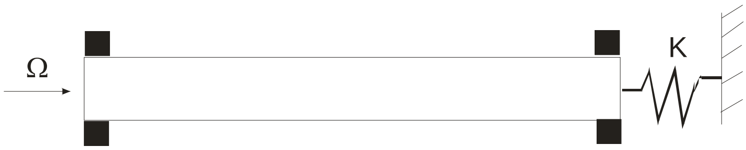

Fig. 6. New rheological model of cylindrical shaft

If we consider cylindrical shafts, the existence of axial force changes the equation of lateral vibration. The equation of lateral vibration of Euler-Bernoulli beam in the presence of axial force P together with the temperature effects can be written as [10, 11]:

$$EI\frac{d^4X}{dx^4}+\left(F_t-P\right)\frac{d^2X}{dx^2}-\rho A\omega^2X=0 \tag{20}$$

Upper equation we can express also with the next expression:

$$X''''(x)+2\gamma X''(x)-\beta^4X(x)=0 \tag{21}$$

In Equation (10), the new symbols represent the following functional relations:

$$\beta^2=\frac{\omega}{c},c^2=\frac{EI}{\rho A},\gamma=\frac{F_t-P}{2EI} \tag{22}$$

If we use 3-D linear elasticity relations we obtain:

$$P=\upsilon\rho I_p\Omega^2 \tag{23}$$

Supported-simply supported beam:

$$sin\left(\Lambda+\Gamma\right)=0 \tag{24}$$

where I_p is the polar moment of inertia, υ presents Poisson's ratio and Ω is the rotational speed of a rotating shaft.

Influence of the gyroscopic effect in combination with the temperature effect [10, 11]:

$$\begin{gathered}EI\frac{d^4X}{dx^4}+\left(F_t-P_G\right)\frac{d^2X}{dx^2}-\rho A\omega^2X=0\\P_G=\rho I\omega\left(2\Omega-\omega\right)\end{gathered} \tag{25}$$

Timoshenko beam model

Timoshenko beam model [4] includes the effect of rotary inertia and shear deformation. The Timoshenko vibrational beam model gives the next expression:

$$\frac{EI}{\rho A}\frac{\partial^4Y}{\partial x^4}+\frac{F_T}{\rho A}\frac{\partial^4Y}{\partial x^4}+\frac{\partial^2}{\partial t^2}-\frac{l}{A}\left(1+\frac{E}{KG}\right)\frac{\partial^4Y}{\partial x^2\partial t^2}+\frac{I}{A}\frac{\rho}{KG}\frac{\partial^4Y}{\partial t^4} \tag{26}$$

$$K=\frac{2(1+\mu)}{4+3\mu} \tag{27}$$

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In the Eq. (26) I - dynamic moment of inertia of the beam, nanotube, K - is the shear coefficient of nanotube, μ - is the Poisson′s ratio. F_t presents additional thermal force:

$$F_t = \alpha TEA \tag{28}$$

The solution of Eq. (26) could be expressed as:

$$Y(x,t) = y(x)e^{-i\omega t} \tag{29}$$

In the Eq. (29) we call ω angular frequency. On the above approximations the following dimensionless forms can be expressed as:

$$\frac{d^4\eta}{d\zeta^4} + \left[(\alpha+\beta)\Gamma^2 + \delta\right]\frac{d^2\eta}{d\zeta^4} - \left(1 - \Gamma^{2\alpha}\alpha\beta\right)\Gamma^2\eta \tag{30}$$

$$\eta = \frac{y}{L};\zeta = \frac{x}{L}, \alpha = \frac{l}{AL^2}, \beta = \frac{El}{KGAL^2} \tag{31}$$

$$\Gamma = \frac{\rho A\omega^2 L^4}{El}, \delta = \frac{F_T L^2}{El} \tag{32}$$

where δ represents the effect of thermal vibration on the frequency of the SWCNT. The general solution of Eq. (30) could be expressed as:

$$\eta(\xi) = C_1\cos\left(\sqrt{\lambda-\gamma}\varepsilon\right) + C_2\cosh\left(\sqrt{\lambda+\gamma}\varepsilon\right) + C_3\sin\left(\sqrt{\lambda-\gamma}\varepsilon\right) + C_4\sinh\left(\sqrt{\lambda+\gamma}\varepsilon\right) \tag{33}$$

When the solution is integrated with the boundary condition for supported-simply supported nanotube model we obtain:

$$sin(\lambda - y) = 0 \tag{34}$$

$$\Gamma^4 - \frac{1+(n\pi)^{2^{(\alpha+\beta)}}}{\alpha\beta}\Gamma^2 + \frac{n^2\pi^2}{\alpha\beta}\left(-n^2\pi^2 + \delta\right) = 0 \tag{35}$$

For the case of Euler-Bernoulli beam ($\alpha = \beta = 0$) we obtain the next equation:

$$\Gamma = n\pi\sqrt{-\delta + (n\pi)^2} \tag{36}$$

Results and discussion

The presented mathematical model was used to calculate thermodynamic properties for the case of pure aluminum microbeam. Table 1 contains the main important data of the beam. The aluminum beam is very interesting, particularly due to relatively high expansion coefficients. In the presented section we have calculated vibrational characteristics for supported-simply supported systems. For carbon nanotubes we have used data for Young's modulus and linear expansion coefficient shown in Prakash thesis [7]. Figures 7 and 8 indicate that when we have relatively long nanotube (L/D>10) the results for Timoshenko and Euler-Bernoulli model give similar results, contrary when we have short nanotubes the Timoshenko model gives much better results. Figure 9 provides the results of oscillation frequency for clamped-free and clamped-simply supported beam. The detailed analysis demonstrates that small changes in temperature cause significant changes in natural frequencies for beams and with the presented models it is possible to perform the research of the microbeams. Figure 10 shows the influence of temperature, rotor speed and gyroscopic effect on angular frequencies up to the sixth mode. The analysis indicates that we have to take into account at high rotor speeds also the temperature effects and the gyroscopic effect, when we need very accurate dynamic calculations. The influence of temperature effects on vibrational characteristics depends on the boundary conditions and also the results are very sensitive on the structure of beam or rotor material. The detailed analysis shows that small changes of temperature cause significant changes in natural frequencies for beams. Figure 11 indicates the temperature variation of fundamental frequency up to the 3rd mode obtained with the Timoshenko model. The detailed analysis demonstrates that the temperature variations have significant impact on vibrational characteristics of the carbon nanotubes.

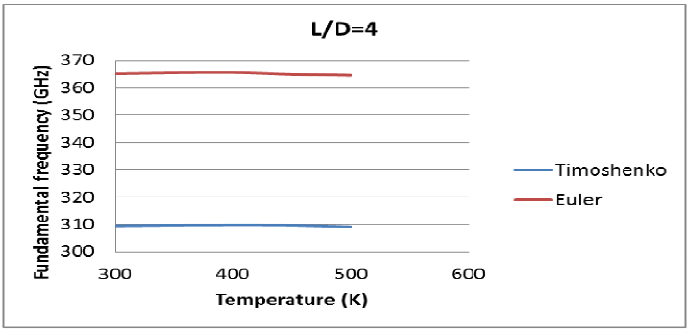

Fig. 7. Fundamental frequency for nanotube with L/D=4 with Timoshenko and Euler - Bernoulli model

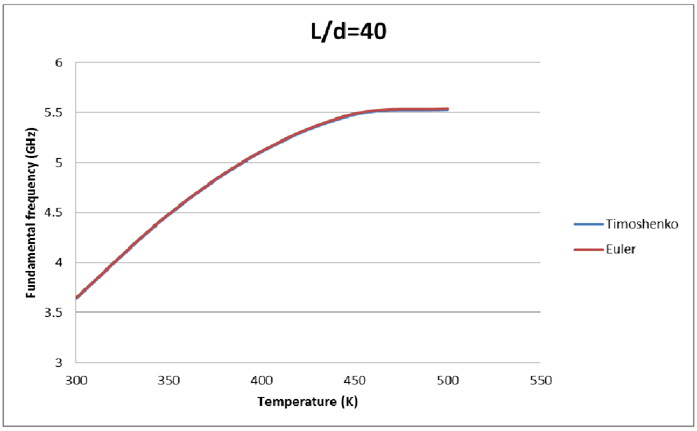

Fig. 8. Fundamental frequency for nanotube with L/D=40 with Timoshenko and Euler - Bernoulli model

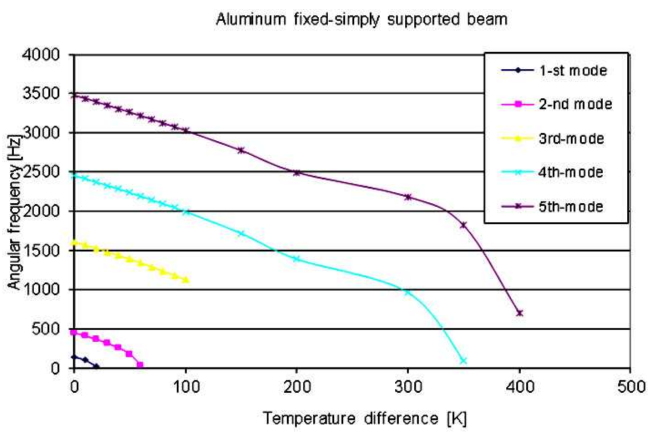

Fig. 9. Aluminum clamped - simply supported beam

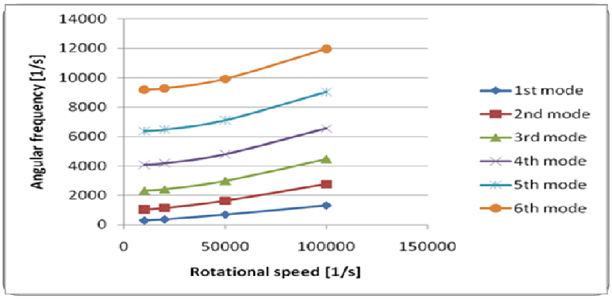

Fig. 10. Vibration of rotor in dependence of rotor speed and gyroscopic effect (θ=330 K)

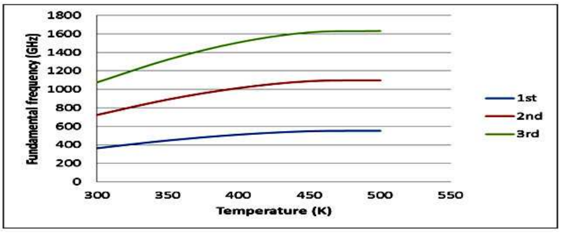

Fig. 11. Fundamental frequency in dependence of temperature for nanotubes for 1st, 2nd and 3rd mode

Conclusions

In the presented paper we have developed a model for vibration of nanotubes, microbeams and minirotors. The thermophysical properties of state, such as modulus of elasticity and linear expansion coefficient, are regarded as constants. The analysis indicates that a minor change in temperature results in a considerable alteration in vibrational behavior of the considered structures.

References

[1] **Marques R. F., Inman D. J.** "An analytical model for a clamped isotropic beam under thermal effects", Proceedings of IMECE 2002, November 2002, p. 17-22.

[2] **Marques R. F., Inman D. J., Rade D. J.** "Assesment of adaptive techniques for the control of structures subject to temperature variations", 12th ICSV Conference, 2005, Lisbon, Portugal.

[3] **Avsec J.** The Influence of Temperature Field on Vibration Characteristics of Beams and Rotating Shafts, Journal of Vibroengineering. Vol. 11, Issue 4, 2009, p. 665–672.

[4] **Hsu J. C., Chang R. P., Chang W. J.** Resonance frequency of chiral single-walled carbon nanotubes using Timoshenko beam theory, 2008, Phys. Let. A, Vol. 372, p. 2757-2759.

[5] **Chang T., Geng J., Guo X.** Prediction of chirality- and size dependent elastic properties of single-walled carbon nanotubes via molecular mechanics model, Proc. Roy. Soc. A, 2006, Vol. 462, p. 2524-2540.

[6] **Weaver W., Timoshenko S., Young, D. H.** Vibration Problems in Engineering, John Wiley & Sons, 1974, New York.

[7] **Prakash N.** Determination of coefficient of thermal expansion of single-walled carbon nanotubes using molecular dynamics simulation, PhD Thesis, Florida State University, 2005.

[8] **Bahzad M., Bastami A. R.** Effect of centrifugal force on natural frequency of lateral vibration of rotating shafts, Journal of Sound and Vibration, 2004, Vol. 274, No. 3, p. 985-995.

[9] **Avsec J., Oblak M.** Vibrational analysis of temperature dependent materials for clamped isotropic beam under thermal effects, Journal of Sound and Vibration, Vol. 308, Issue 3/5, 2007, p. 514-525.

[10] **Chang T., Gao H.** Size-dependent elastic properties of a single walled carbon nanotube via a molecular mechanics model, Journal of Mechanics and Physics of Solids, 2003, Vol. 51, p. 1059-1074.